For Online Publication

Online Appendix

"A Q-Theory of Banks"

Juliane Begenau Saki Bigio

Jeremy Majerovitz Matias Vieyra

A Data Appendix

A.1 Sample Selection

We analyze bank holding companies (BHCs). BHCs file FR-Y-9C forms if they have assets above one billion dollars. Prior to 2015 Q1, this threshold was $500 million and prior to 2006 Q1, this threshold was $100 million. We focus on the sample period from 1990 Q1 to 2021 Q1.

We focus on top-tier bank holding companies that are headquartered in the 50 states or in Washington D.C. For book variables, we use data from the FR Y-9C, downloaded through Wharton Research Data Services (WRDS). We match this to data on market capitalization and returns from the Center for Research in Securities Prices (CRSP) using the PERMCO-RSSD links data set provided by the New York Fed (https://www.newyorkfed.org/research/banking_research/datasets.html). For analyses that use solely book data, we use data for those BHCs that we find in our sample in the FR Y-9C; for analyses that use market data, we use only the observations which we observe in both FR Y-9C and CRSP. In one robustness check, we use information on the dates of, and participants in, bank mergers and acquistions; we obtain data on bank mergers from the Chicago Fed (https://www.chicagofed.org/banking/financial-institution-reports/merger-data). In an additional robustness check, we drop all banks that were ever stress-tested (CCAR and DFAST). We obtain information on whether banks were ever stress tested from the Federal Reserve (The main website is https://www.federalreserve.gov/supervisionreg/stress-tests-capital-planning.htm, and the specific data sets can be found at https://www.federalreserve.gov/supervisionreg/ccar.htm and https://www.federalreserve.gov/supervisionreg/dfa-stress-tests.htm).

When constructing aggregate time series, we drop entrants to correct for the entry of major financial institutions such as Goldman Sachs and Morgan Stanley. Without this correction, aggregate bank assets increase due to the reclassification of large actors such as Morgan Stanley and Goldman Sachs into bank holding companies.

A.2 Motivating Facts

A.2.1 Regulatory Rules

Under Basel II (the regulatory standard in place during the crisis), bank holding companies were subject to regulatory minimums on their total capital ratio and their tier-1 capital ratio. These capital ratios are computed as qualifying capital/risk-weighted assets and, thus, a bank with a higher capital ratio has lower leverage. Basel II required that banks hold a minimum tier-1 capital ratio of 4% and a minimum total capital ratio of 8%. In order to be categorized as "well-capitalized," banks had to meet minimum capital ratios that were two percentage points higher (6% and 10%, respectively). Being categorized as well-capitalized is desirable because banks that are not well-capitalized are subject to additional regulatory scrutiny (Basel Committee on Banking Supervision, 1998; **?**). After the crisis, tighter capital requirements were phased in under Basel III. The minimum total capital ratio stayed at 8% throughout our sample period, but the tier-1 capital ratio rose to 4.5% in 2013, 5.5% in 2014, and finally settled at 6% starting in 2015. Also under Basel III, additional capital ratios (e.g., tier-1 leverage and common equity capital ratio) began being monitored (however these ratios are quite similar to the preexisting tier-1 and total capital ratios) and, starting in 2016, a "capital conservation buffer" and special requirements for systemically important financial institutions were introduced (Basel Committee on Banking Supervision, 2011). Kisin and Manela (2016) study whether banks violate different regulatory constraints and find that typically banks do not violate multiple regulatory constraints.

A.2.2 Bank Accounting Practices

The discrepancy between book and market equity reflects bank accounting practices. Banks can delay acknowledging losses on their books (e.g. Laux and Leuz 2010), because banks are not required to mark-to-market the majority of their assets. There are many incentives to delay book losses. In practice, a key metric for measuring success of a bank is the book return on equity (ROE).[52] Given that ROE is a measure of success, manager compensation is linked to book value performance. Moreover, shareholders and other stakeholders may base their valuations on information from book data. Finally, banks are required to meet capital standards based on book values.

The flexibility of accounting their accounts is studied extensively in the accounting literature (Bushman, 2016 and Acharya and Ryan, 2016 review the literature on this issue, Francis, Hanna and Vincent, 1996 studies the same issue for non-financial firms). In practice, banks can record securities on the books using two methodologies: either amortized historical cost (the security is worth what it cost the bank to buy it with appropriate amortization) or fair value accounting.[53] In addition to mis-pricing securities, another degree of freedom is the extent to which banks can acknowledge impairments: banks have the right to delay acknowledging impairments on assets held at historical cost, if they deem those impairments as temporary (i.e. they believe the asset will return to its previous price). This gives banks substantial leeway, and led banks to overvalue assets on the books during the crisis. Huizinga and Laeven (2012) find that banks used discretion to hold real-estate related assets at values higher than their market value. (Laux and Leuz, 2010) note some notable cases of inflated books during the crisis: Merrill Lynch sold $30.6 billion dollars of CDOs for 22 cents on the dollar while the book value was 65 percent higher than its sale price. Similarly, Lehman Brothers wrote down its portfolio of commercial MBS by only three percent, even when an index of commercial MBS was falling by ten percent in the first quarter of 2008. Laux and Leuz (2010) also document substantial underestimation of loan losses in comparison to external estimates.

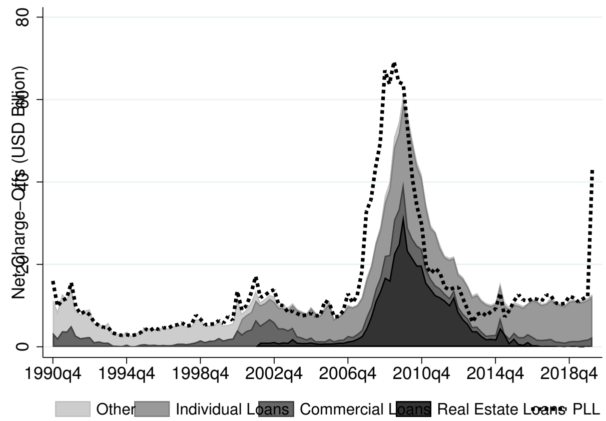
This shows up in our own analysis as well: Figure A.1 shows that provisions for loan losses and net charge-offs only reached their peak in 2009 and 2010 respectively, and remained quite elevated at least through 2011, well after the recession had ended. The decomposition of net charge-offs shows that these losses were heavily driven by real estate, suggesting they were associated with the housing crisis. Loan loss provisions lead net-charge-offs, which can be best seen for the 2008/2009 crisis and in the beginning for the Covid crisis (note we present data until 2020 Q1). Banks' books were only acknowledging in 2011 losses that the market had already predicted when the crisis hit. Harris, Khan and Nissim (2013) construct an index, based on information available in the given time period, that predicts future losses substantially better than the allowance for loan losses.[54] This implies that the allowance for loan losses is not capturing all of the available information to

[52] For example, JP Morgan's 2016 annual report states "the Firm will continue to establish internal ROE targets for its business segments, against which they will be measured" (on page 83 of the report).

[53] Fair value accounting can be done at three levels: Level 1 accounting uses quoted prices in active markets. Level 2 uses prices of similar assets as a benchmark to value assets that trade infrequently. Level 3 is based on models that do not involve market prices (e.g. a discounted cash flow model). Banks are required to use the lowest level possible for each asset. In practice, most assets are recorded at historical cost. The majority of fair value measurements are Level 2 (Goh, Li, Ng and Ow Yong 2015; Laux and Leuz 2010). Recent work has shown that the stock market values fair value assets less if they are measured using a higher level of fair value accounting. This leaves room to mis-price assets on books. Particularly during 2008, Level 2 and Level 3 measures of assets were valued substantially below one. Laux and Leuz (2010) document sizable reclassifications from Levels 1 and 2 to Level 3 during this period. They highlight the case of Citigroup, which moved $53 billion into Level 3 between the fourth quarter of 2007 and the first quarter of 2008 and reclassified $60 billion in securities as held-to-maturity which enabled Citi to use historical costs.

[54] The allowance for loan losses (ALL) is the stock variable corresponding to the provision for loan losses (PLL).

Figure A.1: Decomposition of Net Charge-offs



Notes: This figure shows aggregate net charge-offs for different categories (area chart) and aggregate loan loss provisions (dashed black line) from 1990 Q4 to 2020 Q1. The data source are FR Y-9C reports. Net charge-offs for loans are defined as charge-offs minus recoveries. We decompose the net charge-offs into loans backed by real estate, commercial and industrial (C&I) loans, loans to individuals (e.g., such as credit card loans), and all other loans (e.g. inter-bank loans, agricultural loans, and loans to foreign governments). Net charge-offs of loans to individuals are not separately recorded until 2001 Q1, and net charge-offs of real-estate loans are not separately recorded until 2002 Q1; these categories are thus grouped with "Other" until the date at which they are recorded separately.

estimate losses. This may in part be strategic manipulation, but there may also be a required delay in acknowledging loan losses. Under the "incurred loss model" that was the regulatory standard during the crisis, banks are only allowed to provision for loan losses when a loss is "estimable and probable" (Harris et al., 2013). Thus, even if banks know that many of their loans will eventually suffer losses, they were not supposed to update their books until the loss was imminent.

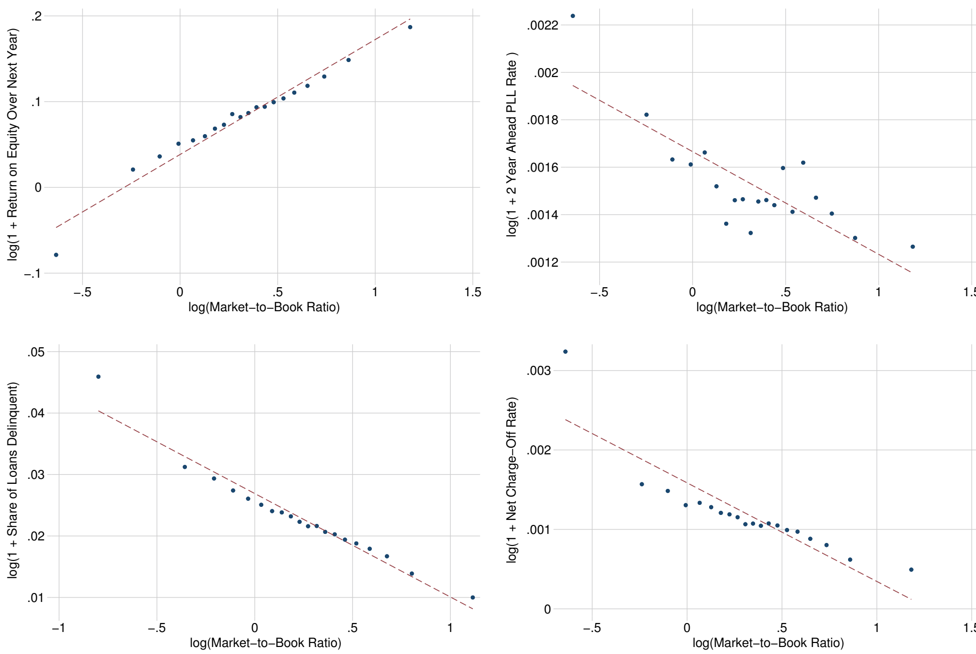
A.2.3 Information content

There are at least two reasons to expect different information content in market and book value measures. One reason is the delayed acknowledgment of known losses, which is a widely documented fact in the accounting literature. As long as banks delay the recognition of losses, or refinance non-performing loans to avoid registering losses (evergreening), book values will not reflect banks' actual losses. If market participants can update their valuations more quickly, detecting these losses, differences in informational content will emerge. This alone can produce differences in the informational content of market and book equity. The other reason is that changes in the underlying market value of loans reflect default expectations while the book value of loans does not (see filing instructions for FR-Y-9C BHCs regulatory reports) at least until January 2020. Before 2020, loan loss expectations were not updated in loan accounting books and loans were only written off once the loss had occurred. Publicly traded banks were supposed to change their accounting system to the new "current expected credit loss" (CECL) accounting system in January 2020.[55] Note that our model can capture these accounting system changes (see Section 4.3) and study their effects on bank lending.

[55] See https://www.occ.treas.gov/news-issuances/bulletins/2021/bulletin-2021-20.html. However, on March 27, 2020, the Fed moved to provide an optional extension of the regulatory capital transition for the new credit loss accounting standard, see https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200327a.htm.

Appendix Section A.2.2Figure A.1 suggests that indeed market values contain more information than book values of equity. Loan loss provisions, denoted as PLL in the figure, peak in early 2009 when market values had already tanked. Loan net charge-offs peaked even later in 2010 when the economy was no longer officially in a recession.[56] The decomposition of net charge-offs shows that these losses were heavily driven by real estate, which is consistent with the nature of the crisis. Loan loss provisions lead net-charge-offs, which can be best seen for the 2008/2009 crisis and in the beginning for the Covid crisis (note we present data until 2020 Q1).

Figure A.2: Market equity contains more cash-flow relevant information than book equity



Notes: This figure presents cross-sectional binned scatter plots of log outcomes on the log Tobin's Q for BHCs. All plots control for log book equity as a proxy for size, the Tier 1 capital ratio of each bank and a quarter-time fixed effect. Data on market equity are from CRSP. All other data are from the FR Y-9C reports. Return on equity over the next year is defined as book net income over the next four quarters divided by book equity in the current quarter. The two-year ahead loan provision rate is calculated as the ratio of eight quarter ahead quarterly loan provisions divided over total loans. The share of delinquent loans is the ratio of 30 days or more past due loans plus loans in non-accrual over total loans. The net charge-off rate is calculated as the difference between loan charge-offs over the next quarter and loan recoveries over the next quarter, divided by total loans this quarter.

Next, we formally show that variation in the cross-section of Tobin's Q reflects differences in the information content of market and book equity values. If market equity values contain more information about bank profitability and credit losses than book equity values, then we expect for Tobin's Q to predict future bank profits and loan losses even after controlling for book equity. In Figure A.2, we show binned scatter plots of logged outcomes on the log market-to-book equity ratio. The plots control for time fixed effects, the Tier 1 regulatory capital ratio, and log book

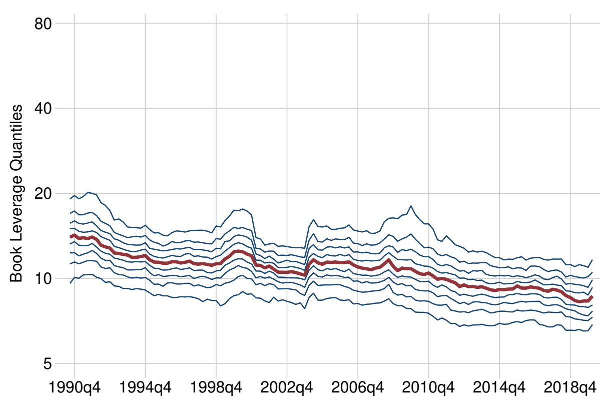
[56]When a bank has a loss that is estimable and probable, it first provisions for loan losses on the income statement, which shows up as PLL in the figure. Later when the loss has realized, the asset is charged off and thus taken off the books, which shows up as charge-offs. Occasionally, the bank can recover the asset later.

equity.[57] The top left panel shows the log return on equity over the next year plotted against the log market-to-book ratio. Banks with higher market-to-book ratios earn higher future profits. A bank with a lower Tobin's Q today is also more likely to have higher loan loss previsions even eight quarters ahead (top right panel). Banks with higher market-to-book ratios also have a lower share of delinquent loans (bottom left panel) and a lower future net charge-off rate on their loans (bottom right). Thus, Tobin's Q predicts future book realized profits and actual loan losses beyond what is reflected in book values, suggesting that book values account for loan losses only very slowly. This is consistent with the fact that book equity did not decline during the crisis, despite widespread issues in credit markets. Note that discount rate variations affect most banks similarly and are therefore unlikely to drive these cross-sectional results. Indeed, our results suggest that banks with lower profitability and more delinquencies have lower Tobin's Q, and that Tobin's Q predicts future loan write-downs and future profitability.

A.2.4 Book Leverage Distribution

Figure A.3 shows that the distribution of book leverage is much less dispersed relative to the market leverage distribution. In fact, it is also relatively stable: the 90th percentile and 10th percentile of the book leverage distribution differed only by a factor of two. This stands in stark contrast to the market leverage distribution in Panel B of Figure 3.

Figure A.3: Quantiles of Book Leverage



Notes: This figure shows the quantiles of book leverage for BHCs on a log scale. Book data (book equity and liabilities) comes from the FR Y-9C. Book leverage is computed as (liabilities + book equity)/book equity. The median value is plotted in red. Each tenth percentile is plotted in blue.

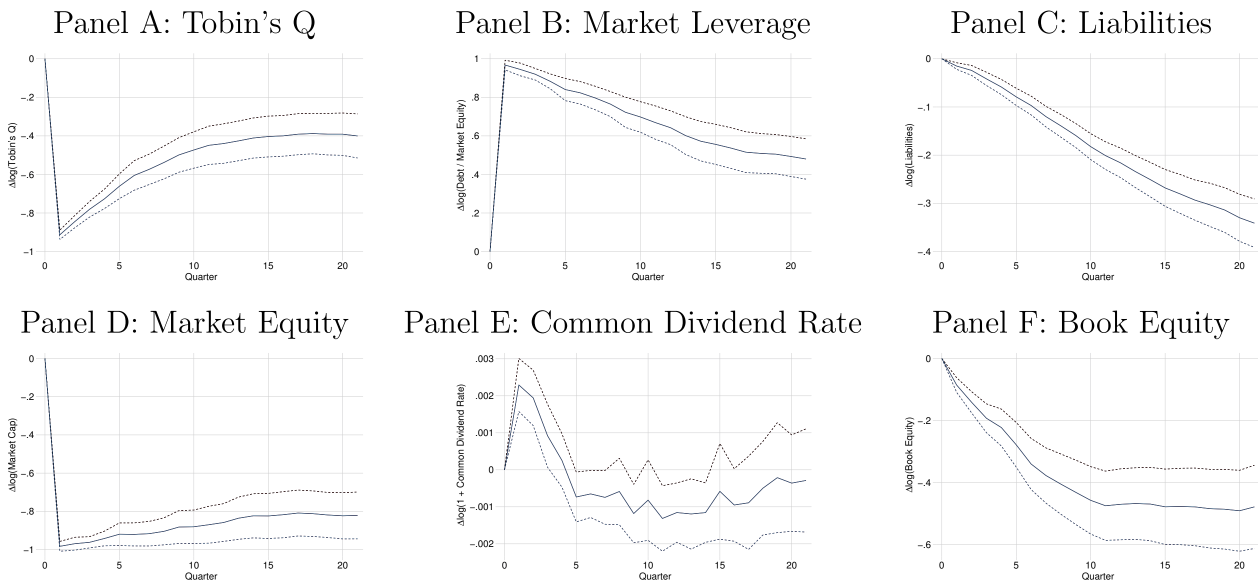
A.2.5 Impulse Response of Banks

In Figure A.4, we show the impulse response of bank holding companies to a negative net worth shock. This figure is the same as Figure 4 in the main text, but it includes additional variables. In particular, panel D shows the impulse response of market capitalization, which drops mechanically on impact and then modestly recovers after a net worth shock. Panel E shows the impulse response of the common dividend rate. We define the log common dividend rate

[57] To control for log book equity, the left and right-hand side variables are residualized on log book equity, and then the mean of each variable is added back to maintain the centering. It is important to control for log book equity to prevent spurious results due to ratio bias (see Kronmal, 1993).

as log(1 + Common Dividends/Market Capitalization). The dividend rate temporarily rises after the shock before returning to a level close to its pre-shock level. This initial rise is driven by the mechanical impact of falling market equity, since market capitalization is in the denominator of the dividend rate. Thus, the response of the dividend rate implies that dividends slowly adjust down after the shock.

Figure A.4: Estimated Impulse Responses



Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated percent impulse response to a 1% negative return shock. For example, in Panel b) we show that market capitalization decreases by roughly 1% in response to a 1% negative return shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log Tobin's Q (Panel A), log market leverage (Panel B), log liabilities (Panel C), log market capitalization (Panel D), the logged common dividend rate (Panel E), and log book equity (Panel F). Market leverage is defined as (Liabilities/Market Capitalization). The logged common dividend rate is defined as log(1 + Common Dividends/Market Capitalization).

A.3 Stylized Facts: Non-Financial Firms

To compare banks to non-financial firms, we use merged data from CRSP-Compustat, excluding firms in finance, insurance, and real estate. We recompute our main results for these non-financial firms; the results are in Figure A.5, A.6, A.7, and A.8. In each figure, we follow the same empirical strategy as in the main text of the paper, but apply these methods to the CRSP-Compustat data for non-financials.

Although our paper does not offer a theory of the behavior of non-financial firms, the regulatory enivironment implies differences in the dynamic optimization problem faced by banks vs. non-financials. In particular, banks have strong incentives to be highly levered, which is held in check by regulatory constraints on their book leverage. This creates the dynamic considerations emphasized by our Q-theory.

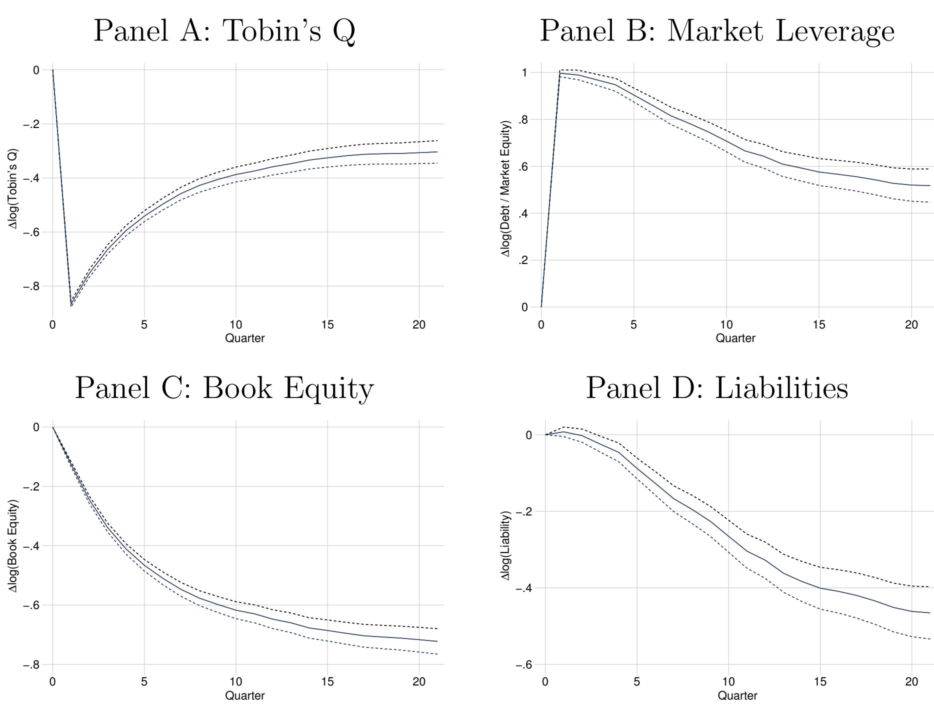
In contrast, these issues are not relevant for most non-financials, who largely hold low leverage and do not face such regulations. Instead, we find that some features of our Q-theory are relevant for non-financial firms, while other features are unique to banks. In particular, the evidence suggests that although historical-cost accounting causes book values at non-financials to lag fundamentals, similarly to banks, this does not interact with those firms' leverage decision, since book leverage regulations are not an important constraint for non-financials. Non-financials do appear to gradually delever in response to net worth shocks, but this is a feature of many dynamic trade-off theory models. Our Q-theory combines a model of accounting with a theory of bank leverage choice; for non-financial firms only the model of accounting is likely to be relevant.

Book values for non-financial firms are often based on historical cost, and thus will lag fundamental values, much like in the banking sector. Thus in Figure A.6, we find that the market-to-book ratio is predictive of future profits, like in the banking sector (Fact 2). Moreover, our estimated impulse responses (Figure A.8; Fact 4) for non-financial firms also find that book equity only slowly reflects net worth shocks, which are (by construction) immediately reflected in market values.

However, non-financial firms face a very different environment when it comes to leverage: they do not have the same incentive to lever up as banks, and are generally not constrained by regulations on leverage. As a result, we do not see the same time series patterns of Tobin's Q and leverage for non-financials as we see for financial firms. Although Tobin's Q fell during the financial crisis for non-financial firms, the drop is smaller than the drop experienced by the banking sector, and is small relative to other fluctuations for non-financials (Figure A.5; Fact 1). Moreover, leverage is low for non-financials, and the distribution of leverage and Tobin's Q is relatively stable throughout the crisis (Figure A.7; Fact 3).

In summary, we find that some of our facts (Facts 2 and 4) are similar for financial and non-financial firms, while others are quite different (Facts 1 and 3).

Figure A.8: Estimated Impulse Responses for Non-Financial Firms



Notes: These figures show the estimated percent impulse response to a 1% negative return shock, for non-financial firms. These figures are the counterpart to Figure 4 in the body of the paper, and are constructed in the same way, but for our sample of non-financial firms in CRSP-Compustat. The y-axis of our plots shows the contemporaneous response ($-\beta_0$) as quarter 0, the cumulative response one quarter later ($-\beta_0 - \beta_1$) as quarter 1, and so on. Dashed lines denote the 95% confidence interval. Standard errors are clustered by firm. The panels display the impulse responses of change in log Tobin's Q (Panel A), change in log market leverage (Panel B), change in log book equity (Panel C), and change in log liabilities (Panel D). Market leverage is defined as (Liabilities/Market Capitalization).

B Details on Impulse Response Function Estimation

B.1 Risk Adjustment

In this subsection, we describe our impulse response estimation procedure in more detail, and establish that it consistently estimates the impulse response to an idiosyncratic net worth shock.

For our main impulse response results, we wish to use risk-adjusted returns, rather than raw returns. More formally, we assume that the market returns of bank i at time t are given by

$$\underbrace{r_{it}}_{\text{Raw Return}} - \underbrace{r_t^f}_{\text{Risk-Free Rate}} = \alpha_i + \underbrace{X_t'}_{\text{factors}} \underbrace{\beta_i}_{\text{loadings}} + \underbrace{\varepsilon_{it}}_{\text{Idiosyncratic Component}}$$

All returns are logged, e.g. r_{it} refers to $\log(1 + \text{Raw Bank Return})$. We wish to isolate variation in the idiosyncratic shocks, ε_{it}, and use this variation to estimate the impulse responses.

A natural, but naive, approach would be to estimate the above model for each bank i using OLS, and then use the estimated residuals, $\hat{\varepsilon}_{it}$, as the regressors in the impulse response estimation. The problem here is that it induces bias: $\hat{\varepsilon}_{it}$ is a noisy measure of the true regressor ε_{it}, which leads to bias as long as T is finite (the bias will shrink as T grows large, because $\hat{\varepsilon}_{it}$ will converge to the true ε_{it}).

Fortunately, there is a simple solution: we estimate $\hat{\varepsilon}_{it}$ using OLS, and then we use $\hat{\varepsilon}_{it}$ as an *instrument* for the unadjusted return, r_{it}. Since our main regressions use contemporaneous returns and twenty lags, this means we use contemporaneous $\hat{\varepsilon}_{it}$ and twenty lags of $\hat{\varepsilon}_{it}$ as instruments. Instrumental variables does not suffer from the same problem of bias under classical measurement error. Instead, to get identification under the assumed model for returns, we need our instrument to be correlated with the "good variation", ε_{it}, and uncorrelated with the "bad variation," $\alpha_i + X_t'\beta_i$. This is exactly what we get when we estimate a bank-level OLS regression of returns on factors, in order to get $\hat{\varepsilon}_{it}$. Although our application of the risk-adjustment to this setting is novel, this procedure (residualizing a potential shock on controls, and using the residual as an instrument) is similar to that of Kanzig (2021), who performs a related procedure to identify oil supply shocks.

Our instrumental variables strategy will give us a consistent estimator of the true impulse response, under the assumption that we have the correct model of returns. Since the OLS regression estimating $\hat{\varepsilon}_{it}$ is conducted at the bank level, which mechanically creates correlation in the instrument at the bank level, we cluster our standard errors at the bank level.

To summarize, our procedure consists of two steps:

1. Estimate a factor model, $r_{it} - r_t^f = \alpha_i + X_t'\beta_i + \varepsilon_{it}$, for each bank i, and generate estimated idiosyncratic shocks, $\hat{\varepsilon}_{it}$.

2. Estimate the IV regression $\Delta \log(y_{i,t}) = \alpha_t + \sum_{h=0}^{k} \beta_h \cdot r_{i,t-h} + \psi_{i,t}$, using the estimated vector of idiosyncratic shocks, $(\hat{\varepsilon}_{i,t-h})_{h=0}^{k}$ as instruments for returns, $(r_{i,t-h})_{h=0}^{k}$.

Proof of Consistency for Risk Adjustment Procedure Below, we provide a formal justification of this procedure, and provide the conditions under which it will provide us consistent estimates as $N \to \infty$. Note the importance of focusing on $N \to \infty$ asymptotics, rather than also assuming that $T \to \infty$: if we had a large number of time periods, then the measurement error in $\hat{\varepsilon}_{it}$ would be small, and we would not need to use the IV procedure (we could just use $\hat{\varepsilon}_{it}$ directly). However, there is meaningful estimation error in $\hat{\varepsilon}_{it}$, and so the IV correction is necessary to ensure that $\hat{\beta} \xrightarrow{p} \beta$.

For expositional simplicity, we will focus on the univariate case ($k = 0$). The extension to include lags of ε_{it} is analagous, and is omitted for brevity. Assume the following:[58]

$$\begin{aligned} y_{it} &= \alpha_t + \beta\varepsilon_{it} + u_{it} \\ r_{it} &= X_{it}'\gamma_i + \varepsilon_{it} \end{aligned}$$

We will make two key assumptions. We will assume that $\mathbb{E}[\varepsilon_{it} \mid X] = 0$: this means that we have correctly specified the factor model of bank returns, r_{it}. Moreover, we will assume $\mathbb{E}[\varepsilon_{is}u_{it} \mid X] = 0 \, \forall s, t$. This assumption means that, regardless of the factor draws X, we will still have that ε and u are orthogonal. This latter assumption is thus a slight strengthening of the typical IV orthogonality condition.

We cannot just regress y_{it} on r_{it}, because X_{it} may be correlated with the error term, u_{it}. We want to isolate the effect of ε_{it}. To get an estimate of ε_{it}, we will first estimate $\hat{\varepsilon}_{it}$ using an OLS regression of r_{it} on X_{it}. This yields:

$$\begin{aligned} \hat{\varepsilon}_{it} &= r_{it} - X_{it}'\hat{\gamma}_i \\ &= r_{it} - X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}r_{is}\right) \\ &= r_{it} - X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\left(X_{is}'\gamma_i + \varepsilon_{is}\right)\right) \\ &= \varepsilon_{it} - X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right) \end{aligned}$$

A naive approach would be to use $\hat{\varepsilon}_{it}$ as our regressor in an OLS regression of y_{it} on $\hat{\varepsilon}_{it}$. However, this will in general not yield consistent estimates of β either, because $\hat{\varepsilon}_{it}$ is a noisy measure of the true ε_{it}. Instead, we will use $\hat{\varepsilon}_{it}$ as an instrument for r_{it}. That is, we will run the regression:

$$y_{it} = \alpha_t + \beta r_{it} + \psi_{it}$$

and we will use $\hat{\varepsilon}_{it}$ as an instrument for the endogenous regressor r_{it}. Of course, our structural equation has β as the effect of ε_{it}, not r_{it}. We can rewrite the original structural equation:

$$\begin{aligned} y_{it} &= \alpha_t + \beta\left(r_{it} - X_{it}'\gamma_i\right) + u_{it} \\ &= \alpha_t + \beta r_{it} + \underbrace{u_{it} - \beta X_{it}'\gamma_i}_{\psi_{it}} \end{aligned}$$

Thus, our IV regression will identify the correct β as long as our instrument, $\hat{\varepsilon}_{it}$, is uncorrelated with the residual, $\psi_{it} = u_{it} - \beta X_{it}'\gamma_i$.

We will now show that $\mathbb{E}[\hat{\varepsilon}_{it}(u_{it} - \beta X_{it}'\gamma_i)] = 0$. We have $\mathbb{E}[\hat{\varepsilon}_{it}(u_{it} - \beta X_{it}'\gamma_i)] = \mathbb{E}[\hat{\varepsilon}_{it}u_{it} - \hat{\varepsilon}_{it}\beta X_{it}'\gamma_i]$.

[58]Note that here, for simplicity, we write r_{it}, but in our implementation we use $r_{it} - r_t^f$ to estimate $\hat{\varepsilon}_{it}$. The results here will go through for $r_{it} - r_t^f$, because the time fixed effect in the main regression will absorb r_t^f.

We will address each component in turn:

$$
\begin{aligned}
\mathbb{E}\left[\hat{\varepsilon}_{it}u_{it}\right] &= \mathbb{E}\left[\varepsilon_{it}u_{it} - X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)u_{it}\right] \\
&= \mathbb{E}\left[-X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)u_{it}\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[-X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)u_{it} \mid X\right]\right] \\
&= \mathbb{E}\left[-X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\frac{1}{T}\sum_s X_{is}\mathbb{E}\left[(\varepsilon_{is}u_{it}) \mid X\right]\right] \\
&= \mathbb{E}\left[-X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\frac{1}{T}\sum_s X_{is}\cdot 0\right] \\
&= 0
\end{aligned}
$$

The other component is:

$$
\begin{aligned}
\mathbb{E}\left[-\hat{\varepsilon}_{it}\beta X_{it}'\gamma_i\right] &= \mathbb{E}\left[X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)\beta X_{it}'\gamma_i\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\varepsilon_{is}\right)\beta X_{it}'\gamma_i \mid X\right]\right] \\
&= \mathbb{E}\left[X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\mathbb{E}\left[(\varepsilon_{is}) \mid X\right]\right)\beta X_{it}'\gamma_i\right] \\
&= \mathbb{E}\left[X_{it}'\left(\frac{1}{T}\sum_s X_{is}X_{is}'\right)^{-1}\left(\frac{1}{T}\sum_s X_{is}\cdot 0\right)\beta X_{it}'\gamma_i\right] \\
&= 0
\end{aligned}
$$

The sum of these components is zero, which proves that our instrument is orthogonal to the residual. Thus, as long as the instrument is relevant (the instrument is correlated with returns), we will consistently estimate β as $N \to \infty$.

B.2 Inferring Impulse Responses from Coefficients

We can infer the impulse response from the coefficients of our model. In particular, the impulse response over k quarters will be equal to $\sum_{h=0}^{k}\beta_h$. To make this clear, we provide a short inductive proof.

We define the impulse response as $\mathbb{E}\left[\log y_{i,t+k} \mid \varepsilon_{i,t} = 1\right] - \mathbb{E}\left[\log y_{i,t+k}\right]$. For $t < 0$, the impulse response is zero, since the bank does not respond to shocks that have not happened yet (shocks are unanticipated). For $t \geq 0$, the impulse response can be backed out by induction.

$$
\begin{aligned}
\underbrace{\mathbb{E}\left[\log y_{i,t+k} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k}\right]}_{\text{Impulse Response for horizon } k} &= \underbrace{\mathbb{E}\left[\log y_{i,t+k-1} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k-1}\right]}_{\text{Impulse Response for horizon } k-1} \\
&+ \underbrace{\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1}\right]}_{\text{Impulse response between horizons } k-1 \text{ and } k}
\end{aligned}
$$

Using stationarity, we have:

$$
\begin{aligned}
&\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1}\right] \\
&= \mathbb{E}\left[\log y_{i,t}-\log y_{i,t-1} \mid \varepsilon_{i,t-k}=1\right]-\mathbb{E}\left[\log y_{i,t}-\log y_{i,t-1}\right]
\end{aligned}
$$

Using our regression equation, we know that this equals

$$
\sum_{h=0}^{k} \beta_h \left(\mathbb{E}\left[\varepsilon_{i,t-h} \mid \varepsilon_{i,t-k}=1\right]-\mathbb{E}\left[\varepsilon_{i,t-h}\right]\right)
$$

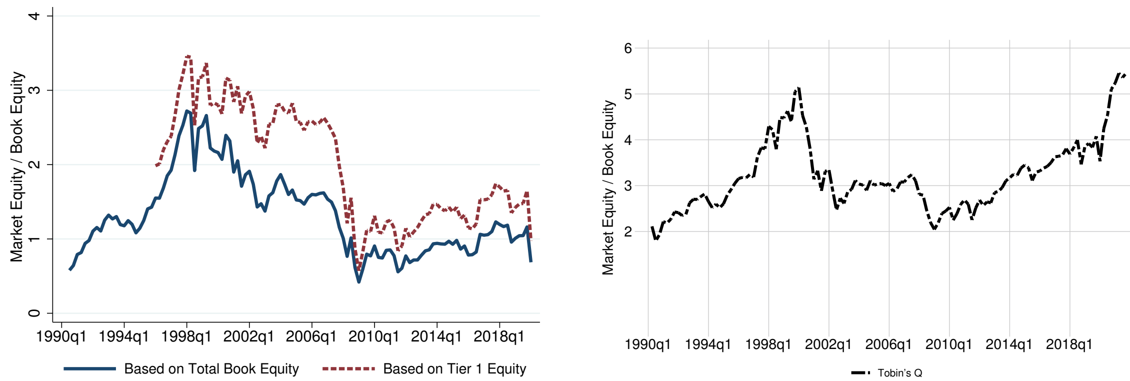
Since the shocks are mean independent, we know that all of these expectation differences are zero, except for the one for $h=k$. Thus, we have:

$$
\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1} \mid \varepsilon_{i,t}=1\right]-\mathbb{E}\left[\log y_{i,t+k}-\log y_{i,t+k-1}\right]=\beta_k
$$

Then, using induction, we find that the impulse response is $\sum_{h=0}^{k} \beta_h$.

Figure A.5: Tobin's Q for Banks and Non-Financial Firms

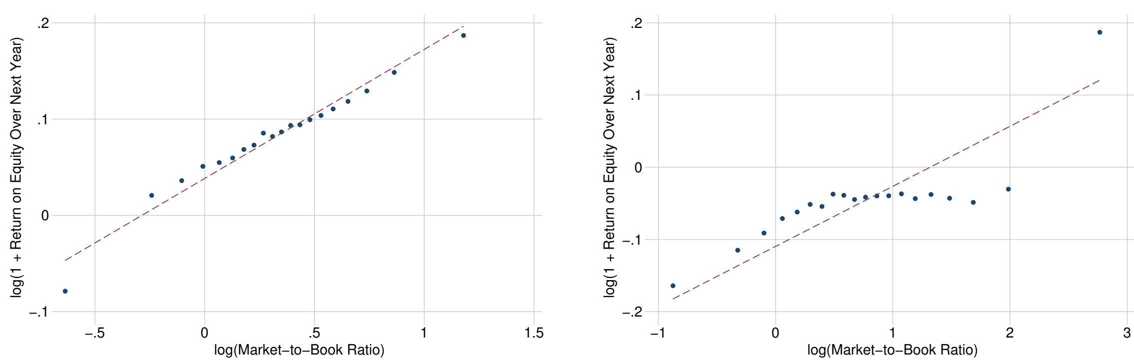
Banks — Non-Financial Firms



Notes: These figures show data on Tobin's Q panel for an aggregate sample of publicly traded Bank holding companies (left panel) and non-financial firms (right panel). The left panel repeats the figure from the body of the paper. Tobin's Q is the ratio of market equity to book equity and the ratio of market equity to Tier 1 equity capital (only available since 1996). Bank's book equity is from the FR Y-9C, and market equity is from CRSP. The right panel shows results for non-financial firms. Non-financial firm's book equity is from Compustat, and market equity is from CRSP.

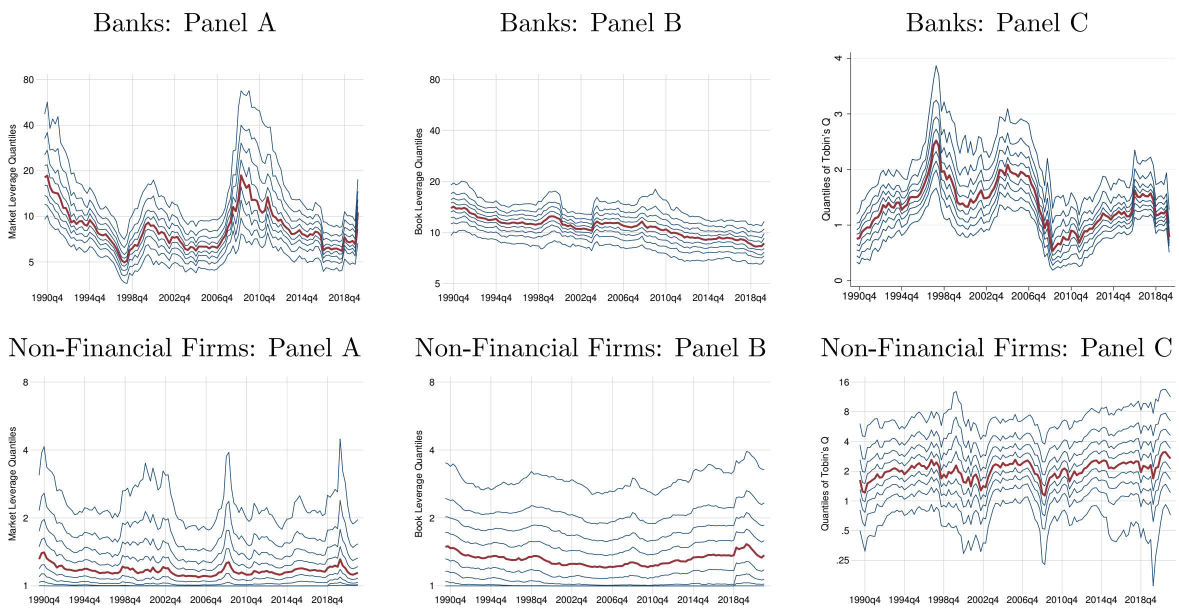
Figure A.6: Tobin's Q Predicts ROE for Banks and Non-Financial Firms

Banks — Non-Financial Firms



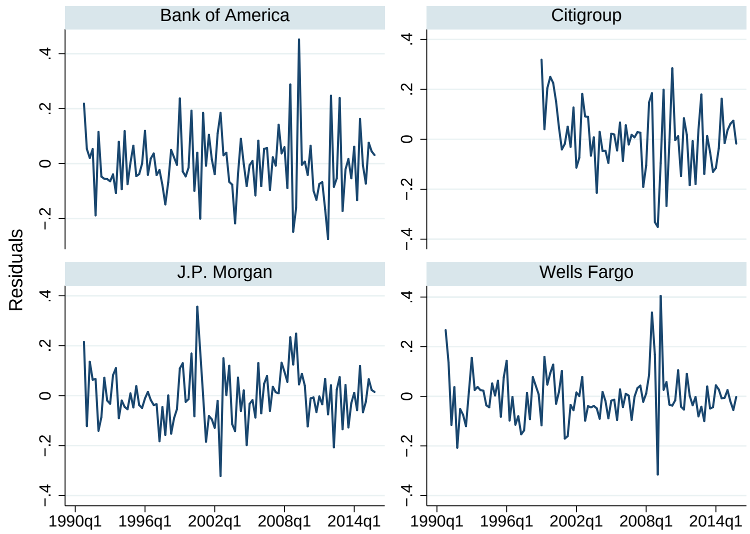
Notes: These figures show cross-sectional binned scatter plots of log outcomes on the log Tobin's Q for BHCs. All plots control for log book equity by residualizing the variables on log book equity, and then adding back the mean of each variable to maintain centering. The left panel shows results for banks, repeating the figure from the body of the paper. Data on market capitalization are from CRSP, and book data are from the FR Y-9C. The right panel shows the same figure but for non-financial firms, with book data from Compustat and market equity from CRSP. ROE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; ROE over the next year is defined the one lead of this variable (i.e. profits over the next four quarters divided by current book equity).

Figure A.7: Quantiles of Market Leverage, Book Leverage, and Tobin's Q for Banks and Non-Financial Firms



Notes: This figure shows the quantiles of market leverage (Panel A), book leverage (Panel B), and Tobin's Q (Panel C) on a log scale. The top row shows results for banks, repeating figures from the body of the paper. Book data (liabilities) comes from the FR Y-9C, and market equity data is from CRSP data. The bottom row shows the same figures but for non-financial firms, where book data comes from Compustat and market data comes from CRSP. Market leverage is computed as (liabilities + market equity)/market equity. Book leverage is computed as (liabilities + book equity)/book equity. Tobin's Q is computed as market equity/book equity. The median value is plotted in maroon. Each tenth percentile is plotted in blue.

Figure B.1: Idiosyncratic Shock Series of Big Four Bank Holding Companies



Notes: This figure plots the idiosyncratic shocks (for the Big Four BHCs) used to estimate the impulse response functions. First, we isolate the idiosyncratic component of returns using the factor model, and then we residualize this on time fixed effects.

B.3 Robustness and Validity of Identification Strategy

In this section, we conduct various tests to check the validity of our identification strategy and robustness of our results.

A narrative approach to corroborate the idiosyncratic shocks To provide corroberating evidence of the validity of our identification strategy, we first show that the estimated return shocks do indeed look like idiosyncratic shocks for the four largest banks (Bank of America, J.P. Morgan Chase, Wells Fargo, Citigroup). To construct the idiosyncratic shocks, we regress each bank's market return on the Fama-French three-factor returns and regress the residual further on time fixed effects. The residuals from this regression represent our idiosyncratic shocks.[59] Figure B.1 presents our estimates of the idiosyncratic shocks. They indeed look like white noise and do not seem to be substantially autocorrelated. Note that the time series for Citigroup starts a little later because Citigroup did not exist until 1998 when Traveler's merged with Citicorp.

We also provide narrative support for the idiosyncratic nature of our estimated shocks using an extensive search of newspaper articles for large idiosyncratic shock value estimates.

[59]We are controlling for the time fixed effects, because they are included in the regression we actually run to get the impulse response function.

Table 2: Narrative Support for Idiosyncratic Shocks

Bank Name	Year-Qtr	idiosyncratic shock	Bank specific events
Bank of America	2000q4	-0.200	Sunbeam (which BofA lended to) posted $86M loss. BofA said net charge-offs in Q4 will double. BofA issues warning on $1B uncollectible debt, may miss the December quarter profit forecast by as much as 27%.
	2003q4	-0.218	BofA agrees to pay $47 to buy FIeetBoston Financial "hefty premium" & "could dilute earnings."
	2008q3	0.288	BofA to buy Merrill for $50B (Sept 15)
	2009q2	0.452	Stress test: BofA needs to address $34B capital shortfall, better than expectation.
	2011q4	-0.275	Merrill Lynch has agreed to pay $315 million to end a mortgage-securities lawsuit (Dec 7)
	2012q4	0.248	BofA considered better buy after increase in house prices that (given its portfolio composition) particularly benefited BofA.
Citigroup	1999q1	0.319	Citigroup Profit Fell 53% in 4th period, but still topped analysts' expectations
	1999q3	0.205	Citigroup posts an unexpected increase of 9.3% in net income for second quarter (July 20)
	1999q4	0.250	Citigroup's citibank unit is marketing credit card for the internet to millions
	2000q1	0.226	Citi Intelligent Technology Receives Investment; Dividends increase from $1.05 to $1.20
	2003q4	-0.215	Citi to repay certain funds $16 mln plus interest; Citigroup Asset Management faces federal probe.
	2009q1	-0.351	Citigroup had $2B in direct gross exposure to LyondellBasell Industries, who filed for bankruptcy protection last week. Fitch cuts Citi preferred to junk
	2009q3	0.199	Citi reports profit after gain from Smith Barney. Citigroup's mortgage mitigation rises 29% in second quarter.
	2009q4	-0.267	Citi fined in tax crackdown. Abu Dhabi's sovereign wealth fund is demanding that Citigroup scraps a deal that would see the fund make a heavy loss on a $7.5 billion investment in the bank.
	2010q2	0.285	Citi reported quarterly earnings of $4.4B exceeding expectations
J.P. Morgan Chase	1997q2	-0.182	J.P. Morgan particularly large exposure to 1997 Asian Financial Crisis. https://www.imf.org/external/pubs/ft/wp/1999/wp99138.pdf
	2000q1	0.169	J.P. Morgan told investors on Monday that January and February had topped performance levels seen in the fourth quarter. Dividends increase from $0.2733 to $0.3200 on March 21.
	2000q3	0.357	Chase buying J.P. Morgan.
	2001q2	-0.185	J.P. Morgan Chase disclosed this week that their venture capital portfolios had incurred significant losses.
	2002q3	-0.322	JPMorgan Partners Reports $165M Operating Loss for Q2. J.P. Morgan sees third-quarter shortfall.
	2004q4	-0.198	JPMorgan Chase profit falls 13%.
	2008q3	0.234	J.P. Morgan profit falls 53%, but tops Wall Street target.
	2009q1	0.249	J.P. Morgan net falls sharply, but tops Wall Street view. J.P. Morgan to sell Bear Wagner to Barclays Capital: WSJ
	2012q2	-0.207	J.P. Morgan: London Whales $2 Billion Losses. Two Shareholder Suits Filed Against J.P. Morgan
Wells Fargo	2001q2	-0.161	Wells Fargo disclosed that their venture capital portfolios had incurred significant losses. Wells Fargo to take $1.1 billion charge
	2008q3	0.338	Wells Fargo's net dropped 21% as it set aside $3 billion for loan losses, better than expected. Earnings declined but beat estimates.
	2009q1	-0.315	Wells Fargo posted a surprise $2.55B Q1 loss, later revised to $2.77B. Wells Fargo added a pretax $328.4M impairment of perpetual preferred securities to its fourth-quarter loss.
	2009q2	0.405	Wells Fargo sees record Q1 profit, projections easily exceed expectations (expects earnings of $3 billion).

Table 2 shows that large absolute idiosyncratic shock values are consistent with good or bad bank specific events, such as "Wells Fargo sees record Q1 profit, projections easily exceed expectations," or "Citi fined in tax crackdown." The table shows that large positive or negative idiosyncratic shocks can be corroborated with specific events that appear bank specific, which supports the validity of our identification strategy.

Placebo Tests To test the validity of our identification strategy, we conduct placebo tests where we include ten leads of returns (in addition to the contemporary value and twenty lags as before). If the returns really are unanticipated shocks, then the leading values should not affect current behavior. This is similar to testing for pre-trends. We are testing whether the banks that will experience higher returns in the future are already acting differently today. Overall, the placebo test are encouraging, and suggest that our results are not driven by prior differences in the behavior of banks which experience return shocks.

Identification robustness We provide a few additional pieces of evidence that corroborate the validity and robustness of our identification strategy.

First, we verify that our results are robust to excluding the crisis years 2008 and 2009 from our sample. This rules out the notion that our results are related to specific events during the crisis (e.g. the realization that the government might not guarantee that a bank wouldn't fail, or that this was somehow about exposure to Lehman). Doing this made no noticeable difference to the results, since these years are only a small part of the sample.

We also check whether bank mergers drive the results. To this end, we drop the quarter of the merger as well as the quarter before and after the merger. Again, this made no noticeable difference to the results, since only a small number of observations were dropped.

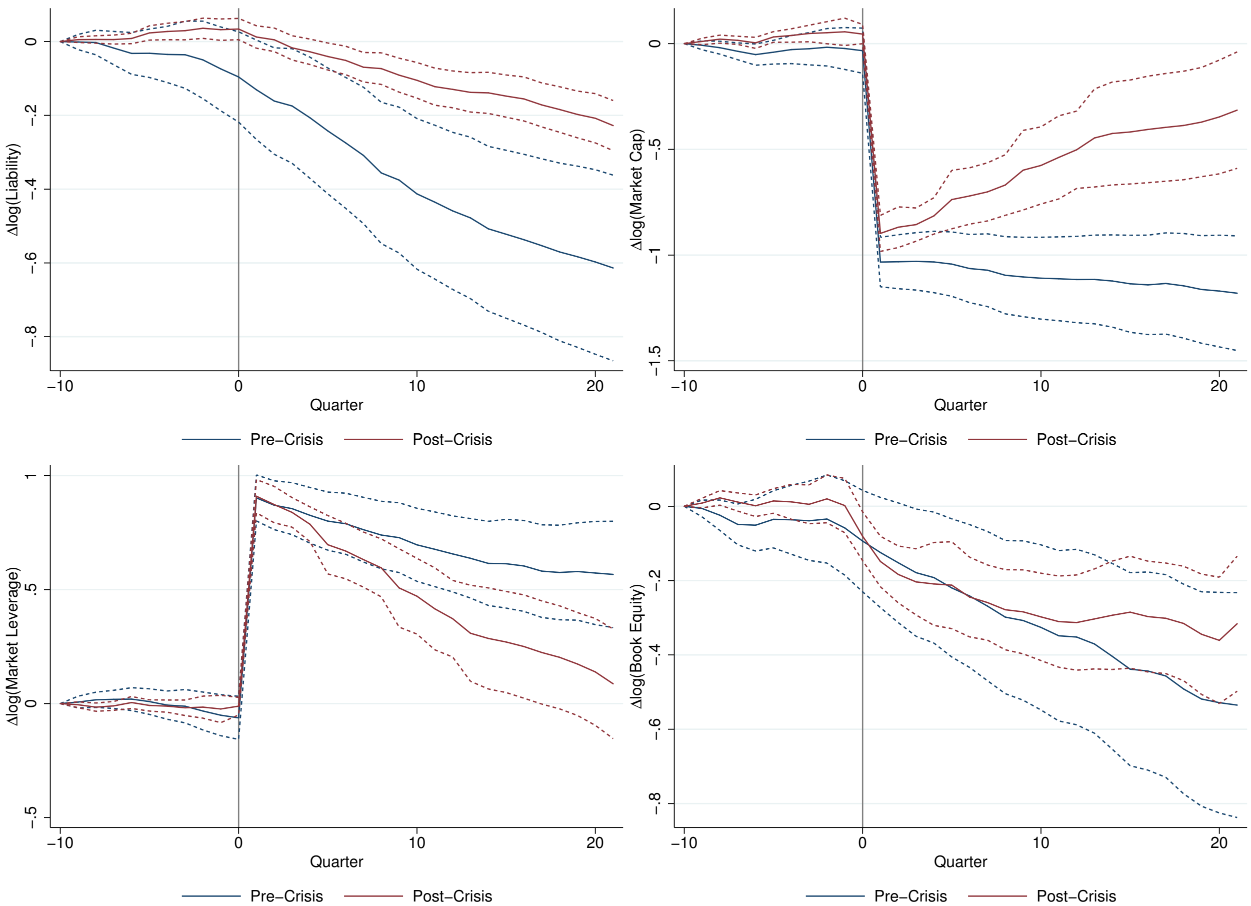
Similarly, we check whether the results are driven by the stress tests performed by banks: these stress tests were implemented after the onset of the crisis, and encouraged or mandated that banks raise additional capital. To show that the stress tests do not drive the results, we drop all banks that ever participated in a stress test (e.g. Bank of America participated in the stress tests, and so we drop Bank of America from our sample in all periods). Again, this makes no noticeable difference to the results, because only a handful of banks in the sample were ever subject to stress tests.

Another potential concern is that the return shocks could be picking up shocks to future investment opportunities, rather than default shocks. To test this concern, we check the response of the liquid assets ratio: if negative return shocks indeed predict lower future investment opportunities rather than current cash flows, we would expect banks to respond to these shocks by moving their portfolio into liquid assets. The results are in Figure B.3. Panel A shows the response of our main measure of the liquidity ratio, which we define as (Cash + Treasury Bills) /Total Assets); within the regression sample, the average liquidity ratio is 5.7%. We find a small, temporary response that begins to reverse after two years. Panel B shows an alternative measure of the liquid assets ratio, defined as (Cash + Federal Funds Sold + Securities Purchased Under Agreement to Resell + Securities)/Total Assets; the average of this liquidity ratio is 28%. The impulse response function has a significant but quantitatively small response: the impulse response implies that a 10% negative shock to market returns would cause the liquidity ratio to rise by just 0.2 percentage points over the course of two years. We take this as evidence against the hypothesis that return shocks reflect shocks to investment opportunities.

B.4 Heterogeneity

We explore heterogeneity in impulse response functions by dividing banks into two groups based on a variable, and estimating impulse responses separately for each group. We divide banks by size

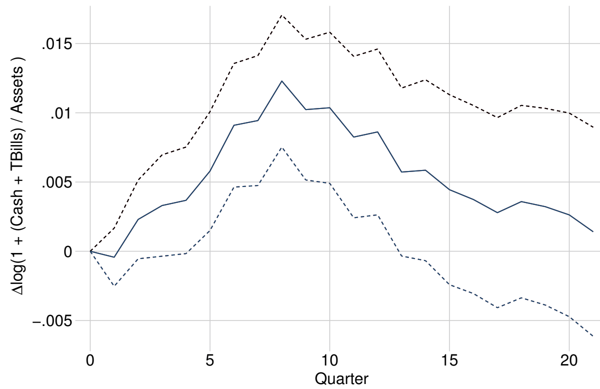
Figure B.2: Estimated Impulse Responses for Stock Variables (Risk-Adjusted, with Placebo)



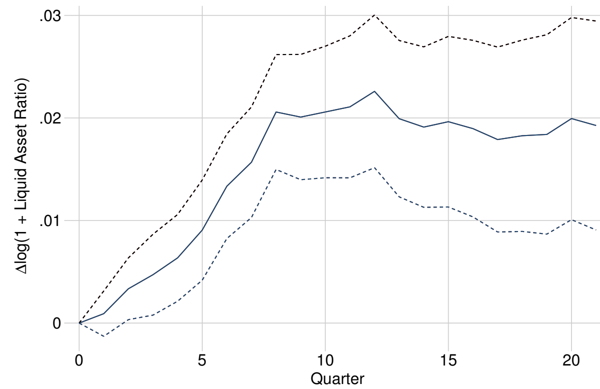
Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar).

Figure B.3: Estimated Impulse Responses of the Liquidity Ratio

a)



b)



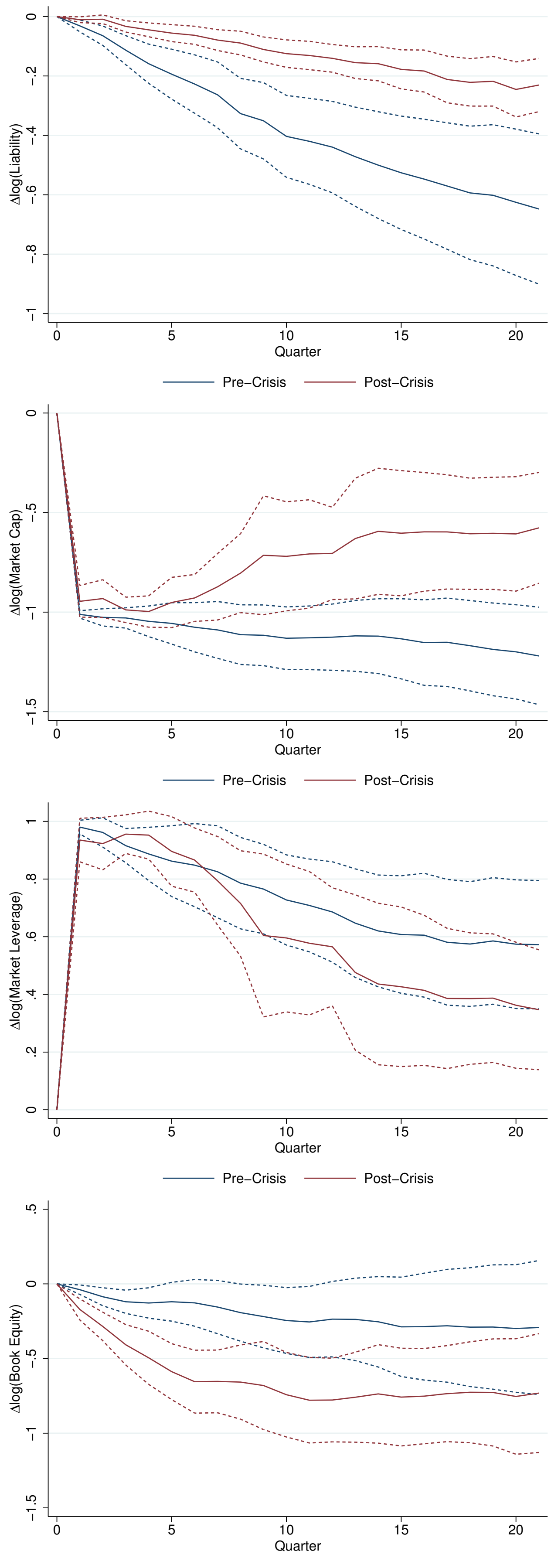
Notes: This figure shows the estimated impulse response function for BHCs to a 1% negative return shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. Panel A shows our main measure of the liquid assets ratio, defined as log((Cash + Treasury Bills) /Total Assets). Panel B shows results for our alternative, broader measure of the liquid assets ratio, defined as log((Cash + Fed Funds Sold + Securities Purchased Under Agreement to Resell + Securities) /Total Assets).

(total assets), by trading assets ratio (trading assets as a share of total assets), by the risk-weighted asset ratio (risk-weighted assets as a share of total assets), and by the mortgage ratio (real estate loans as a share of total assets). We use the value of the variable in 2000 Q1 to sort banks into two groups: above-median and below-median. We report the results in this section. Broadly, we do not find strong evidence of differential responses, but we lack statistical power to rule out some meaningful differences.

Since bank size is among the most important differences across different banks, we begin by discussing the results for heterogeneity by size. The results are shown in Figures B.4 and B.5. Visually, these impulse responses look remarkably similar to each other. However, the standard errors are sufficiently large that we cannot rule out meaningful differences in the impulse responses.

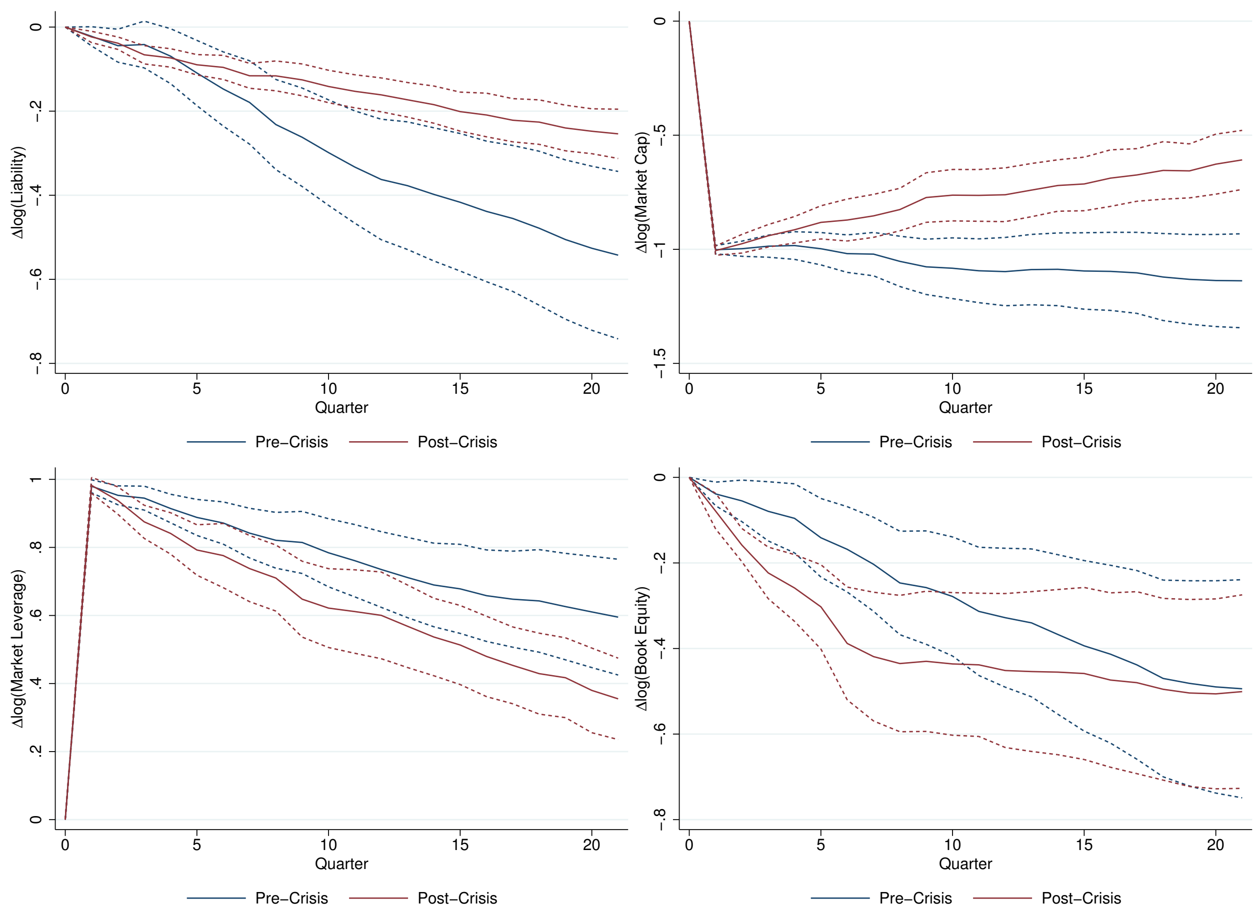
We summarize the results of these impulse responses, as well as of the other potential groupings (by trading assets ratio, risk-weighted assets ratio, and mortgage ratio) in Tables 3, 4, 5, and 6 below. For each grouping, we report the cumulative impulse response for the high and low groups after 10 quarters and after 20 quarters, and we also report the p-value of a test of equality between the impulse responses of the two groups. In a table of 64 tests, only one of the tests rejects the null at the 5% level. As before, we take this to suggest that there is not strong evidence in favor of sizable heterogeneity, but we caution that the standard errors are too large to rule out meaningful heterogeneity.

Figure B.4: Impulse Responses for Small Banks



Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence

Figure B.5: Impulse Responses for Large Banks



Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities + Market Capitalization)/Market Capitalization) are extremely similar).

Table 3: Heterogeneity in Impulse Responses: Small vs. Large Banks

		Response After 10 Quarters			Response After 20 Quarters		
		Small	Large	p-value on Equality	Small	Large	p-value on Equality
Market Equity	Pre-Crisis	-1.13	-1.09	0.75	-1.22	-1.14	0.61
		(0.08)	(0.07)		(0.12)	(0.11)	
	Post-Crisis	-0.71	-0.76	0.71	-0.58	-0.61	0.84
		(0.14)	(0.06)		(0.14)	(0.07)	
Liabilities	Pre-Crisis	-0.42	-0.33	0.39	-0.65	-0.54	0.52
		(0.07)	(0.07)		(0.13)	(0.10)	
	Post-Crisis	-0.13	-0.15	0.49	-0.23	-0.25	0.67
		(0.02)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.71	0.76	0.59	0.57	0.60	0.87
		(0.08)	(0.05)		(0.11)	(0.09)	
	Post-Crisis	0.58	0.61	0.81	0.35	0.35	0.95
		(0.13)	(0.06)		(0.11)	(0.06)	
Book Equity	Pre-Crisis	-0.25	-0.31	0.68	-0.29	-0.49	0.44
		(0.12)	(0.08)		(0.23)	(0.13)	
	Post-Crisis	-0.78	-0.44	0.04	-0.73	-0.50	0.32
		(0.15)	(0.09)		(0.20)	(0.12)	

Notes: The table compares impulse responses of small vs. large BHCs. BHCs are categorized into the small vs. large group based on their total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for small banks. The second column shows the same results, but for large banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for small banks vs. large banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

Table 4: Heterogeneity in Impulse Responses: Low vs. High Trading Asset Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.10	-1.20	0.60	-1.15	-1.32	0.54
		(0.05)	(0.19)		(0.07)	(0.28)	
	Post-Crisis	-0.76	-0.57	0.11	-0.60	-0.47	0.33
		(0.09)	(0.08)		(0.10)	(0.09)	
Liabilities	Pre-Crisis	-0.36	-0.36	0.99	-0.56	-0.63	0.78
		(0.05)	(0.14)		(0.08)	(0.22)	
	Post-Crisis	-0.14	-0.15	0.73	-0.24	-0.25	0.91
		(0.02)	(0.04)		(0.03)	(0.05)	
Market Leverage	Pre-Crisis	0.74	0.84	0.35	0.59	0.70	0.50
		(0.05)	(0.10)		(0.07)	(0.14)	
	Post-Crisis	0.63	0.42	0.08	0.37	0.22	0.25
		(0.08)	(0.09)		(0.08)	(0.09)	
Book Equity	Pre-Crisis	-0.25	-0.42	0.35	-0.28	-0.75	0.17
		(0.07)	(0.17)		(0.13)	(0.32)	
	Post-Crisis	-0.62	-0.45	0.39	-0.66	-0.54	0.64
		(0.10)	(0.16)		(0.13)	(0.20)	

Notes: The table compares impulse responses of low vs. high trading asset ratio BHCs. BHCs are categorized into the low vs. high group based on their trading assets as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

Table 5: Heterogeneity in Impulse Responses: Low vs. High Risk-Weighted Asset Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.09	-1.15	0.59	-1.10	-1.22	0.43
		(0.07)	(0.08)		(0.11)	(0.12)	
	Post-Crisis	-0.72	-0.79	0.64	-0.52	-0.66	0.44
		(0.12)	(0.09)		(0.14)	(0.11)	
Liabilities	Pre-Crisis	-0.29	-0.41	0.21	-0.47	-0.66	0.20
		(0.06)	(0.07)		(0.10)	(0.11)	
	Post-Crisis	-0.13	-0.17	0.17	-0.25	-0.25	0.97
		(0.03)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.80	0.73	0.47	0.63	0.56	0.59
		(0.07)	(0.06)		(0.09)	(0.10)	
	Post-Crisis	0.59	0.62	0.82	0.27	0.41	0.31
		(0.11)	(0.09)		(0.11)	(0.09)	
Book Equity	Pre-Crisis	-0.19	-0.35	0.23	-0.24	-0.45	0.40
		(0.10)	(0.09)		(0.16)	(0.18)	
	Post-Crisis	-0.49	-0.74	0.17	-0.51	-0.81	0.23
		(0.09)	(0.16)		(0.11)	(0.23)	

Notes: The table compares impulse responses of low vs. high risk-weighted asset ratio BHCs. BHCs are categorized into the low vs. high group based on their risk-weighted assets as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

Table 6: Heterogeneity in Impulse Responses: Low vs. High Mortgage Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.04	-1.21	0.17	-1.07	-1.27	0.26
		(0.05)	(0.11)		(0.07)	(0.17)	
	Post-Crisis	-0.75	-0.75	0.98	-0.61	-0.56	0.79
		(0.13)	(0.08)		(0.17)	(0.09)	
Liabilities	Pre-Crisis	-0.28	-0.46	0.11	-0.45	-0.73	0.09
		(0.05)	(0.10)		(0.08)	(0.15)	
	Post-Crisis	-0.17	-0.11	0.09	-0.28	-0.19	0.13
		(0.02)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.76	0.75	0.92	0.62	0.54	0.55
		(0.06)	(0.07)		(0.08)	(0.11)	
	Post-Crisis	0.59	0.64	0.72	0.34	0.37	0.81
		(0.11)	(0.09)		(0.13)	(0.07)	
Book Equity	Pre-Crisis	-0.20	-0.36	0.32	-0.27	-0.42	0.63
		(0.07)	(0.15)		(0.09)	(0.30)	
	Post-Crisis	-0.66	-0.56	0.57	-0.70	-0.59	0.65
		(0.10)	(0.14)		(0.13)	(0.19)	

Notes: The table compares impulse responses of low vs. high mortgage ratio BHCs. BHCs are categorized into the low vs. high group based on their real estate loans as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

C General Equilibrium and Derivation of Social Welfare Function

In this section, we embed our Q theory of banks into a general equilibrium setting. The purpose is to derive a welfare function in order to produce normative implications. The non-financial sector resembles is akin to the two-sector models of He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014). The key distinction is that we introduce for Poisson capital destruction events and bank liquidations. We furthermore consider a single household that owns all firms in the economy.

C.1 Environment

A representative risk-neutral household holds wealth in bank stocks and firms. There is a homogeneous good produced with capital in two productive sectors. Capital is freely mobile across sectors. One sector (banked firms) produces goods borrowing loans from banks to buy capital. These banked firms operate part of the capital stock and are financed entirely through banks. Capital operated by banked firms yield an immediate return A^L. The rest of the capital stock is operated by firms that are directly owned by households. Output in this latter sector is A^D per unit of capital. Banks are the entities encountered in the text.

Each capital unit depreciates at rate δ. In addition, the capital operated by banked firms can be destroyed rate $\sigma\varepsilon$. From an aggregate perspective $\sigma\varepsilon$ is like additional depreciation in the banked sector. We assume that: $A^L - \sigma\varepsilon > A^D$ so that considering depreciation, it is economically efficient to allocate capital to the banked sector. Capital is fully reversible and investment transforms one unit of goods into capital. The price of capital is, therefore, equal to one unit of goods. Loans are fully collateralized by the capital bought by banked firms. The capital destruction shocks leads to loan defaults as in the model.

Notation. We use script variables to denote variables aggregated across banks. For example, W represents the equity in a given bank whereas $\mathcal{W}$ is the aggregate equity of all banks.

Social Costs of Bank Liquidations. To relate capital destruction to loan default events, we assume banks do not diversify loans across banked firms. For simplicity, we assume each bank lends to a single firm—we can generalized this to multiple firms with correlated shocks. Thus, σ can be viewed as the arrival rate of capital destruction event that affects the customer of a bank. Under this assumption, as in the body of the text, σ captures the intensity of a default event at a given bank (and its firm) and ε the share of the capital that gets destroyed. When there is no liquidation of the bank, the fraction $(1-\varepsilon)$ is the recovery of the collateral that backs the bank loans, which is entirely seized by the bank. Thus, $\varepsilon = 1 - (1-\varepsilon)$ is the loan recovery rate.

When banks are liquidated, the bank only recovers the fraction $\psi < 1$ capital that backs the loan. Thus, total losses are:

$$(1 - \psi(1-\varepsilon)) = \varepsilon + (1-\psi)(1-\varepsilon).$$

We assume bank liquidations are socially inefficient:

$$A^L - \sigma(\varepsilon + (1-\psi)(1-\varepsilon)) < A^D.$$

That is, whereas economically there should only be $\lambda W\varepsilon$ in bank wealth losses, when banks are liquidated, the restructuring costs amount to $(1-\psi)(1-\varepsilon)\lambda W$ in additional losses. It is convenient to account for the wealth remaining at the bank after the default event $W+\bar{J}^W W$:

$$W + \bar{J}^W (\lambda) W = \underbrace{\psi (1-\varepsilon) \lambda W}_{\text{recovered loans}} - \underbrace{(\lambda - 1) W}_{\text{deposits}} = (1 - (1 - \psi (1-\varepsilon)) \lambda) W.$$

This wealth is used to form new banks. Per unit of wealth, the remaining bank wealth is, $1 - (\varepsilon + (1-\psi)(1-\varepsilon)) \lambda = \psi (1-\varepsilon) \lambda < \lambda$. The value of social losses are $\bar{J}^W (\lambda) = -(\varepsilon + (1-\psi)(1-\varepsilon)) \lambda$ per unit of wealth. The recovered assets minus liabilities, $W + \bar{J}^W W$, are used to form new banks, that start with $z = 0$. Thus, the entry per unit of wealth loss is:

$$W^{new} = W + \bar{J}^W W.$$

Notice that $\bar{J}^W$ is the jump in societies wealth after the bank is liquidated whereas J^W is the private losses. As in the text, we think of the remaining value of the bank after liquidation, v_0, as some fixed share that the bank gets to keep after liquidations. The banker views this amount as returned to its shareholders and does not internalize the social losses. We impose a parametric restriction to guarantee that there is value left at the surviving banks. Thus:

$$v_0 \leq 1 - (1 - \psi (1-\varepsilon)) \lambda, \quad \forall \lambda \to v_0 \leq 1 - (1 - \psi (1-\varepsilon)) \kappa.$$

Equilibrium Rates. We assume a competitive environment. Given the linear technologies:

$$r_t^L = A^L - \delta, \quad \Pi_t^L = 0.$$

The user cost of capital of unbanked firms must equal, the deposit rate:

$$r_t^D = A^d - \delta.$$

De facto, banks face a perfectly elastic supply of deposits and demand for loans. Implicitly, this assumes that unbanked firms are plenty and deposits are never scarce. By contrast, banks have limited wealth.

Aggregation. We now consider the heterogeneity across banks that occurs when there is delayed accounting. Let $g_t(z, W)$ denote the joint distribution of z and W. Then, aggregating across banks we obtain that total loans are:

$$\mathcal{L}_t = \int_0^\infty \int_0^\infty \lambda (z) W g_t (z, W) \, dz dW,$$

and likewise for deposits:

$$\mathcal{D}_t = \int_0^\infty \int_0^\infty (\lambda (z) - 1) W g_t (z, W) \, dz dW.$$

The total equity at banks is:

$$\mathcal{W}_t = \int_0^\infty \int_0^\infty W g_t (z, W) \, dz dW.$$

Households hold wealth, N, in equity in non-banked firms and in the banking sector $\mathcal{W}$. The sum of their sectoral wealth adds to the total capital stock:

$$K_t = \mathcal{W}_t + N_t;$$

Operated capital differs from the allocation of equity. In particular, the capital stock is divided into:

$$K_t = K_t^L + K_t^D,$$

where K_t^D is the capital stock operated by households directly and K_t^L is the capital managed by banked firms, financed by borrowing loans. This latter capital equals the stock of bank loans because loans are used to finance the holdings of capital by banked firms: $K_t^L = \mathcal{L}_t$. Here, $\mathcal{L}_t$ is the aggregation of the individual loans L_t of a banks encountered earlier. Each bank chooses $D_t \geq 0$ standing for borrowed funds from non-banked firms. Thus, $\mathcal{D}_t = K_t^L - \mathcal{W}_t$, which is determined by the choice of leverage. Naturally, borrowed funds comes from firms, $K_t^D + \mathcal{D}_t = N_t$.

Individual bank accounting profits are:

$$\Pi_t^B = r^L L_t - r^D D_t.$$

The profits of unbanked firms is:

$$\Pi_t^D = r^D D_t + A^D \left(N_t - D_t\right),$$

which implicitly counts deposits at banks and the remainder of operated capital. Banked-firm profits are:

$$\Pi_t^L = \left(A^L - r^L\right) K_t^L - \delta K_t^L.$$

The unbanked firms and banks payout exogenous dividend rates, $\bar{c}$ and c^D, respectively. Banked firms payout all their profits as dividends: Π^L. Hence, total consumption is:

$$\mathcal{C}_t = \bar{c}\mathcal{W}_t + c^D N_t + \Pi_t^L. \tag{22}$$

The change in the aggregate wealth stored in banks is:

$$\dot{\mathcal{W}}_t = \Pi_t^B - \bar{c}\mathcal{W}_t - \sigma \underbrace{\left(\omega + (1-\omega)\left(\varepsilon + (1-\psi)(1-\varepsilon)\right)\right)}_{\equiv \eta} K_t^L,$$

where ω is the share of banks that suffer a default event and survive. Wealth in unbanked firms evolves as:

$$\dot{N}_t = \Pi^D - c^D N_t - \delta K_t^D.$$

Aggregate income:

$$\begin{aligned} \mathcal{Y}_t &= \Pi^B + \Pi^D + \Pi^L \\ &= A^L K_t^L + A^D K_t^D, \end{aligned} \tag{23}$$

which sums to aggregate production. Capital investment is done by banked firms, as we show next.

Using (22) and (23), we have that:

$$\begin{aligned} \mathcal{Y}_t - \mathcal{C}_t &= A^L K_t^L + A^D K_t^D - c^D N_t - \bar{c}\mathcal{W}_t - \Pi_t^L \\ &= \left(r_t^L + \delta\right) K_t^L + A^D K_t^D - c^D N_t - \bar{c}\mathcal{W}_t \\ &= \left(r_t^L + \delta\right) K_t^L - r^D D_t - \bar{c}\mathcal{W}_t + A^D K_t^d + r^D D_t - c^D N_t \\ &= \int \int_0^\infty \int_0^\infty \left(\Pi_t^B g_t(z, W) - \bar{c}W\right) dz dW + \Pi_t^D + \delta K_t^L - c^D N_t \\ &= \dot{\mathcal{W}}_t + \dot{W}_t^d + \eta K_t^b + \delta K_t^L + \delta K_t^D \\ &= \dot{K} + (\eta + \delta) K_t^L + \delta K_t. \end{aligned}$$

Thus, defining $\mathcal{I}_t$ as the change in capital plus total capital depreciated:

$$\dot{K} = \mathcal{I}_t - (\eta + \delta) K_t^b - \delta K_t.$$

Hence, we verify the income identity, $\mathcal{I}_t \equiv \mathcal{Y}_t - \mathcal{C}_t$. We are now ready to introduce the welfare function.

C.2 Social Welfare Function

To work with aggregate bank decisions, we introduce the following notation: $\mathcal{W}_t$, represents aggregate bank equity. Consistent with the paper, we set $\bar{c}$ to be the dividend rate of a bank as in the text. Also, to avoid confusion, define $N_t \equiv N_t$, to be the aggregate household net-worth held directly at firms.

We work directly with the household's linear benefit from consumption as the notion of social welfare:

$$\mathcal{V}(N_t, \mathcal{W}_t) \equiv \mathbb{E}\left[\int_0^\infty \exp(-\rho t)\, \mathcal{C}_t dt | \mathcal{W}_0, N_0\right].$$

Immediate Accounting. Let's start with the immediate accounting case, assuming that all banks behave the same. We have that the social welfare function has an HJB representation:

$$\rho\mathcal{V}(N, \mathcal{W}) \equiv c^d N + \bar{c}\mathcal{W} + \mathcal{V}_N(N, \mathcal{W})\left(A^D - \delta - c^D\right) N + \mathcal{V}_{\mathcal{W}}(N, \mathcal{W})\left(r^L \lambda \mathcal{W} - (\lambda - 1) r^D - \bar{c}\mathcal{W} - \eta\lambda\mathcal{W}\right).$$

This HJB is additive:

$$\mathcal{V}(N, \mathcal{W}) = \mathcal{F}(N) + \mathcal{P}(\mathcal{W})$$

where:

$$\rho\mathcal{F}(N) = c^d N + \mathcal{F}_N(N)\left(A^d - \delta - c^d\right) N$$

and,

$$\rho\mathcal{P}(\mathcal{W}) = \bar{c}\mathcal{W} + \mathcal{P}_{\mathcal{W}}(\mathcal{W})\underbrace{\left(r^L\lambda - (\lambda - 1) r^D - \bar{c} - \eta\lambda\right)}_{\equiv \mu^{\mathcal{W}},}\mathcal{W}.$$

In turn, this value function is linear and thus:

$$\mathcal{P}(\mathcal{W}) = p\mathcal{W}$$

where p satisfies

$$\rho p = \bar{c} + p\mu^{\mathcal{W}},$$

and unpacking $\mu^{\mathcal{W}}$,

$$\mu^{\mathcal{W}} = \left(r^L\lambda - (\lambda - 1) r^D - \bar{c} - \sigma\varepsilon\lambda \times \mathbb{I}_{[\lambda \leq \min\{\Lambda, \Xi\}]} - \sigma(\varepsilon + (1 - \psi)(1 - \varepsilon))\lambda \times \mathbb{I}_{[\lambda \leq \min\{\Lambda, \Xi\}]}\right)\mathcal{W}.$$

Thus, we have the following result:

Problem 2 *[Planner Optimal Leverage] Under immediate accounting, the maximization of social welfare requires the maximization of*

$$\left(r^L\lambda - (\lambda - 1) r^D - \bar{c} - \sigma\varepsilon\lambda \times \mathbb{I}_{[\lambda \leq \Lambda]} - \sigma(\varepsilon + (1 - \psi)(1 - \varepsilon))\lambda \times \mathbb{I}_{[\lambda > \Lambda]}\right).$$

Delayed Accounting. In sequential form, the objective function is:

$$\mathcal{V}\left(N_t, \mathcal{W}_t\right) \equiv \overbrace{\int_0^\infty \exp\left(-\rho t\right) c^d N_t dt}^{\mathcal{F}(N_0)} + \mathbb{E}\left[\int_0^\infty \int_0^\infty \int_0^\infty \exp\left(-\rho t\right) \bar{c} W_t g_t\left(z, W\right) dz dW dt\right].$$

We know that:

$$g_t\left(z, W\right) = \int_0^\infty \int_0^\infty G_t\left(z_t, W_t; z_0, W_0\right) \times g_0\left(z_0, W_0\right) dz_0 dW_0,$$

for some function $G_t\left(z_t, W_t; z_0, W_0\right)$ that yields a density of $\{z_t, W_t\}$ conditional on an initial condition $\{z_0, W_0\}$. Thus, the term in the objective is written as:

$$\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \exp\left(-\rho t\right) \bar{c} W_t G_t\left(z_t, W_t; z_0, W_0\right) dz_t dW_t dt g_0\left(z_0, W_0\right) dz_0 dW_0$$

In turn, the term:

$$\int_0^\infty \int_0^\infty W_t G_t\left(z_t, W_t; z_0, \mathcal{W}_0\right) dz_0 dW_0 = \int_0^\infty \int_0^\infty \mathbb{E}\left[W_t | W_0 = W, z_0 = z\right] g_0\left(z_0, W_0\right) dz_0 dW_0.$$

Hence, the expected value of dividends by banks can be written as:

$$\mathcal{P}\left[\{g_0\}\right] \equiv \int_0^\infty \int_0^\infty P\left(z, W\right) g_0\left(z, W\right) dz dW.$$

where implicitly we have defined:

$$P\left(z, \mathcal{W}\right) = \mathbb{E}\left[\int_0^\infty \exp\left(-\rho t\right) \bar{c} W_t dt | \mathcal{W}_0 = \mathcal{W}, z_0 = z\right].$$

Next, we work with the Feynman-Kac formula to consider bank liquidations and the formation of new banks. In this case of jumps:

$$\rho P\left(z, W\right) = cW + P_W\left(z, W\right) \mu^W\left(z\right) W + P_z\left(z, W\right) \dot{z} + \sigma\left[P\left(z + \bar{J}^z, W + \bar{J}^W W\right) - P\left(z, W\right)\right].$$

In this case, we have that:

$$\mu^W\left(z\right) \equiv \left(r^L \lambda\left(z\right) - \left(\lambda - 1\right) r^D - \bar{c}\right) W_t,$$

whereas the jump terms are:

$$\bar{J}^W = -\sigma\varepsilon\lambda \times \mathbb{I}_{\lambda(z)\leq\Lambda(z)} - \sigma\left(\varepsilon + \left(1-\psi\right)\left(1-\varepsilon\right)\right)\lambda \times \mathbb{I}_{\lambda(z)>\Lambda(z)}$$

$$\bar{J}^z = J^z \times \mathbb{I}_{\lambda(z)\leq\Lambda(z)} - z \times \mathbb{I}_{[\lambda(z)>\Lambda(z)]}.$$

As before, we verify that $P\left(z, W\right)$ is scale independent. Thus:

Problem 3 *[Bank Optimization Problem] Under delayed accounting, the social value of an individual bank is given by $P\left(z, W\right) = p\left(z\right) W$ where:*

$$\rho p\left(z\right) = c + p\left(z\right) \mu^W\left(z\right) + p_z\left(z\right) \dot{z} + \sigma\left[p\left(z + \bar{J}^z\right)\left(1 + \bar{J}^W\right) - p\left(z\right)\right],$$

where

$$\bar{J}^W = -\sigma\varepsilon\lambda \times \mathbb{I}_{\lambda(z)\leq\Lambda(z)} - \sigma\left(\varepsilon + \left(1-\psi\right)\left(1-\varepsilon\right)\right)\lambda \times \mathbb{I}_{\lambda(z)>\Lambda(z)}$$

and

$$\bar{J}^z = J^z \times \mathbb{I}_{\lambda(z) \leq \Lambda(z)} - z \times \mathbb{I}_{[\lambda(z) > \Lambda(z)]}.$$

D Additional Model Discussions

D.1 In Detail: Model Timing

Timing. To clarify the timing assumption, Figure D.1 plots an example of a sample path of N_t and dN_t and the implied behavior of book leverage $\bar{\lambda}_t$ and Z_t. Assume that the bank arbirtrarilly decides to set book leverage to a constant, $\bar{\lambda}$. The Figure depicts a hypothetical scenario of a default event at time τ. At τ, the process N_t jumps following a path that is continuous from the right. This reflects in the discontinuous point in dN_t. The next panel depicts the path of $\bar{\lambda}$: the discontinuity represents the jump after the default event. In this example, $\bar{\lambda} + J^{\bar{\lambda}} < \Xi$, so the bank remains solvent. Critically, this happens because the bank cannot control its book leverage, via D, all the time. The panel next to it shows that Z_t also jumps the date of the shock. However, this variable is continuous from the left, which is why $\bar{\lambda}$ jumps for that instant. After the shock, the bank sells loans and transfers its losses into the stock of Zombie loans to return back to a constant book leverage path. The bank could have been liquidated if the jump lead to some point $\bar{\lambda} + J^{\bar{\lambda}} > \Xi$, even though the violation would have been for an infinitesimal period of time. This assumption is equivalent to the discrete time assumption that the shock occurs between periods. It can also be obtained as a limit process, where we have adjustment costs to selling loans that are taken to zero.

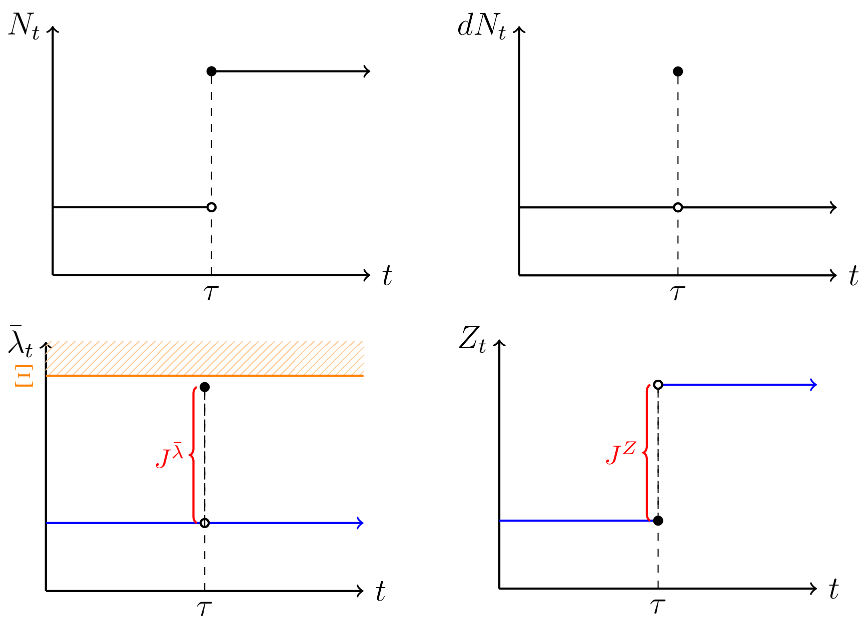


Figure D.1: Example: Timing Assumption

Notation and Definitions. We begin by presenting some definitions and deriving the laws of motion of the state variables of the model and other variables of interest. Recall that, for level variables, μ^x and J^x refer to the drift and jump components of the path of a variable x scaled by wealth W, respectively. For financial ratios, μ^x and J^x refer to the drift and jump components of the path of a variable x scaled without scaling.

Throughout the paper we use the following relationships that allow us to recover the original state variables $\{L, \bar{L}, D\}$ from the triplet $\{\lambda, z, W\}$:

$$
\begin{aligned}
L &= \lambda \cdot W && (24)\\
D &= (\lambda - 1) \cdot W && (25)\\
\bar{L} &= \lambda W + zW. && (26)\\
\bar{W} &= W + zW. && (27)
\end{aligned}
$$

We express the dividend-to-equity ratio as:

$$c \equiv C/W.$$

D.2 Variable Paths

Let τ be the time of the realization of default event. Next, we describe the paths of variables in their canonical (integral) form.

Discontinuity Points at Default Events. We describe the behavior of each model variable at its discontinuity point.

Loans. We have that the controlled part of loans is:

$$\lim_{t \to \tau^-} L_t = \lim_{t \to \tau^-} \lambda_t W_t,$$

along a path where λ_t follows a continuous path prior to the default event.

Upon a default event, the fraction ε of loans is lost:

$$L_\tau = \lim_{t \to \tau^-} L_t - \varepsilon L_t.$$

Immediately after the jump, the bank sells loans at the amount S_τ. Therefore,

$$\lim_{t \to \tau^+} L_t = L_\tau - S_\tau.$$

Discrete loan events occur when there are no defaults, but z reaches z^o. The date τ of a loan sale event, we also have the same equation.

Deposits. Default events do not change the bank's liabilities. Thus,

$$\lim_{t \to \tau^-} D_t = D_\tau,$$

After a default event, the bank sells loans and reduces its stock of liabilities:

$$\lim_{t \to \tau^+} D_t = D_\tau - S_\tau.$$

Fundamental Equity. At the instant of the default event, equity and loans satisfy:

$$W_\tau = \lim_{t \to \tau^-} W_t - \varepsilon L_t$$

Fundamental equity remains unchanged with a loan sale:

$$\lim_{t \to \tau^+} W_t = W_\tau.$$

This limits always satisfy $W_t = L_t - D_t, \quad \forall t.$

Zombie Loans. As a result of a default event, zombie loans satisfy:

$$Z_\tau = \lim_{t\to\tau^-} Z_t = \lim_{t\to\tau^-} z_t W_t,$$

at the instant of the default event. The banker can choose to hide losses in the amount $H_\tau = \varepsilon \lim_{t\to\tau^-} \lambda_t W_t$. It optimally does so. Thus, zombie loans change with the hiding of losses immediately after the default event:

$$\lim_{t\to\tau^+} Z_t = Z_\tau + H_\tau.$$

Fundamental Leverage. For convenience, we define:

$$F_\tau \equiv \frac{S_\tau}{\lim_{t\to\tau^-} W_t}.$$

As a result of a default event, leverage is:

$$\lambda_\tau = \lim_{t\to\tau^-} \frac{L_t - \varepsilon L_t}{W_t - \varepsilon L_t} = (1-\varepsilon) \lim_{t\to\tau^-} \frac{\lambda_t}{1-\varepsilon\lambda_t},$$

at the instant of the default event. The jump in leverage corresponds to the uncontrolled part of leverage

$$\lambda_\tau - \lim_{t\to\tau^-} \lambda_t.$$

Considering the asset sales, fundamental leverage is follows:

$$\lim_{t\to\tau^+} \lambda_t = \frac{L_\tau + S_\tau}{W_\tau} = \frac{L_t - \varepsilon L_t + S_\tau}{W_t - \varepsilon L_t} = \lim_{t\to\tau^-} \frac{\lambda_t - \varepsilon\lambda_t}{1-\varepsilon\lambda_t} + \frac{1}{\lim_{t\to\tau^-} W_t} \frac{S_\tau}{(1-\varepsilon\lambda_t)}$$

Thus, leverage jumps to:

$$\lim_{t\to\tau^+} \lambda_t = \lim_{t\to\tau^-} \frac{\lambda_t - \varepsilon\lambda_t}{1-\varepsilon\lambda_t} + \frac{S_\tau}{(1-\varepsilon\lambda_t)},$$

considering the adjustment of loans.

Zombie Ratios. At the time of a default event, the zombie ratio satisfies:

$$z_\tau = \lim_{t\to\tau^-} \frac{Z_t}{W_t - \varepsilon L_t} = \lim_{t\to\tau^-} \frac{z_t}{1-\varepsilon\lambda_t},$$

at the instant of the default event. The zombie ratio remains unchanged with a loan sale:

$$\lim_{t\to\tau^+} z_t = z_\tau + \lim_{t\to\tau^-} \frac{\varepsilon\lambda_t}{1-\varepsilon\lambda_t}.$$

Book Loans. At the time of a default event, book loans satisfy:

$$\bar{L}_\tau = \lim_{t\to\tau^-} \bar{L}_t + \varepsilon\lambda_t W_t,$$

reflecting that the losses can be detected at the instant of the default event. However, immediately after the jump, the bank sells loans the amount $-S_\tau$,the sale is registered in the books. Thus,

$$\lim_{t\to\tau^+} L_t = L_\tau - S_\tau + H_\tau.$$

Book Equity. Book equity jumps the instant of a default event. Thus:

$$\bar{W}_{\tau} = \lim_{t \to \tau^{-}} \bar{W}_t - \varepsilon \lambda_t W_t,$$

But it increases with the hiding of losses immediately after:

$$\lim_{t \to \tau^{+}} \bar{W}_t = \bar{W}_{\tau} + H_{\tau}.$$

Book Leverage. Book leverage remains jumps the instant of a default event. Thus:

$$\bar{\lambda}_{\tau} = \lim_{t \to \tau^{-}} \bar{\lambda}_t - \frac{\varepsilon \lambda_t}{1 - \varepsilon \lambda_t},$$

but reverts back immediately after to:

$$\lim_{t \to \tau^{+}} \bar{\lambda}_t = \bar{\lambda}_{\tau} + \lim_{t \to \tau^{-}} \frac{\varepsilon \lambda_t}{1 - \varepsilon \lambda_t} - \frac{S_{\tau}}{1 - \varepsilon \lambda_t}$$

Little q. At the time of a default event, little q satisfies:

$$q_{\tau} = \lim_{t \to \tau^{-}} \frac{W_t - \varepsilon \lambda_t W_t}{\bar{W}_{\tau} - \varepsilon \lambda_t W_t}.$$

:

$$\lim_{t \to \tau^{+}} q_{\tau} = \lim_{t \to \tau^{-}} \frac{W_t - \varepsilon \lambda_t W_t}{\bar{W}_{\tau}},$$

reverts back with the jump in losses.

Discontinuity Points at Unforced Deleveraging Events. If at some date τ, the bank reaches some point in the stat space $\{z^o, W\}$ an decides to switch its leverage position discretely, i.e., if the optimal policy λ^* has a discontinuity pint, the paths of variables are as above, setting $\varepsilon = 0$ and

$$S_{\tau} = (1 - \varepsilon \lambda) \left(\lim_{t \to \tau^{+}} \lambda_t - \lim_{t \to \tau^{-}} \lambda_t \right).$$

D.3 From Leverage to Loan Growth Ratios

While leverage is a control variable for banks, along a continuous path of the bank's state variables and controls, we can also describe growth rate of loans ι. Along a continuous path, the net investment in loans by a the bank is:

$$\iota \equiv I/L - \delta,$$

where δ can represent the repayment share of past loans and I a flow of new loans. The flow of new loans and, thus, ι should be consistent with the bank's leverage decision.

We note the relationship between the growth in loans and the state variable z, along a continuous path for loans.

$$L = \lambda W \Rightarrow \mu^L W = \dot{\lambda} W + \lambda \mu^W W \Rightarrow \dot{\lambda} = \mu^L - \lambda \mu^W.$$

Replacing the values for these variables, we obtain:

$$\dot{\lambda} = \iota \lambda - \lambda \mu^W \Rightarrow \iota = \frac{\dot{\lambda}}{\lambda} + \mu^W.$$

Thus, the growth rate of loans is the sum of growth of leverage plus the growth in equity.

Consistency. Along a continuous path, if the solution to the bank's problem is given by some optimal leverage decision, $\lambda^*(z)$. In this case:

$$\dot{\lambda} = \lambda_z^*(z)\,\dot{z}.$$

Thereby, we obtain that the growth in loans is given by:

$$\iota = \frac{\lambda_z^*(z)}{\lambda^*(z)}\dot{z} + \mu^W(z).$$

Likewise, upon a default event:

$$\left(\lambda + J^\lambda + \bar{J}^\lambda\right)\left(W + J^\lambda\right) - \lambda W = SdN_t.$$

D.4 Derivation of Laws of Motion from Discrete-Time Analogs

Summary Table. The following table summarizes the drift and jump terms of different variables as functions of $\{\lambda, z, W\}$.

Variable	Drift	Jumps	Sale
State Variables and Financial Ratios			
W	$\left[r^L\lambda - r^D(\lambda-1) - c\right]W$	$-\varepsilon\lambda W$	0
Z	$-\alpha Z$	$\varepsilon\lambda W$	0
z	$-z\left(\alpha+\mu^W\right)$	$\varepsilon\lambda\left(\frac{z+1}{1-\varepsilon\lambda}\right)$	0
λ	$\iota\lambda - \lambda\mu^W$	$\frac{\varepsilon\lambda}{1-\varepsilon\lambda}(\lambda-1)$	$F(z)/(1-\varepsilon\lambda)$
Deduced Variables			
L	$\iota\lambda W$	$-\varepsilon\lambda W$	$F(z)$
D	$\left[r^D(\lambda-1) - r^L\lambda + \iota\lambda + c\right]W$	0	$F(z)$
$\bar{L}$	$(\iota\lambda - \alpha z)W$	0	$F(z)$
$\bar{W}$	$\left(r^L\lambda - r^D(\lambda-1) - \alpha z\right)W$	0	0
q	$\frac{\alpha z}{(1+z)^2}$	$-\varepsilon\lambda q$	0
$\bar{\lambda}$	$\frac{1}{1+z}\left(\iota\lambda - \lambda\mu^W - \alpha z\frac{1-\lambda}{\lambda+z}\right)$	0	$F(z)/q$
s	$-s'(z)\left(\alpha+\mu^W\right)z$	$s\left(z + \varepsilon\lambda\left(\frac{z+1}{1-\varepsilon\lambda}\right)\right) - s(z)$	0

Table 7: Drifts and Jumps of Variables

We present some observations that derive these laws of motion and provided preliminary results that aid the proof of the main propositions in the paper.

Preliminary Results 1: derivations of laws of motion. Here, we provide an explicit derivation of the law of motion of bank equity, starting from a discrete time formulation. In a discrete time formulation, with time interval Δ, the bank receives a default shock $\varepsilon < 1$ with probability $\sigma\Delta$. Let:

$$N_{t+\Delta} - N_t = \begin{cases} 0 & \text{with prob } 1-\sigma\Delta \\ 1 & \text{with prob } \sigma\Delta \end{cases}$$

denote a default event process. Recall that dN is a Poisson process.

Loans. Now consider a time interval of length Δ. The law of motion for fundamental loans satisfies:

$$L_{t+\Delta} = (1-\delta\Delta) L_t + I_t\Delta - \varepsilon L_t (N_{t+\Delta} - N_t) + F_t,$$

with the interpretation that the first term is the non-maturing fraction of loans, the second are loan issuances, and the third are losses in a time interval. Subtracting L_t from both sides and taking $\Delta \to 0$, we obtain the following law of motion:

$$dL = (I - \delta L)\, dt - \varepsilon L dN + F^L.$$

The interpretation of dL is important. As all differentials, it is a stand in for notation. We should interpret as a limit of a rate of change as $\Delta \to 0$; likewise, dN as the limit Poisson event corresponding to $N_{t+\Delta} - N_t$, as $\Delta \to 0$ and F^L as the process of asset sales.[60] The key is that this differentials must represent the paths in D.2.

We express this law of motion in terms of net-worth, replacing (24), to obtain:

$$dL = \iota\lambda W dt - \varepsilon\lambda W dN + F^L. \tag{28}$$

Consistent with our notation, we define the drift and jumps relative to wealth are given by:

$$\mu^L \equiv \iota\lambda \text{ and } J^L \equiv -\varepsilon\lambda.$$

Deposits. For deposits we have that:

$$D_{t+\Delta} = \left(1 + r^D\Delta\right) D_t - \left(r^L\Delta + \delta\Delta\right) L_t + I_t\Delta + C_t\Delta + F_t$$

with the interpretation that the first term is the increase in deposits that results from paying interest with deposits; the second term is the reduction in deposits by the interest and principal payments on outstanding loans; the third term is the increase in deposits as a result of loan issuances; and the final term is dividend payments, all paid with deposits. Taking $\Delta \to 0$, again we obtain the following law of motion:

$$dD = \left[r^D D - \left(r^L + \delta\right) L + I + C\right] dt + F_t.$$

We express this law of motion in terms of wealth, by using (27), to obtain:

$$dD = \left[r^D (\lambda - 1) - r^L\lambda + \iota\lambda + c\right] W dt + F_t. \tag{29}$$

We define the growth rate of deposits relative to net-worth:

$$\mu^D \equiv r^D (\lambda - 1) - r^L\lambda + \iota\lambda + c \text{ and } J^D = 0.$$

[60] Likewise, we may wish to consider the path of loans conditional on no Poisson events, as wells as the expected path of loans, respectively:

$$\dot{L} = (I - \delta L) \quad \dot{L} = (I - \delta L) - \varepsilon L\sigma + F.$$

Fundamental Equity. Next, we present the evolution of fundamental equity:

$$
\begin{aligned}
dW &= dL - dD \\
&= \left[\mu^L - \mu^D\right] W dt + J^L dN && (30)\\
&= \left[\underbrace{r^L\lambda - r^D(\lambda - 1)}_{\text{levered returns}} - \underbrace{c}_{\text{dividend rate}}\right] W dt - \underbrace{\varepsilon \cdot \lambda}_{\text{loss rate}} W dN. && (31)
\end{aligned}
$$

where the second line uses the laws of motion in (28) and (29). The interpretation of this expression is natural: the terms multiplying rates represent the net interest margin on the bank, which are the banks levered return; the second term are the capital gains that are accounted immediately as the bank creates an asset that can be worth more or less than a liability; the third term is the banks' dividend rate; and the final term is the loss rate, which scales with leverage.

Define the drift of the growth rate of bank equity as:

$$\mu^W \equiv r^L\lambda - r^D(\lambda - 1) - c$$

and denote the jump component of wealth as:

$$J^W \equiv -\varepsilon\lambda W = J^L W.$$

This verifies the following result.

Lemma 2 *If λ is independent of W, the model satisfies growth independence.*

Importantly, we note that the bank accumulates wealth over time, but does not make accounting profits by issuing deposits.

Zombie loans. The law of motion for zombie loans satisfies:

$$dZ = -\alpha\Delta Z - \varepsilon L_t\left(N_{t+\Delta} - N_t\right)$$

Taking $\Delta \to 0$, we obtain the following law of motion:

$$dZ = -\alpha Z dt + \varepsilon L dN = -\alpha z W dt + \varepsilon\lambda W dN. \quad (32)$$

Thus,

$$\mu^Z \equiv -\alpha Z \textbf{ and } J^Z \equiv -\varepsilon\lambda W = J^L W.$$

Book Loans. The law of motion for book loans satisfies:

$$\bar{L}_{t+\Delta} = (1 - \delta\Delta) L_t + I_t\Delta - \alpha\Delta\left(\bar{L}_t - L_t\right) - \varepsilon L_t\left(N_{t+\Delta} - N_t\right) + F_t,$$

with the interpretation that the first term represents how book loans fall as the principal of fundamental loans gets repaid; the second term increases book loans by newly issued loans; the third term decreases book loans at the speed of loan loss recognition α times the gap in the book versus fundamental loans; and the final term is the fraction of losses recognized in books upon receiving a default shock. Taking $\Delta \to 0$, we obtain the following law of motion:

$$d\bar{L} = (-\delta L + I)\, dt - \alpha\left(\bar{L} - L\right) dt + F_t.$$

We express this law of motion, by using (26), in terms of wealth to obtain:

$$d\bar{L} = [\iota\lambda - \alpha z]\, W dt + F_t. \tag{33}$$

We define the growth rate of book loans and the jump relative to net-worth accordingly:

$$\mu^{\bar{L}} \equiv \iota\lambda - \alpha z.$$

Book Equity. Book equity is the difference between book loans and deposits:

$$\bar{W}_t = \bar{L}_t - D_t.$$

Thus, the differential is:

$$d\bar{W}_t = \left(r^L\lambda - r^D(\lambda - 1) - c - \alpha z\right) W dt,$$

whereby

$$\mu^{\bar{W}} \equiv \left(r^L\lambda - r^D(\lambda - 1) - c - \alpha z\right).$$

Law of motion of zombie ratio. Employing the formula for the differential of a ratio we get:

$$\begin{aligned} \mu^z &\equiv z\left(\frac{\mu^Z W}{Z} - \frac{\mu^W W}{W}\right) \\ &= z\left(\frac{-\alpha z W}{Z} - \frac{\mu^W W}{W}\right) \\ &= -z\left(\alpha + \mu^W\right). \end{aligned} \tag{34}$$

Next, we derive the two possible jumps for z.

Upon a default event, we have that:

$$J^z \equiv \frac{Z + \varepsilon L}{W - \varepsilon L} - z = \frac{z + \varepsilon\lambda}{1 - \varepsilon\lambda} - z = \varepsilon\lambda\left(\frac{z+1}{1-\varepsilon\lambda}\right) = -J^W\left(\frac{z+1}{1-\varepsilon\lambda}\right).$$

Law of motion for q. Next, we produce the law of motion for leverage q. Recall that

$$q = \frac{W}{\bar{W}} = \frac{W}{W+Z} = \frac{1}{1+z}.$$

Thus, the continuous portion of q satisfies:

$$\mu^q \equiv \frac{1}{1+z}\cdot\frac{-\mu^z}{1+z} = \frac{\alpha z}{(1+z)^2}. \tag{35}$$

The jump upon an unrecognized default event is:

$$J^q \equiv \frac{W - \varepsilon L}{W - \varepsilon L + Z + \varepsilon L} - q = \frac{1-\varepsilon\lambda}{1+z} - \frac{1}{1+z} = -\varepsilon\lambda q = J^w q.$$

Leverage. Next, we derive the law of motion for leverage λ given a choice of ι and c, along the continuous path of bank's variables. Employing the formula for the differential of a ratio we get:

$$\begin{aligned}
\mu^{\lambda} &= \lambda\left(\frac{\mu^{L}W}{L}-\frac{\mu^{W}W}{W}\right) \\
&= \lambda\left(\frac{\iota\lambda W}{L}-\frac{\mu^{W}W}{W}\right) \\
&= \lambda\left(\iota-\mu^{W}\right).
\end{aligned} \tag{36}$$

Upon a default shock, the discontinuous jump in leverage is given by:

$$J^{\lambda}=\frac{L-\varepsilon\lambda W}{W-\varepsilon\lambda W}-\frac{L}{W}=\left(\frac{(1-\varepsilon)\cdot\lambda}{1-\varepsilon\lambda}-\lambda\right)=\varepsilon\lambda\cdot\frac{\lambda-1}{1-\varepsilon\lambda}.$$

Therefore, combining the drift and jump portions of the law of motion, we obtain:

$$d\lambda=\left(\iota-\mu^{W}\right)\lambda dt-\varepsilon\lambda\cdot\frac{\lambda-1}{1-\varepsilon\lambda}dN. \tag{37}$$

The interpretation of this law of motion is that leverage increases with the issuance rate, falls as loans mature and falls as the bank makes earns income on its current portfolio, μ^{W}. We thus have:

$$\mu^{\lambda}=\left(\iota-\mu^{W}\right)\lambda,$$

and for the jump term, we obtain

$$J^{\lambda}=\varepsilon\lambda\cdot\frac{\lambda-1}{1-\varepsilon\lambda}=-J^{W}\frac{\lambda-1}{1-\varepsilon\lambda}.$$

Naturally, leverage jumps with defaults, and more so the more levered the bank is.

Next, immediately after a jump in leverage, the bank sells a number of loans F.

Book Leverage. Next, we produce the law of motion for book leverage $\bar{\lambda}$. Recall that

$$\bar{\lambda}=\frac{\bar{L}}{\bar{W}}=\frac{L+Z}{W+Z}=\frac{\lambda+z}{1+z}.$$

Thus, the continuous portion of $\bar{\lambda}$ satisfies:

$$\mu^{\bar{\lambda}}\equiv\bar{\lambda}\left(\frac{\mu^{\lambda}}{\lambda+z}+\mu^{z}\left(\frac{1}{\lambda+z}-\frac{1}{1+z}\right)\right)=\frac{1}{1+z}\left(\mu^{\lambda}+\mu^{z}\frac{1-\lambda}{\lambda+z}\right). \tag{38}$$

The jump upon an unrecognized default event is:

$$J^{\bar{\lambda}}\equiv\frac{L-\varepsilon L+Z+\varepsilon L}{W-\varepsilon L+Z+\varepsilon L}-\bar{\lambda}=\frac{\lambda+z}{1+z}-\bar{\lambda}=0.$$

D.5 In Detail: Solution Under Immediate Accounting

Recall Proposition 1 in the body of the text. Figure D.2 depicts the objective function $\Omega(\lambda)$ in (11), as a function of λ. The figure illustrates the trade-off between levered returns and liquidation risk. Notice that $\Omega(\lambda)$ displays three segments: (I) In the *liquidation region* (i.e., $\lambda > \kappa$) the bank is immediately liquidated—the value of $\Omega(\lambda)$ is zero. (II) In the *solvency region,* leverage is lower than the shadow-boundary leverage, (i.e., $\lambda \in [1, \Lambda]$). The bank effectively circumvents liquidation risk, as its leverage remains below the liquidation threshold, even in the event of a default. (III) In the *liquidation-risk region* (i.e., $\lambda \in (\Lambda, \kappa]$), the bank is liquidated once a loan default event occurs.

Panel a) shows $\Omega(\lambda)$ maximized at the shadow boundary, while Panel b) depicts its maximization at the liquidation boundary. The same pattern is true with delayed accounting.

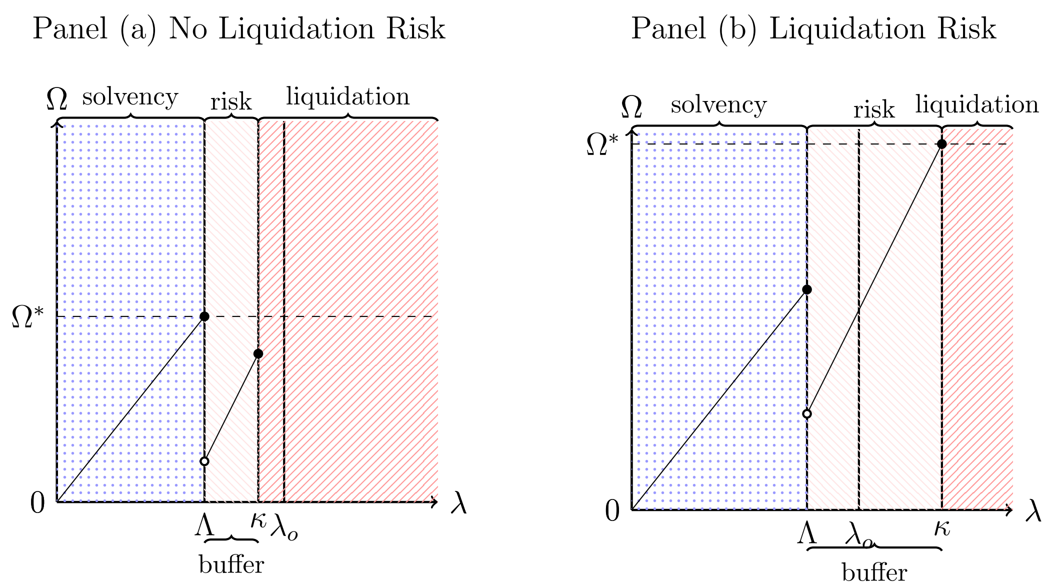


Figure D.2: Return vs. Liquidation Risk Tradeoff

Notes: The two panels plot the value of the bank's objective for different values of λ for the case with immediate loan loss recognition for different values of the fundamental leverage constraint, κ—under $\kappa < \Xi$. In the left panel the bank prefers to set leverage at the shadow boundary and not risk liquidation. In the right panel, the bank risks liquidation.

In the solvency region, both the levered return, expressed as $\left(r^L - r^D\right)\lambda$, and the default losses, denoted as $-\sigma\varepsilon\lambda$, are proportional to leverage. According to Assumption 1, the expected levered return is positive, leading to an increase in the objective function within this segment. In contrast, in the liquidation risk region, while the levered return $\left(r^L - r^D\right)\lambda$ continues to be linearly related to leverage, loan losses no longer influence the bank's objective, resulting in an increased slope of the objective function. However, the fixed expected liquidation cost, represented by $\sigma\left(\frac{v^o}{v^*} - 1\right)$, is deducted from the objective function. When leverage transitions from the solvency region to the liquidation risk region, there is a discontinuous drop in the objective, reflecting the bank's anticipated liquidation cost. Consequently, $\Omega(\lambda)$ exhibits two local maxima: one at the shadow boundary, Λ, and another at the liquidation boundary, κ. Therefore, the leverage that maximizes expected returns, Ω^*, is situated either at the shadow boundary or at the liquidation boundary

$$\Omega^* = r^D + \max\left\{ \overbrace{\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right)}^{\text{liquidation boundary}}, \quad \overbrace{\Lambda\left(\left(r^L - r^D\right) - \varepsilon\sigma\right)}^{\text{shadow boundary}} \right\},$$

as noted in the body of the text. Figure D.2 illustrates the two scenarios: Panel a) shows $\Omega(\lambda)$ maximized at the shadow boundary, while Panel b) depicts its maximization at the liquidation boundary. The same pattern is true with delayed accounting. Corollary 1 dictates parametric conditions under which one case is the optimal solution.

D.6 In Detail: Market-based Liquidations and Insolvency

Next, we explain the connection between insolvency and liquidations. The bank is solvent after a default shock if:

$$(L - \varepsilon L) - D \geq 0 \rightarrow (1 - \varepsilon)\lambda - (\lambda - 1) \geq 0.$$

Or, re-arranging:

$$\lambda \leq 1/\varepsilon.$$

The bank satisfies the market-based constraint and, thus, avoids liquidation after the shock if:

$$\lambda \leq \frac{1}{\kappa^{-1} + \varepsilon\left(1 - \kappa^{-1}\right)} \equiv \Lambda < \kappa.$$

If $\kappa \rightarrow \infty$, then, the condition es equivalent to a solvency condition, $\lambda \leq 1/\varepsilon$

We have the following relations:

- if $\kappa \rightarrow \infty$, the shadow boundary is equivalent to a solvency condition after a shock
- for finite values of κ, the bank is solvent if banks stay at the shadow boundary
- if κ is set below $1/\varepsilon$, the bank may be liquidated while always being solvent
- if κ is set above $1/\varepsilon$, and λ is chosen above $1/\varepsilon$, the bank is insolvent after the first shock
- if $\kappa = 1/\varepsilon$, the shadow boundary is 1. In that case, the bank is always liquidated after the first shock if it levered.

D.7 In Detail: Epstein-Zin Preferences

In the quantitative sections, we use Duffie-Epstein preferences. Under these preferences:

$$V_t = \mathbb{E}_t \left[\int_t^{\infty} f\left(C_s, V_s\right) ds \right],$$

where the f is given by:

$$\begin{aligned} f\left(C, V\right) &\equiv \frac{\rho}{1-\theta} \left[\frac{C^{1-\theta} - \left\{\left(1-\psi\right) V + \psi\right\}^{\frac{1-\theta}{1-\psi}}}{\left\{\left(1-\psi\right) V + \psi\right\}^{\frac{1-\theta}{1-\psi}-1}} \right] \\ &= \frac{\rho}{1-\theta} \left\{\left(1-\psi\right) V + \psi\right\} \left[\frac{C^{1-\theta}}{\left\{\left(1-\psi\right) V + \psi\right\}^{\frac{1-\theta}{1-\psi}}} - 1 \right]. \end{aligned}$$

In this representation, V_t stands for flow utility. We have some limits of interest.

CRRA Case. Consider the following limit:

$$\lim_{\psi \to \theta} \frac{\rho}{1-\theta} \left[\frac{C^{1-\theta} - \left\{\left(1-\psi\right) V + \psi\right\}^{\frac{1-\theta}{1-\psi}}}{\left\{\left(1-\psi\right) V + \psi\right\}^{\frac{1-\theta}{1-\psi}-1}} \right] = \rho \frac{C^{1-\theta} - \theta}{1-\theta} - \rho V.$$

Thus, the HJB equation (54) becomes:

$$\rho V = \rho \frac{C^{1-\theta} - \theta}{1-\theta} + V_W \mu^W + V_Z \mu^Z + \theta J^V.$$

This formulation is consistent with standard time-separable utility of the integral:

$$V_t = \rho \mathbb{E}_t \left[\int_t^{\infty} \exp\left(-\rho s\right) \frac{C^{1-\theta} - \theta}{1-\theta} ds \right],$$

the standard representation of utility expressed in flows.

Smoothed Dividends and Risk Neutrality Case. First, the limit as risk-aversion vanishes:

$$\lim_{\psi \to 0} f\left(C, V\right) = \frac{\rho}{1-\theta} V \left[\frac{C^{1-\theta}}{V^{1-\theta}} - 1 \right].$$

and for the derivative with respect to dividends, we obtain:

$$\lim_{\psi \to 0} f_c\left(C, V\right) = \rho C^{-\theta} V^{\theta}.$$

Thus, the HJB equation in this case is:

$$0 = f\left(C_s, V_s\right) + V_W \mu^W + V_Z \mu^Z + \theta J^V.$$

We use this representation in the quantitative analysis.

Baseline case. Consider the further limit:

$$\lim_{\psi, \theta \to 0} f\left(C, V\right) = \rho C - \rho V. \tag{39}$$

In the baseline case of the model we assume $C = cW$ is given by a constant dividend policy. Then, the (54) becomes:

$$\rho V = \rho cW + V_W \mu^W + V_Z \mu^Z + \theta J^V,$$

which is a scalar transformation of the one in the main text.

D.8 Microfoundation: Regulatory Liquidations

In the body of the paper we argue that whereas regulators cannot directly observe the amount of zombie loans held by banks, they could infer these through market prices. Moreover, we argue that during the instants where banks suffer defaults, regulators have a window of opportunity to intervene and liquidate them. Even though regulators do not observe bank accounting books in real time, market values, which would anticipate a successful intervention, could reveal the violation of regulatory constraints. In turn, if the intervention did not happen, banks would continue operating and immediately hide losses, only to reveal them later. This explains why even if regulators have the same information, they could only liquidate banks when default events make bank leverage cross the liquidation boundary. Here, we present a sequential form game, based on ideas on the costly-state verification model of Townsend (1979), which provides a micro-foundation for these features of the model.

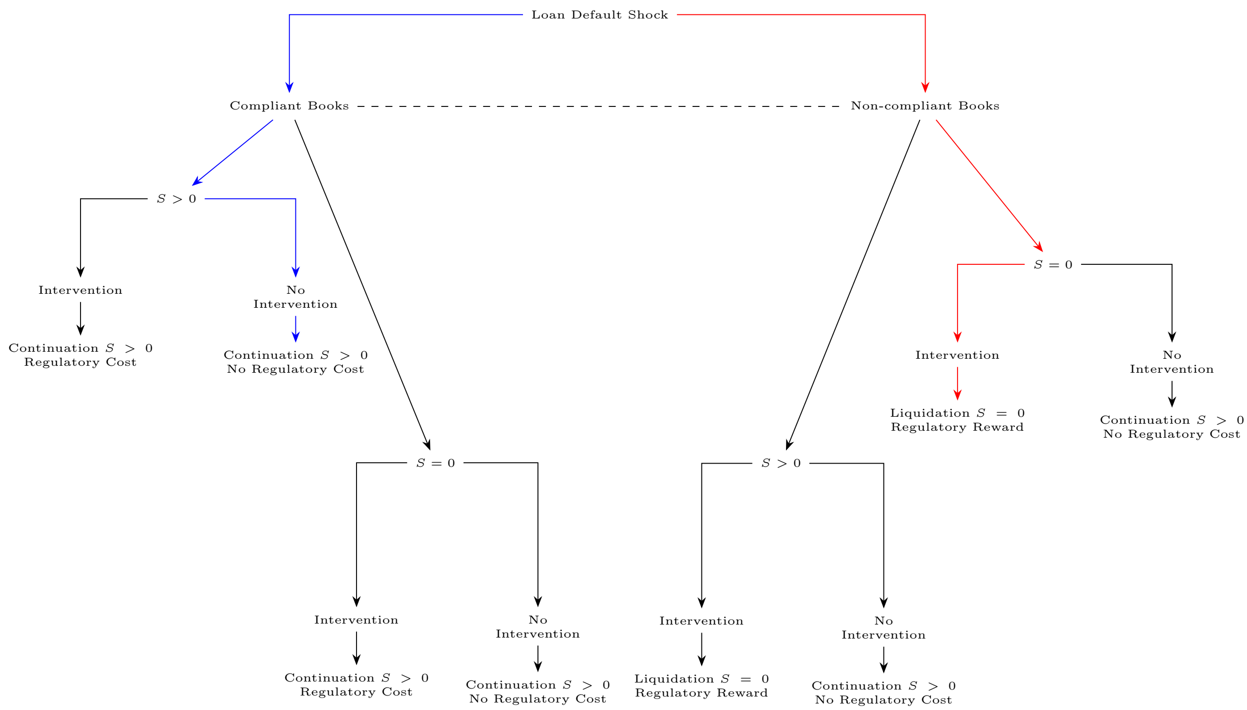


Figure D.3: Microfoundation for Regulatory Liquidations

The extensive form of this game is depicted in Figure D.3. Consider an instant of time t which unfolds starting with a loan default shock, determining the bank's condition as either maintaining regulatory compliant books or non-compliant books, considering current losses, but not past losses which are already hidden. The state of solvency is unknown to the regulator, but known to investors. The initial event sets the sequences of decisions by investors and the regulator, leading to distinct outcomes.

Following the shock, investors evaluate the bank's equity value, reflecting rational expectations regarding the continuation of the bank:

- $S > 0$: if investors assign a positive value to the bank's equity, anticipating the regulator's actions and the outcome of those actions.

- $S = 0$: Investors consider the bank's equity valueless, anticipating the regulators actions and the outcome of those actions.

Then, the regulator decides to intervene, conditioning on whether the price is positive or not:

- Intervention: the bank is intervened and, based on its books, the bank continues to operate only if it is solvent in its books.
- No Intervention: the regulator decides against taking any action, and the bank continues to operate regardless of its state.

The game's outcomes and payoff depend on the state of the bank and the actions of the regulator.

- Intervention of a compliant bank: The regulator incurs a cost for intervening. Investors do not loose anything.
- Intervention of an non compliant bank: The bank is liquidated and the regulator receives a reward for taking timely action to mitigate further financial system risks. Investors get a value of zero and the price jumps to zero.
- No Intervention: the regulator does not gain or lose anything. The value of the bank remains positive.

We consider sub-game perfect equilibria of this game. The color-coded path represents one sub-game perfect equilibrium within the context of the game. Specifically, the path depicted in blue is a sub-game perfect equilibrium consistent with rational prices. If the bank is solvent the price is positive, signaling that a regulatory intervention will only lead to a cost to the regulator. The continuity of the bank guarantees that a positive price is indeed rational.

Likewise, the path colored in red is also a sub-game perfect equilibrium, delineating the optimal course of action in the event the bank is revealed to have insolvent books by low prices. In anticipation of a regulatory intervention, the price jumps to zero, thereby revealing to regulators the possibility of a successful intervention.

We note that another sub-game perfect equilibrium occurs when the price is always positive, revealing no information, and leaving the regulator oblivious. We do not consider this case, because, as we noted, it is inconsistent with a regulatory buffer.

E Proofs and Derivations

E.1 Proof of Lemma 1

This section derives the liquidation and shadow boundaries.

Notation. To simplify notation, let τ define the instant of a jump in X_t. We use:

$$X_{\tau^-} = \lim_{t \to \tau^-} X_t \quad \text{and} \quad X_{\tau^+} = \lim_{t \to \tau^+} X_t,$$

and X_τ is the value of the variable at the moment of the jump.

Liquidation Boundary. The regulatory constraint is

$$\bar{L}_t \leq \Xi \cdot \bar{W}_t \Leftrightarrow \bar{\lambda}_t \leq \Xi \quad , \forall t, \tag{40}$$

as we noted in the main body of the text. We express the regulatory capital requirement in terms of the pair $\{\lambda, z\}$. Recall that:

$$\bar{\lambda} \equiv \frac{\bar{L}}{\bar{W}} = \frac{\lambda W + Z}{W + Z} = \frac{\lambda + z}{1 + z}. \tag{41}$$

Combining (41) with (40), we obtain:

$$\frac{\lambda_t + z_t}{1 + z_t} \leq \Xi \Rightarrow \lambda_t \leq \Xi + (\Xi - 1)\, z_t. \tag{42}$$

Next, consider the market-based constraint:

$$\lambda_t \leq \kappa. \tag{43}$$

Thus, combining (42) and (43), we obtain:

$$\lambda \leq \Gamma(z) = \min\{\kappa, \Xi + (\Xi - 1)\, z\}. \tag{44}$$

Hence, the liquidation boundary can be written independent of W. The market based constraint is tighter than the regulatory constraint whenever:

$$\kappa \leq \Xi + (\Xi - 1)\, z \to z \geq \frac{\kappa - \Xi}{\Xi - 1}.$$

Thus, for any

$$z_t \geq z^{\ell} \equiv \frac{\kappa - \Xi}{\Xi - 1},$$

the market based constraint is tighter.

Shadow Boundary. The regulatory constraint can be expressed in terms of the capital buffer:

$$X_t \equiv \Xi \cdot \bar{W}_t - \bar{L}_t \, .$$

The bank satisfies the constraint whenever:

$$X_t \geq 0, \quad \forall t.$$

Let τ be be the instant of a default. Assume that the regulator could verify the fraction β of losses on impact, i.e., it observes the losses $\beta\varepsilon L_{\tau}$.

- When $\beta = 1$, we are in the main case studied in the paper. In this case, the bank can hide losses immediately after. Zombie loans increase immediately after τ.

- When $\beta = 0$, we are in alternative case where regulators never observe losses. Zombie loans increase immediately at τ.

The capital buffer at τ is:

$$\begin{aligned} X_{\tau} &= \Xi\left(\bar{W}_{\tau} - \bar{L}_{\tau}\right) \\ &= \Xi\left(\bar{W}_{\tau^-} + \beta\varepsilon L_{\tau^-} - \left(\bar{L}_{\tau^-} + \beta\varepsilon L_{\tau^-}\right)\right). \end{aligned}$$

Next, we write the capital buffer at τ in terms of fundamental variables:

$$\begin{aligned} \frac{X_{\tau}}{W_{\tau}} &= \Xi\cdot\left(1 + z_{\tau-} + \beta\varepsilon\lambda_{\tau^-}\right) - \left(\lambda_{\tau^-} + z_{\tau-} + \beta\varepsilon\lambda_{\tau^-}\right) \rightarrow \\ &= \Xi + \left(\Xi - 1\right) z_{\tau^-} - \lambda_{\tau^-} - \left(\Xi - 1\right)\beta\varepsilon\lambda_{\tau^-}. \end{aligned}$$

The bank can guarantee solvency at τ if $X_{\tau}/W_{\tau} \geq 0$. Hence, for any t, the bank avoids liquidation after a shock as long as:

$$0 \leq \Xi + \left(\Xi - 1\right) z_{\tau^-} - \left(1 + \left(\Xi - 1\right)\beta\varepsilon\right)\lambda_{\tau^-}.$$

Re-arranging, this implies that the bank remains solvent if

$$\lambda_{\tau^-} \leq \frac{\Xi + \left(\Xi - 1\right) z_{\tau^-}}{1 + \left(\Xi - 1\right)\beta\varepsilon}. \tag{45}$$

Setting $\beta = 1$ delivers the condition in the body of the paper. Next, we derive the shadow boundary, for $\beta \in \{0, 1\}$.

Shadow Boundary in Case $\beta = 1$. The bank satisfies the market-based constraint at τ if:

$$\lambda_{\tau^-} + J^{\lambda}\left(\lambda_{\tau^-}\right) = \lambda_{\tau^-}\frac{\left(1 - \varepsilon\right)}{1 - \varepsilon\lambda_{\tau^-}} < \kappa.$$

Thus, the is survives the shock if:

$$\lambda_{\tau^-} \leq \frac{\kappa}{1 - \varepsilon + \varepsilon\kappa}. \tag{46}$$

Combining (45), for $\beta = 1$, and (46), we obtain the shadow boundary; the bank survives a default event as long as

$$\lambda \leq \Lambda\left(z\right) \equiv \min\left\{\frac{\kappa}{1 - \varepsilon + \varepsilon\kappa}, \frac{\Xi + \left(\Xi - 1\right) z}{1 - \varepsilon + \Xi\varepsilon}\right\},$$

for z being the zombie ratio at the instant of a default.

In fact, the shadow boundary can be written as the pair $\{z, \lambda\}$ satisfying:

$$\lambda + J^{\lambda}\left(\lambda\right) = \min\left\{\kappa, \Xi + \left(\Xi - 1\right) J^{z}\left(z, \lambda\right)\right\}.$$

Recall that the value of $J^{z}\left(z, \lambda\right) = z\frac{1}{1 - \varepsilon\lambda}$ and the value of $\lambda + J^{\lambda}\left(\lambda\right) = \lambda\frac{(1-\varepsilon)}{1-\varepsilon\lambda}$. For values of

$\kappa = \Xi + (\Xi - 1) J^z(z, \lambda)$, we have that:

$$\lambda \frac{(1-\varepsilon)}{1-\varepsilon\lambda} = \Xi + (\Xi - 1) \frac{z}{1-\varepsilon\lambda} \rightarrow \lambda = \frac{\Xi + (\Xi - 1) z}{1 + (\Xi - 1) \varepsilon},$$

thus verifying the case where the bank avoids liquidation under the regulatory constraint with $\beta = 1$.

Next, we compute the values of z such that the market-based liquidation boundary is smaller than the regulatory counterpart. This value is

$$z^s \equiv \frac{1-\varepsilon}{1-\varepsilon+\varepsilon\kappa} \times \frac{\kappa - \Xi}{\Xi - 1} = \frac{1-\varepsilon}{1-\varepsilon+\varepsilon\kappa} \times z^m.$$

and satisfies:

$$\frac{\Xi + (\Xi - 1) z^s}{1-\varepsilon+\varepsilon\Xi} = \frac{\kappa}{1-\varepsilon+\varepsilon\kappa}.$$

Hence, we have that the shadow boundary is:

$$\Lambda(z) = \begin{cases} \frac{\kappa}{1-\varepsilon+\varepsilon\kappa} & \text{if } z > z^s \\ \frac{\Xi + (\Xi-1)z}{1-\varepsilon+\varepsilon\Xi} & \text{if } z \leq z^s. \end{cases}$$

We verify the continuity of the shadow boundary at z^s:

$$\Lambda(z^s) = \frac{\Xi + (\Xi - 1) z^s}{1-\varepsilon+\Xi\varepsilon} = \frac{\Xi + \frac{\kappa - \Xi}{1-\varepsilon+\varepsilon\kappa}}{1-\varepsilon+\Xi\varepsilon} = \frac{\Xi + \frac{\kappa-\Xi-\varepsilon(\kappa-\Xi)}{1-\varepsilon+\varepsilon\kappa}}{1-\varepsilon+\Xi\varepsilon} = \frac{\frac{\Xi(\varepsilon\kappa)+\kappa-\varepsilon\kappa}{1-\varepsilon+\varepsilon\kappa}}{1-\varepsilon+\Xi\varepsilon} = \frac{\kappa}{1-\varepsilon+\varepsilon\kappa},$$

which equals the value at the portion $z > z^s$.

Shadow Boundary in Case $\beta = 0$. Next, we solve the case when regulators are oblivious to any information. Consider the set of values of $\{\lambda, z\}$ that guarantee the bank's solvency after a default event. We show next that these set of values satisfy

$$\lambda + J^\lambda(\lambda) \quad \leq \min\left\{\kappa,\ \Xi + (\Xi - 1) \cdot (z + J^z(z, \lambda))\right\},$$

guarantee solvency after the shock. Recall that the value of $z + J^z(z, \lambda) = \frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}$ and the value of $\lambda + J^\lambda(\lambda) = \lambda \frac{(1-\varepsilon)}{1-\varepsilon\lambda}$. For values of $\kappa \geq \Xi + (\Xi - 1)(z + J^z(z, \lambda))$, we have that:

$$\lambda \frac{(1-\varepsilon)}{1-\varepsilon\lambda} \leq \Xi + (\Xi - 1) \frac{z + \varepsilon\lambda}{1-\varepsilon\lambda} \rightarrow \lambda \leq \Xi + (\Xi - 1) z,$$

thus verifying the case where the bank avoids liquidation under the regulatory constraint with $\beta = 0$. For values of $\kappa < \Xi + (\Xi - 1) J^z(z, \lambda)$, we have that:

$$\lambda \leq \frac{\kappa}{1-\varepsilon+\varepsilon\kappa}.$$

This case corresponds to the market based constraint. Thus, any $\lambda \leq \Lambda(z) \equiv \min\left\{\frac{\kappa}{1-\varepsilon+\varepsilon\kappa}, \Xi + (\Xi - 1) z\right\}$ survives a default event.

Next, we compute the values of z such that the market-based liquidation boundary is smaller than the regulatory counterpart. That is, the values of z such that:

$$\kappa < \Xi + (\Xi - 1)(z + J^z(z, \Xi + (\Xi - 1) z)).$$

In this case, we obtain:

$$\kappa < \Xi + (\Xi - 1) \frac{z + \varepsilon (\Xi + (\Xi - 1) z)}{1 - \varepsilon (\Xi + (\Xi - 1) z)}.$$

Define $x(z) \equiv \varepsilon (\Xi + (\Xi - 1) z)$, thus:

$$\begin{aligned}
\kappa &< \Xi + (\Xi - 1) \frac{z + x(z)}{1 - x(z)} \rightarrow \\
\kappa &< \frac{\Xi - \Xi x(z) + (\Xi - 1) z + (\Xi - 1) x(z)}{1 - x(z)} \rightarrow \\
\kappa &< \frac{\Xi + (\Xi - 1) z - x(z)}{1 - x(z)}. \\
\kappa &< \frac{x(z)/\varepsilon - x(z)}{1 - x(z)}
\end{aligned}$$

Re-arranging terms, we obtain:

$$\begin{aligned}
\frac{\kappa\varepsilon}{1 - \varepsilon} &< \frac{x(z)}{1 - x(z)} \rightarrow \\
x(z) &> \frac{\kappa\varepsilon}{1 - \varepsilon + \kappa\varepsilon}.
\end{aligned}$$

Replacing $x(z)$, we obtain:

$$\begin{aligned}
(\Xi - 1) z &> \frac{\kappa}{1 - \varepsilon + \kappa\varepsilon} - \Xi \rightarrow \\
z &> \frac{1}{(\Xi - 1)} \frac{\kappa - \Xi (1 - \varepsilon + \kappa\varepsilon)}{1 - \varepsilon + \kappa\varepsilon}.
\end{aligned}$$

Thus, for any $z > z^s$, the market-based liquidation is tighter, where:

$$z^s \equiv \frac{1}{\Xi - 1} \frac{\kappa - \Xi (1 - \varepsilon + \kappa\varepsilon)}{1 - \varepsilon + \kappa\varepsilon}.$$

Hence, we have that the shadow boundary is:

$$\Lambda(z) = \begin{cases} \frac{\kappa}{1 - \varepsilon + \varepsilon\kappa} & \text{if } z > z^s \\ \Xi + (\Xi - 1) z & \text{if } z \leq z^s. \end{cases}$$

We verify the continuity of the shadow boundary at z^s:

$$\Lambda(z^s) = \Xi + (\Xi - 1) z^s = \frac{\kappa}{(1 + (\kappa - 1) \varepsilon)} = \frac{\kappa}{1 - \varepsilon + \varepsilon\kappa},$$

which equals the value at the portion $z > z^s$.

Any point at the shadow boundary of the regulatory constraint, either jumps to another point at the shadow boundary or to the market based constraint. This condition is intuitive: since the regulator never observes a loss when $\beta = 0$, any shock that starts at the regulatory limit should puts the bank at the regulator limit because neither book loans nor book equity change with the shock.

E.2 Immediate Accounting Characterization: Proof of Propositions 1 and Corollary 1

The proof of the first part of Proposition 1 follows a special case of the proof of Proposition 2 which, in turn, is further proved for general preferences on bank dividends.

Main Result. In this Appendix we prove the following result that characterizes the solution to the bank's problem under immediate accounting. We specialize to linear objectives and exogenous dividends in the body of the paper:

Proposition 5 *[Bank's Problem] The bank's value function when $\alpha \to \infty$ is*

$$0 = \max_{\{c\}} \underbrace{f(c, v^*)}_{\text{dividend choice}} + v^* \cdot (\Omega^* - c) \tag{47}$$

where Ω^* is the maximal expected leveraged bank return,

$$\Omega^* = r^D + \underbrace{\max_{\lambda \in [1, \min\{\Xi, \kappa\}]} \underbrace{\left(r^L - r^D\right)}_{\text{levered return}} \lambda + \sigma \left\{(1 - \varepsilon\lambda) \mathbb{I}_{[\lambda \leq \Lambda]} + v^o \mathbb{I}_{[\lambda > \Lambda]} - 1\right\}}_{\text{leverage choice}}.$$

Derivation of the Main Result. Under immediate accounting, $v_z = 0$, $v_z \mu_z = 0$ in Proposition 2. If $\lambda > \kappa$, the bank is liquidated immediately and its value is $v^o < \bar{c}/\rho$, which is suboptimal. Thus, the problem in Proposition 2 simplifies to:

$$0 = \max_{\{c\}} f(c, v^*) + v^* \mu^W + \sigma \left\{v^* \left[1 + J^w \mathbb{I}_{[\lambda \leq \Lambda]}\right] + v^o \mathbb{I}_{[\lambda \in (\Lambda, \kappa]]} - 1\right\}.$$

Thus, replacing μ^W, we can write:

$$0 = \max_{\{c\}} f(c, v^*) + v^* \cdot (\Omega^* - c)$$

where:

$$\Omega^* = r^D + \max_{\lambda \in [1, \kappa]} \underbrace{\left(r^L - r^D\right)}_{\text{levered return}} \lambda + \sigma \left\{J \mathbb{I}_{[\lambda \leq \Lambda]} + \left(\frac{v^o}{v^*} - 1\right) \mathbb{I}_{[\lambda > \Lambda]}\right\}$$

and

$$J \equiv -\varepsilon\lambda.$$

Specializing to the case of constant dividends and using the limit representation in (39), the objective simplifies to:

$$f(c, v^*) \to \bar{c} - v^* \rho,$$

where $\bar{c}$ is any constant rate. This corresponds to the special case in the Propositions 1. Rearranging terms, in the constant dividend case we obtain:

$$v^* = \frac{\bar{c}}{\rho - \Omega^*}.$$

Proving Corollary 1. We prove the result first for $\kappa = \min\{\Xi, \kappa\}$ and then generalize. Since

$$r^L - r^D > \sigma\varepsilon > 0,$$

the objective is piece-wise linear in λ:

- For any $\lambda < \Lambda$, the objective in Ω^* increases with leverage linearly with slope $r^L - r^D - \sigma\varepsilon$. Thus, setting $\lambda = \Lambda$ is optimal within $\lambda \in [1, \Lambda]$.
- For any, $\lambda \in (\Lambda, \kappa]$, the objective in Ω^* increases with leverage linearly with slope $r^L - r^D$. Thus, setting $\lambda = \kappa$ is optimal within $\lambda \in (\Lambda, \kappa]$.

The objective as a negative discontinuity at $\lambda = \Lambda$. Thus, without loss of generality, the bank must choose between two values $\lambda = \{\Lambda, \kappa\}$. Thus, we obtain:

$$\Omega^* = r^D + \max\left\{ \left(r^L - r^D\right)\kappa + \sigma\left(\frac{v^o}{v^*} - 1\right), \quad \Lambda\left(\left(r^L - r^D\right) - \varepsilon\sigma\right) \right\}.$$

Setting leverage at the liquidation boundary is optimal if:

$$\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right) > \frac{\kappa}{1 + \varepsilon(\kappa - 1)}\left(r^L - r^D - \varepsilon\sigma\right).$$

This is the condition presented in the statement of Corollary 1. Next, we solve for the values of κ such that the condition above holds.

We express this inequality in terms of a quadratic equation. First, multiplying both sides by the denominator in the right:

$$\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right) + \varepsilon(\kappa - 1)\left(\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right)\right) > \kappa\left(r^L - r^D - \varepsilon\sigma\right)$$

Clearing terms:

$$-\sigma\left(1 - \frac{v^o}{v^*}\right) + \varepsilon(\kappa - 1)\left(\left(r^L - r^D\right)\kappa - \sigma\left(1 - \frac{v^o}{v^*}\right)\right) > -\varepsilon\kappa\sigma.$$

Reorganizing, this expression into its polynomial components, we obtain:

$$\kappa^2\varepsilon\left(r^L - r^D\right) + \kappa\varepsilon\left(\sigma\frac{v^o}{v^*} - \left(r^L - r^D\right)\right) - (1 - \varepsilon)\sigma\left(1 - \frac{v^o}{v^*}\right) > 0.$$

Next, we solve for the critical roots:

$$\begin{aligned}
\kappa^o &= \frac{\varepsilon\left(\left(r^L - r^D\right) - \sigma\frac{v^o}{v^*}\right) \pm \sqrt{\varepsilon^2\left(r^L - r^D - \sigma\frac{v^o}{v^*}\right)^2 + 4\varepsilon\left(r^L - r^D\right)(1 - \varepsilon)\sigma\left(1 - \frac{v^o}{v^*}\right)}}{2\varepsilon\left(r^L - r^D\right)} \\
&= \frac{1}{2}\left(1 - \frac{\sigma}{\left(r^L - r^D\right)}\frac{v^o}{v^*} \pm \sqrt{\left(1 - \frac{\sigma}{\left(r^L - r^D\right)}\frac{v^o}{v^*}\right)^2 + 4\frac{(1 - \varepsilon)}{\varepsilon}\frac{\sigma}{\left(r^L - r^D\right)}\left(1 - \frac{v^o}{v^*}\right)}\right).
\end{aligned}$$

Since the discriminant is positive, because

$$4\varepsilon\left(r^L - r^D\right)(1 - \varepsilon)\sigma\left(1 - \frac{v^o}{v^*}\right) > 0,$$

there are two real roots, one negative and, at most, one positive. Only positive solutions are valid since $\kappa > 1$. The intercept is negative while the quadratic coefficient is positive. Thus, the bank risks liquidation if $\kappa > \kappa^o$. Hence, for any

$$\kappa > \kappa^o = \frac{1}{2}\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*} + \sqrt{\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 + 4\frac{(1-\varepsilon)}{\varepsilon}\frac{\sigma}{(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right)}\right) \tag{48}$$

the bank will risk liquidation setting leverage to the shadow boundary. This yields the value of λ_0 in Corollary 1. It's value is unique.

Next, we look for the combination of parameters such that $\kappa^o \leq 1$, so that the threshold is relevant. For such set of parameters, we have that the bank risks liquidation for any level of leverage constraints. The solution κ^o is less than 1 if

$$1 + \sqrt{\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 + 4(1-\varepsilon)\frac{\sigma}{\varepsilon(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right)} - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*} \leq 2.$$

Re-arranging terms, the condition is true if:

$$\sqrt{\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 + 4(1-\varepsilon)\frac{\sigma}{\varepsilon(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right)} \leq 1 + \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}.$$

Since both sides are strictly positive, we have that:

$$\left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 + 4(1-\varepsilon)\frac{\sigma}{\varepsilon(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right) \leq \left(1 + \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2$$

$$4(1-\varepsilon)\frac{\sigma}{\varepsilon(r^L - r^D)}\left(1 - \frac{v^o}{v^*}\right) \leq \left(1 + \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 - \left(1 - \frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}\right)^2 = 4\frac{\sigma}{(r^L - r^D)}\frac{v^o}{v^*}.$$

Canceling terms, the condition becomes:

$$(1-\varepsilon)\left(1 - \frac{v^o}{v^*}\right) \leq \varepsilon\frac{v^o}{v^*} \to \varepsilon \geq \left(1 - \frac{v^o}{v^*}\right).$$

This guarantees that the threshold κ^o is greater than 1. Thus, λ^o is unique and greater than 1.

We finally generalize to the case where $\Xi = \min\{\Xi, \kappa\}$. Note that the threshold λ^o is independent of κ. The optimal leverage decision is thus:

$$\lambda = \min\{\Xi, \kappa\} \quad \text{if} \quad \min\{\Xi, \kappa\} > \lambda^o,$$

and

$$\lambda = \frac{\min\{\Xi, \kappa\}}{1 + (\min\{\Xi, \kappa\} - 1)\varepsilon} \quad \text{if} \quad \min\{\Xi, \kappa\} \leq \lambda^o.$$

E.3 Proof of Proposition 2

In this Appendix we prove the following general version of version of Proposition 2 where dividends are endogenous. We use this solution in the quantitative section. The solution is identical for the case of exogenous dividends. The general version of the proposition is the following.

Proposition 6 *[Bank's Problem] Given $\{z\}$, $V(Z,W) = v(z)W$, where v is the solution to the following HJB equation:*

$$0 = \max_{\{c,\lambda\}} f(c,v) + v_z \mu^z + v\mu^W + \sigma J^v \tag{49}$$

where J^v is the jump in the bank's value given a default event:

$$J^v = \left(\underbrace{\left(v\left(z+J^v\right)\left(1+J^W\right) - v(z)\right)}_{\text{jump in wealth}} \cdot \mathbb{I}_{[\lambda \le \Lambda(z)]} + \underbrace{[v^o - v]}_{\text{liquiditation}} \cdot \mathbb{I}_{[\lambda > \Lambda(z)]} \right).$$

The optimal policies are given by: $C(Z,W) = c(z) \cdot W$.

The market value satisfies $S(Z,W) \equiv s(z) \cdot W$, where s solves:

$$\rho^I s = c(z) + s_z \mu^z + s\mu^W + \sigma J^s, \tag{50}$$

where J^s is given by:

$$J^s = \left(\underbrace{\left(s\left(z+J^z\right)\left(1+J^W\right) - s\right)}_{\text{jump in wealth}} \mathbb{I}_{[\lambda \le \Lambda(z)]} + \underbrace{[s^o - s]}_{\text{liquidation}} \cdot \mathbb{I}_{[\lambda > \Lambda(z)]} \right)$$

Finally, Tobin's Q is given by:

$$Q(z) = s(z) \times q(z, \lambda(z)). \tag{51}$$

We can re-arrange the terms in the objective and obtain a reformulation:

Corollary 3 *[Bank's Problem] Given $\{z\}$, $V(Z,W) = v(z) \cdot W$, where v is the solution to the following HJB equation:*

$$0 = \max_{\{c\}} f(c,v) - (v - v_z z)\, c - v_z \alpha z + (v - v_z z)\, \Omega^* \tag{52}$$

where the optimal portfolio is:

$$\Omega^* = r^D + \max_{\{\lambda\}} \left(r^L - r^D\right)\lambda + \frac{J^v}{v(z) - v_z(z)},$$

and where:

$$J^v \equiv \left(\left[v\left(z+J^z\right)(1-\varepsilon\lambda) - v(z)\right] \cdot \mathbb{I}_{[\lambda \le \Lambda(z)]} + \left(v^o - v(z)\right) \cdot \mathbb{I}_{[\lambda > \Lambda(z)]}\right) W.$$

Further specializing to the version with constant dividends yields Proposition 1 in the body of the text:

Proposition 7 *[Bank's Problem] Consider the problem of the bank with exogenous dividends, c. Given $\{z\}$, $V(Z,W) = v(z) \cdot W$, where v is the solution to the following HJB equation:*

$$0 = c + (v + v_z)\,\mu_z + (v - v_z z) \max_{\lambda} \left(\mu^W + \frac{J^v}{v - v_z}\right) \tag{53}$$

where

$$J^v \equiv \left(\left[v\left(z+J^z\right)(1-\varepsilon\lambda) - v(z)\right] \cdot \mathbb{I}_{[\lambda \le \Lambda(z)]} + \left(v^o - v(z)\right) \cdot \mathbb{I}_{[\lambda > \Lambda(z)]}\right) W.$$

This condition is identical to the one that appears in the body of the paper. To prove the result, we present a formulation and then guess and verify the solution.

Formulation. Next, we prove Proposition 6. The primitive bank HJB equation, (10), is:

$$0 = \max_{\{C,\lambda\}} f\left(C, V\left(Z, W\right)\right) + \frac{E\left[dV\left(Z, W\right)\right]}{dt}. \tag{54}$$

Using a standard result in stochastic calculus for Jump processes:

$$\frac{E\left[dV\left(Z, W\right)\right]}{dt} = V_Z\left(Z, W\right)\mu^Z W + V_W\left(Z, W\right)\mu^W W + \sigma\mathbb{E}\left[J^V\right],$$

where J^V is given by:

$$J^V = \left[V\left(Z + J^Z, W + J^W\right) - V\left(Z, W\right)\right]\hat{\mathbb{I}} + \left(v^o W - V\left(Z, W\right)\right)\left(1 - \hat{\mathbb{I}}\right)$$

where

$$\hat{\mathbb{I}} = \begin{cases} 1 & \text{if } \lambda \leq \Lambda\left(Z/W\right) \\ 0 & \text{otherwise.} \end{cases}$$

Conjecture. We conjecture a solution to the value function and verify that it satisfies the HJB equation. The conjecture is:

$$V\left(Z, W\right) = v\left(z\right) W, \tag{55}$$

for a suitable candidate $v\left(z\right)$. Under this conjecture, we verify that $C\left(Z, W\right) = c\left(z\right) \cdot W$.

Factorization. We perform some useful calculations on the guess (55). In particular, we factorize equity from every term in the HJB equation. Under the conjecture,

$$\begin{aligned} f\left(C, V\right) &= f\left(c\left(z\right) W, v\left(z\right) W\right) \\ &= \frac{\rho}{1-\theta} v\left(z\right) W\left[\frac{c\left(z\right)^{1-\theta} W^{1-\theta}}{\left(v\left(z\right) W\right)^{1-\theta}} - 1\right] \\ &= \frac{\rho}{1-\theta} v\left(z\right) W\left[\frac{c\left(z\right)^{1-\theta}}{v\left(z\right)^{1-\theta}} - 1\right] \\ &= f\left(c\left(z\right), v\left(z\right)\right) W. \end{aligned} \tag{56}$$

The change in the value function with respect to zombie loans is:

$$\begin{aligned} V_Z &= \partial\left[v\left(Z/W\right) W\right]/\partial Z \\ &= v_z. \end{aligned}$$

The derivative of the value function with respect with respect to W is given by:

$$\begin{aligned} V_W &= \partial\left[v\left(Z/W\right) W\right]/\partial W \\ &= -v_z \frac{Z}{W} + v\left(z\right). \end{aligned} \tag{57}$$

Next, we collect terms to construct a modified drift for the value function:

$$\begin{aligned} V_Z \mu^Z W + V_W \mu^W W &= v_z(z) \cdot \left(-\alpha \frac{Z}{W}\right) W + \left(-v_z(z)\frac{Z}{W} + v(z)\right) \mu^W W \\ &= -v_z(z)\left(\alpha + \mu^W\right) zW + v(z) \cdot \mu^W W \\ &= \left(v_z(z)\mu^z + v(z) \cdot \mu^W\right) W. \end{aligned}$$

Finally, under the conjectured solution, the jump in the value function after an unrecognized default event is:

$$\begin{aligned} J^V &= \left[v\left(\frac{Z+J^Z}{W+J^W}\right)\left(W+J^W\right) - v(z) W\right]\hat{\mathbb{I}} + \left(v^o - v(z)\right) W \left[1-\hat{\mathbb{I}}\right] \\ &= \left[v\left(z+J^z\right)(1-\varepsilon\lambda) W - v(z) W\right]\hat{\mathbb{I}} + \left(v^o - v(z)\right)\left[1-\hat{\mathbb{I}}\right] W \\ &= \left(\left[v\left(z+J^z\right)(1-\varepsilon\lambda) - v(z)\right]\hat{\mathbb{I}} + \left(v^o - v(z)\right)\left[1-\hat{\mathbb{I}}\right]\right) W, \\ &= \left(\left[v\left(z+J^z\right)(1-\varepsilon\lambda) - v(z)\right]\hat{\mathbb{I}} + \left(v^o - v(z)\right)\left[1-\hat{\mathbb{I}}\right]\right) W. \end{aligned}$$

Verification. We verify that the conjecture satisfies its HJB equation. We need to combine the pieces together. With the factorization above, (54) can be written as:

$$\begin{aligned} 0 &= \max_{\{c,\lambda\}} f(c,v) W \ldots \\ &+ \underbrace{\left[\begin{array}{cc} v_z(z) & v(z) \end{array}\right] \times \left[\begin{array}{c} \mu^z \\ \mu^W \end{array}\right]}_{\equiv \mu^v} \cdot W \ldots \\ &+ \sigma J^V W. \end{aligned}$$

where we used the fact that any choice of C can be expressed as a choice of $c(z) W$ as there is a one to one map from the $\{z, W\}$ space to the original space—by change of coordinates. Then, we can factor wealth from this HJB equation to express it as:

$$0 = \left[\max_{\{c,\lambda\}} f(c,v) + \mu^v + J^v\right] \cdot W,$$

and since the maximization is independent of W, this verifies the conjecture. Hence we have the proof of Proposition 6.

Collecting terms the solution to the HJB equation:

$$0 = \max_{\{c\}} f(c,v) - (v - v_z z) c - \alpha v_z z + (v - v_z z)\Omega(z), \tag{58}$$

where

$$\Omega(z) = r^D + \max_{\lambda \in [1,\Gamma(z)]} \left(r^L - r^D\right)\lambda - \sigma \frac{J^v}{v - v_z z}.$$

we verify the conjecture that the formula (55) satisfies the HJB equation (49). The factorization is valid as long as $v(z) - v_z(z) z > 0$. This is true since:

$$0 < V_W = -v_z(z)\frac{Z}{W}\frac{W}{W} + v(z) = v(z) - v_z(z) z.$$

This proves Proposition 3.

Finally, when the dividend is set to a constant rate and $\theta \to 0$, the value function specializes to:

$$0 = \rho c - \rho v - (v - v_z z)\, c - \alpha v_z z + (v - v_z z)\, \Omega(z), \tag{59}$$

where

$$\Omega(z) = r^D + \max_{\lambda \in [1, \Gamma(z)]} \left(r^L - r^D\right) \lambda - \sigma \frac{J^v}{v - v_z z}.$$

Since, the factor ρ is a monotone transformation of utility, this proves Proposition 3.

E.4 Proof of Corollary 2

We derive the first-order conditions of this problem. The optimal leverage is independent of dividends. We thus optimize this problem on its own. The next steps show that the solution is indeed bang-bang.

Optimal Leverage. Assume that indeed the value function $v(z)$ is concave. Consider the optimal leverage choice given by:

$$\Omega(z) = r^D + \max_{\lambda \in [1, \Gamma(z)]} \Omega^o(z, \lambda),$$

where

$$\Omega^o(z, \lambda) \equiv r^D + \left(r^L - r^D\right) \lambda + \sigma \left\{ \frac{[v(z + J^z)(1 - \varepsilon\lambda)] - v(z)}{v(z) - v_z(z)\, z} \right\}.$$

We now investigate the solution to the optimal λ.

Region $\lambda < \Lambda(z)$. The derivate of the objective with respect to λ is:

$$\frac{\partial \Omega^o(z, \lambda)}{\partial \lambda} \equiv \left(r^L - r^D\right) + \left[\frac{(1 - \lambda\varepsilon)\, v_z(z + J^z(\lambda, z))\, J^z_\lambda(\lambda, z) - \varepsilon v(z + J^z(\lambda, z))}{v(z) - v_z(z)\, z} \right]. \tag{60}$$

Recall that the jump of the zombie ratio is:

$$J^z(\lambda, z) = \varepsilon\lambda \left(\frac{z + 1}{1 - \varepsilon\lambda} \right).$$

Therefore. the derivative of the jump with respect to λ is:

$$\begin{aligned} J^z_\lambda(\lambda, z) &= J^z(\lambda, z) \left(\frac{1}{\lambda} + \frac{\varepsilon}{1 - \lambda\varepsilon} \right) \\ &= J^z(\lambda, z) \left(\frac{1}{\lambda} \cdot \frac{1}{1 - \varepsilon\lambda} \right) \end{aligned}$$

and, thus,

$$(1 - \lambda\varepsilon)\, v_z(z + J^z(\lambda, z))\, J^z_\lambda(\lambda, z) = \varepsilon v_z(z + J^z(\lambda, z)) \frac{1}{\lambda} J^z(\lambda, z).$$

Hence, substituting this last expression into (60), we obtain:

$$\begin{aligned}\frac{\partial\Omega^{o}(z,\lambda)}{\partial\lambda} &= \left(r^{L}-r^{D}\right)-\sigma\varepsilon\left(\frac{v\left(z+J^{z}(\lambda,z)\right)-v_{z}\left(z+J^{z}(\lambda,z)\right)\frac{1}{\varepsilon\lambda}J^{z}(\lambda,z)}{v(z)-v_{z}(z)z}\right)\\ &= \left(r^{L}-r^{D}\right)-\sigma\varepsilon\left(\frac{v\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)-v_{z}\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)\left(\frac{z+1}{1-\varepsilon\lambda}\right)}{v(z)-v_{z}(z)z}\right).\end{aligned}$$

The derivative $\partial\Omega^{o}(z,\lambda)/\partial\lambda$ is positive if:

$$\frac{\left(r^{L}-r^{D}\right)}{\sigma\varepsilon}>\left(\frac{v\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)-v_{z}\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)\left(\frac{z+1}{1-\varepsilon\lambda}\right)}{v(z)-v_{z}(z)z}\right). \tag{61}$$

Recall that by assumption: $\lambda<\kappa<\frac{1}{\varepsilon}$. Thus, the term on the right of (61) satisfies:

$$\left(\frac{v\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)-v_{z}\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)\left(\frac{z+\varepsilon\lambda}{1-\varepsilon\lambda}\right)}{v(z)-v_{z}(z)z}\right)>\left(\frac{v\left(z+J^{z}(\lambda,z)\right)-v_{z}\left(z+J^{z}(\lambda,z)\right)\left(z+J^{z}(\lambda,z)\right)}{v(z)-v_{z}(z)z}\right).$$

A sufficiently condition for (61) is to show that:

$$\frac{r^{L}-r^{D}}{\sigma\varepsilon}\geq\frac{v(z')-v_{z}(z')z'}{v(z)-v_{z}(z)z} \tag{62}$$

for any pair z',z. We find bounds for the ratio of the right. Recall that:

$$v(z)-v_{z}(z)z=\frac{\partial V(1,z)W}{\partial W}=\frac{\partial V(W,Z)}{\partial W}.$$

We can bound, for any z, :

$$\frac{\partial V(W,zW)}{\partial W}\leq\lim_{\Xi\to\kappa}\frac{\partial V(W,Z')}{\partial W}=\frac{c}{\rho-\left(r^{D}+\kappa\left(r^{L}-r^{D}-\sigma\varepsilon\right)-c\right)}. \tag{63}$$

The bound follows because the marginal value of equity is higher if there is no regulation. The second equality follows directly from Proposition 1.

Next, we bound the value from below:

$$\frac{\partial V(W,Z')}{\partial W}\geq\lim_{\alpha,\Xi,\to\infty}\frac{\partial V(W,Z')}{\partial W}=\lim_{\alpha,\Xi,\to\infty}v(z)-v_{z}(z)z=\frac{c}{\rho-r^{D}-c}. \tag{64}$$

The bound follows because the marginal value of wealth is lowest when leverage is no admissible, as happens when $\alpha,\Xi\to0$. The second equality follows directly from Proposition 1.

Combining the bounds, (63) and (64),

$$\frac{\rho-r^{D}-c}{\rho-\left(r^{D}+\kappa\left(r^{L}-r^{D}-\sigma\varepsilon\right)-c\right)}\geq\frac{v(z')-v_{z}(z')z'}{v(z)-v_{z}(z)z}.$$

Thus, a sufficient condition for (62), is to show that:

$$\frac{r^{L}-r^{D}}{\sigma\varepsilon}\geq\frac{\rho-r^{D}-c}{\rho-\left(r^{D}+\kappa\left(r^{L}-r^{D}-\sigma\varepsilon\right)-c\right)}.$$

Subtracting one from both sides and cancelling terms we obtain that this solution is guaranteed as long as:

$$\rho \geq \left(r^{D} + \kappa \left(r^{L} - r^{D}\right) - c\right).$$

This condition holds by assumption. Thus, if $\lambda < \Lambda(z)$, setting $\lambda = \Lambda(z)$ increases the bank's value. The bank must be at a corner.

Region $\lambda \in (\Lambda(z), \quad \Gamma(z))$. Let $\lambda > \Lambda(z)$. The derivative of the objective is:

$$\left(r^{L} - r^{D}\right) > 0.$$

Thus, if $\lambda > \Lambda(z)$, setting $\lambda = \Gamma(z)$ increases the bank's value.

Summary. Since leverage is either at $\lambda = \Gamma(z)$ or $\lambda = \Lambda(z)$, as shown above, the solution must be bang-bang. That is:

$$\Omega(z) = r^{D} + \max\left\{\Omega^{\Gamma}(z), \quad \Omega^{\Lambda}(z)\right\},$$

where

$$\Omega^{\Gamma}(z) \equiv \left(r^{L} - r^{D}\right)\Gamma(z) + \sigma\frac{v^{o} - v(z)}{v(z) - v_{z}(z)\,z}$$

and

$$\Omega^{\Lambda}(z) \equiv \left(r^{L} - r^{D}\right)\Lambda(z) + \sigma\frac{v\left(z + J^{z}\left(\Lambda(z), z\right)\right)\left(1 - \varepsilon\Lambda(z)\right) - v(z)}{v(z) - v_{z}(z)\,z}$$

Optimal Dividend. When dividends are chosen, we have we have that the first-order condition for dividends is given by:

$$f_{c}(c, v) = v - v_{z}z,$$

we can solve this to obtain:

$$c = \rho^{1/\theta}\left[\frac{v}{\left(v - v_{z}z\right)^{1/\theta}}\right]. \tag{65}$$

For the case of log utility, $\theta \to 1$, and we verify that $c = \rho$ if v is constant, as in the case of immediate accounting.

E.5 Proof of Proposition 3: Optimal Regulation with Immediate Accounting Case

We prove the result in two steps. First, showing the solution to the first-best and then the one of the second best.

First Best. In the first best, we directly choose λ, ignoring any regulatory liquidations, $\Xi \geq \kappa$. Hence, from Proposition 2, the social optimal value of λ under immediate accounting solves:

$$r^{D} - c + \max_{\lambda}\left(\left(r^{L} - r^{D}\right)\lambda - \sigma\varepsilon\lambda \times \mathbb{I}_{[\lambda \leq \Lambda]} - \sigma\left(\varepsilon + (1 - \psi)(1 - \varepsilon)\right)\lambda \times \mathbb{I}_{[\lambda > \Lambda]}\right).$$

By assumptions 1 and 2,

$$\left(r^{L} - r^{D}\right) - \sigma\varepsilon > 0 > \left(r^{L} - r^{D}\right) - \sigma\left(\varepsilon + (1 - \psi)(1 - \varepsilon)\right).$$

Thus, the optimal leverage is increasing up to Λ and then decreasing.

$$\lambda^{fb} = \Lambda = \kappa \left(1 + \varepsilon \left(\kappa - 1\right)\right)^{-1}.$$

Second Best. Recall the bank's optimal response to regulation, equation (13):

$$\lambda^* \left(\Xi, \kappa\right) = \max\left\{\kappa, \Xi\right\} \times \mathbb{I}_{\left[\max\left\{\kappa, \Xi\right\} > \lambda^o\right]} \ldots + \max\left\{\kappa, \Xi\right\} \left(1 + \varepsilon \left(\max\left\{\kappa, \Xi\right\} - 1\right)\right)^{-1} \times \mathbb{I}_{\left[\max\left\{\kappa, \Xi\right\} \leq \lambda^o\right]},$$

where λ^o is given by (12). From Proposition 2, the social optimal value of λ under immediate accounting solves:

$$r^d - c + \max_{\Xi} \left(\left(r^L - r^D\right) - \sigma\varepsilon \times \mathbb{I}_{\left[\lambda \leq \Lambda\right]} - \sigma \left(\varepsilon + \left(1 - \psi\right)\left(1 - \varepsilon\right)\right) \times \mathbb{I}_{\left[\lambda > \Lambda\right]}\right) \lambda^* \left(\Xi, \kappa\right),$$

as described in the Proposition.

We study two cases:

Case I. $\kappa \leq \lambda^o$. If $\kappa \leq \lambda^o$,then for any $\Xi \geq \kappa$, we obtain:

$$\lambda^* \left(\Xi, \kappa\right) = \kappa \left(1 + \varepsilon \left(\kappa - 1\right)\right)^{-1} = \lambda^{fb}.$$

In this case, the first-best can be implemented with laissez faire regulation.

Case II. $\kappa > \lambda^o$. If $\kappa > \lambda^o$ then for any $\Xi \geq \kappa$, we obtain:

$$\lambda^* \left(\Xi, \kappa\right) = \kappa > \lambda^{fb},$$

and the value of the objective is:

$$r^D - c + \left(\left(r^L - r^D\right) - \sigma \left(\varepsilon + \left(1 - \psi\right)\left(1 - \varepsilon\right)\right)\right) \kappa.$$

Case II.a. $\Xi \in \left(\lambda^o, \kappa\right]$. If $\Xi \in \left(\lambda^o, \kappa\right]$, then $\lambda^* \left(\Xi, \kappa\right) = \Xi > \Lambda = \Xi \left(1 + \varepsilon \left(\Xi - 1\right)\right)^{-1}$ and the value of the regulatory constraint is:

$$r^D - c + \max_{\Xi} \left(\left(r^L - r^D\right) - \sigma \left(\varepsilon + \left(1 - \psi\right)\left(1 - \varepsilon\right)\right)\right) \Xi.$$

By Assumption 2, the objective is decreasing in Ξ. Thus, $\Xi = \lambda^o$ in this region.

Case II.b. $\Xi \in \left[0, \lambda^o\right]$. If $\Xi \in \left[0, \lambda^o\right]$, then $\lambda^* \left(\Xi, \kappa\right) = \Lambda = \Xi \left(1 + \varepsilon \left(\Xi - 1\right)\right)^{-1}$, the value of the regulatory constraint is:

$$r^D - c + \max_{\Xi} \left(\left(r^L - r^D\right) - \sigma\varepsilon\right) \Xi \left(1 + \varepsilon \left(\Xi - 1\right)\right)^{-1}.$$

By Assumption 2, the objective is increasing. Thus, within the range $\Xi \in \left[0, \kappa\right]$, the local maximum is $\Xi = \lambda^o$.

There is a discontinuity at λ^o. We must chose between setting Ξ to the right or to the left of λ^o. We set it to the left:

$$\left(\left(r^L - r^D\right) - \sigma\varepsilon\right) \lambda^o \left(1 + \varepsilon \left(\lambda^o - 1\right)\right)^{-1} \geq \left(\left(r^L - r^D\right) - \sigma \left(\varepsilon + \left(1 - \psi\right)\left(1 - \varepsilon\right)\right)\right) \lambda^o,$$

The right-hand side is negative By Assumption 2. Hence, in a second best, we have that $\Xi = \lambda^o$.

Summary. We conclude that when $\kappa > \lambda^o$, regulation is needed and is second best as the social value is:

$$\Omega^{sb} = r^D - c + \left(\left(r^L - r^D\right) - \sigma\varepsilon\right)\lambda^o \left(1 + \varepsilon\left(\lambda^o - 1\right)\right)^{-1} < r^d - c + \left(\left(r^L - r^D\right) - \sigma\varepsilon\right)\lambda^o = \Omega^{fb}.$$

When $\kappa \leq \lambda^o$, regulation is not necessary and $\Xi = \kappa$ achieves the first best Ω^{fb}.

E.6 Model Version with Loan Adjustment Costs

In this section, we derive a version of the model with adjustment costs on loans,

$$\Phi(I, L) = I + \frac{\gamma}{2}\left(\frac{I}{L} - \delta\right)^2 L.$$

We can factor out L and employing the definition of ι to obtain:

$$\begin{aligned}\Phi(I, L) &= \left(\iota + \delta + \frac{\gamma}{2}\iota^2\right) L \\ &= \Phi(\iota, 1) L + \delta L.\end{aligned}$$

Thus, we can express the funding cost relative to equity as:

$$\Phi(I, L)/W = (\Phi(\iota, 1) + \delta)\lambda, \tag{66}$$

which is a function independent of the bank's size and depends on leverage and the investment rate.

Observation 1: Derivations of Laws of Motion. Now consider a time interval of length Δ. The law of motion for fundamental loans satisfies:

$$L_{t+\Delta} = (1 - \delta\Delta) L_t + I_t\Delta - \varepsilon L_t (N_{t+\Delta} - N_t),$$

with the interpretation that the first term is the non-maturing fraction of loans, the second are loan issuances, and the third are losses in a time interval. Taking $\Delta \to 0$, we obtain the following law of motion:

$$dL = (I - \delta L) dt - \varepsilon L dN.$$

We express this law of motion in terms of net-worth to obtain:

$$dL = \iota\lambda W dt - \varepsilon\lambda W dN. \tag{67}$$

To ease the notation, we define the growth rate of fundamental loans and the jump relative to net-worth:

$$\mu^L \equiv \iota\lambda \text{ and } J^L \equiv -\varepsilon\lambda.$$

Similarly, for deposits we have that:

$$D_{t+\Delta} = \left(1 + r^D\Delta\right) D_t - \left(r^L\Delta + \delta\Delta\right) L_t + \Phi(I_t, L_t)\Delta + C_t\Delta$$

with the interpretation that the first term is the increase in deposits that results from paying interest with deposits; the second term is the reduction in deposits by the interest and principal payments on outstanding loans; the third term is the increase in deposits as a result of loan issuances; and the final term is dividend payments, all paid with deposits. Taking $\Delta \to 0$, we obtain the following law of motion:

$$dD = \left[r^D D - \left(r^L + \delta\right) L + \Phi(I, L) + C\right] dt.$$

We express this law of motion in terms of wealth to obtain:

$$dD = \left[r^D(\lambda - 1) - \left(r^L + \delta\right)\lambda + (\Phi(\iota, 1) + \delta)\lambda + c\right] W dt. \tag{68}$$

We define the growth rate of deposits relative to net-worth:

$$\mu^D \equiv r^D(\lambda - 1) - \left(r^L + \delta\right)\lambda + (\Phi(\iota, 1) + \delta)\lambda + c.$$

The evolution of Z is identical.

Observation 2: growth independence. Next, we present the evolution of net-worth with adjustment costs:

$$\begin{aligned} dW &= dL - dD \\ &= \left[\underbrace{\left(r^L + \delta\right)\lambda - r^D(\lambda - 1)}_{\text{levered returns}} + \underbrace{(\iota - (\Phi(\iota, 1) + \delta))\lambda}_{\text{capital loss from adjustment}} - \underbrace{c}_{\text{dividend rate}}\right] W dt \\ &= \underbrace{-\varepsilon\lambda}_{\text{loss rate}} \; W dN. \end{aligned} \tag{69}$$

where the second line uses the laws of motion in (67) and (68), and employed observation 1.

F Model Appendix: Parametrization

This appendix section describes our calibration and estimation procedures in Section 4 in more detail. We use quarterly data from 1990 Q3 to 2021 Q1 to produce the target moments. Model moments are therefore also at quarterly frequency. To produce model moment counterparts, for each parameter draw, we simulate a panel of 10,000 banks for the same number of quarters as the data. Each sample simulation starts from the model's analytic stationary distribution, which we obtain by solving the Kolmogorov-Forward equation. From each sample, we calculate the cross-sectional average moments and construct the impulse-response functions (IRF).

F.1 Matching Facts

We estimate a more flexible version of the model presented in Section 3, by allowing for an endogenous dividend rate choice and assuming Duffie-Epstein preferences (rather than linear preferences) for the banker. The banker's risk aversion parameter is denoted as ψ and the intertemporal elasticity of substitution as $1/\theta$. To keep the parametrization tractable, we calibrate $\left\{r^L, r^D, \Xi\right\}$ independently, matching model moments to target moments in the data. Then, conditional on these calibrated parameters, we jointly estimate $\left\{\rho, \rho^I, \theta, \varepsilon, \alpha, \kappa\right\}$ using simulated method of moments together with the calibration of σ and v_0. The parameter values are listed in Table 1 in the main text. Table 8 presents both the targeted and untargeted moments in the data and the corresponding model moment.

Calibrated parameters. The exogenous returns on loans and deposits, r^L and r^D, are respectively set to 1.01% and 0.51%, consistent with the quarterly yield on loans (total interest income on loans divided by total loans) and the rate banks pay on their debt (total interest expenses divided by interest-bearing liabilities) in bank call reports. These values are also consistent with the calibration in Corbae and D'Erasmo (2021). We set the capital requirement parameter Ξ to 12.5, reflecting a Tier-1 risk-based capital ratio requirement of 8% at which a bank is considered well capitalized.[61]

Model Fit and Interpretation. Table 8 compares the moments generated by the model and those obtained from the data: our model fits most data moments well, with the exceptions of log market returns (which in the data includes other aggregate factors), the growth rate of book equity, and the common dividend rate. The model fits market leverage (8.596 in the data vs 8.274 in the model), book leverage (11.361 in the data vs 11.098 in the model), the log market return (2% in the data and 3.4% in the model), the market to book equity ratio (exact fit at 1.316), and the net charge-off rate (0.1% in the model vs 0.1% in the data) very tightly. Note that the capital requirement constraint limits banks' book leverage ratio to at most 12.5. Hence, in our model banks keep an equity buffer over the capital requirement constraint as in the data. [62] The model overshoots the dividend rate (0.6% in the data vs 3% in the model) and undershoots the growth rate of book equity (2% in the data vs. 0.4% in the model). In the data, banks can also repurchase shares to return cash to their shareholders, leading to a higher dividend rate in the model.

Table 8 presents unobservable model variables, such as fundamental leverage λ and q (fundamental equity/accounting value of equity). Fundamental leverage is 16.5, substantially higher than the book leverage value of 11.1. The average value for $q = W/\bar{W}$ is 0.69, implying that the fundamental value and the accounting value of equity differ by 31 percent. In terms of loans, zombie loans represent 3% of the total loans of banks.

[61] See the Federal Reserve Supervision and Regulation Report of November 2018, available here.

[62] In Appendix G, Figure G.1 presents the stationary distribution of fundamental leverage λ and the zombie loan to equity ratio z together with the liquidation set. It also shows that banks keep an equity buffer over the liquidation boundary determined by the regulatory constraint.

Table 8: Model and Data Moments

	Data	Model
Log Market Returns	0.020	0.034
	(0.176)	(0.032)
Market leverage	8.596	8.274
	(0.590)	(0.823)
Book Leverage	11.361	11.098
	(0.361)	(0.132)
Market to Book Equity	1.316	1.316
	(0.545)	(0.131)
Growth Rate of Book Equity	0.020	0.004
	(0.112)	(0.003)
Log Common Dividend Rate	0.006	0.030
	(0.006)	(0.001)
Charge-Off Rate	0.001	0.001
	(0.003)	(0.001)

	mean/(s.e.)
q	0.687
	(0.144)
z	0.540
	(0.438)
Zombie Loans/Total Loans	0.029
	(0.014)
λ	16.544
	(4.358)
c	0.061
	(0.009)
dW/W	0.004
	(0.071)
s	1.972
	(0.283)

Notes: The data uses the full sample from 1990 Q3 to 2021 Q1. The moments from the model are generated from a panel of 10,000 banks with the same number of quarters as in the data. The first row for each variable shows the mean. The second row shows the standard error of the mean in parenthesis. For market leverage, book leverage and market-to-book equity, the mean and standard error are computed on the logs, but when reporting the mean we apply exponential to show the mean in levels. All rates are quarterly.

G Model Appendix: Numerical Solution

We solve the model using the finite-differences method with an upwind scheme for the choice of forward or backward differences. Specifically, we compute the numerical derivatives of the value function $v(z)$ using finite differences and use the first order conditions to solve for c, and iterate on the HJB equation. A detailed description of this algorithm can be found in Achdou et al. (2020).

Our model is simple to solve because prices are constant, but presents two complications:

1. The size of the jump depends on the endogenous state variable. Starting from a point z, upon receiving a Poisson shock, the bank jumps to $z + J^z$. We use linear interpolation to get the value function off the grid, and to build the

2. The bank must choose which boundary to operate on. Rather than using a linear complementarity problem solver, we instead follow the relaxation approach of Cioffi (2021). We assume that the choice of λ is taken infrequently: the bank can only switch from one boundary to the other upon arrival of an exogenous Poisson process with arrival rate $\mathcal{P}$. We take $\mathcal{P}$ to be large enough so that none of the results are affected by this approximation. In the limit, as $\mathcal{P} \to \infty$, the value function that we get converges to the desired one.

Let $b \in \{S, L\}$ denote the choice of the bank to operate in the shadow (S) or liquidation (L) boundary, so that:

$$b = \begin{cases} S & \text{if } \lambda = \Lambda(z) \\ L & \text{if } \lambda = \Gamma(z) \end{cases}$$

The HJB of the bank is:

$$\begin{aligned} 0 = \max_c f(c, v(z,S)) + \mu^z(z, \Lambda(z)) v_z(z,S) + \mu^W(\Lambda(z)) v(z,S) \\ + \sigma J^v(z,S) + \mathcal{P} \max\{v(z,L) - v(z,S), 0\} \end{aligned}$$

$$\begin{aligned} 0 = \max_c f(c, v(z,L)) + \mu^z(z, \Gamma(z)) v_z(z,L) + \mu^W(\Gamma(z)) v(z,L) \\ + \sigma J^v(z,L) + \mathcal{P} \max\{v(z,S) - v(z,L), 0\} \end{aligned}$$

where:

$$J^v(z,b) \equiv \begin{cases} v(z + J^z, S)(1 - \varepsilon\Lambda(z)) - v(z,S) & \text{if } b = S \\ v^o - v(z,L) & \text{if } b = L \end{cases}$$

Note that the value function $v(z,b)$ shows up non-linearly in the terms $f(c, v(z,b))$—due to the assumption of Duffie-Esptein preferences—and in $\max\{v(z,L) - v(z,S), 0\}$. For these two terms we use the value function of the previous iteration. This circumvents the problem of having to solve for the value function non-linearly at every iteration, at the cost of having to choose a smaller time step when updating the value function—because the method used is semi-implicit.

The first order condition used for solving for the optimal policy $c(z)$ is:

$$f_c(c, v(z,b)) + z v_z(z,b) - v(z,b) = 0$$

where $f_c(c, v) = \beta(v/c)^\theta$.

G.1 Model Stationary distribution.

To compute the stationary distribution, we use the Kolmogorov Forward equation. Let $g(z,b)$ be the invariant distribution over z conditional on choice of boundary b. Let $P_{b \to b'} = \mathcal{P} \cdot \mathbb{I}_{[v(z,b') > v(z,b)]}$

We have:

- in the shadow boundary:

$$\begin{aligned}\dot{g}(z,S) &= -\frac{\partial}{\partial z}\left[\mu^{z}(z,S)\,g(z,S)\right] \\ &\quad -\sigma g(z,S)+\sigma\int_{0}^{\infty} g\left(z',S\right)\mathbb{I}_{[z'+J^{z}(z',S)=z]}dz' \\ &\quad -P_{S\to L}\cdot g(z,S)+P_{L\to S}\cdot g(z,L)\end{aligned}$$

  for $z>0$. The first line represents changes in the density due to z drifting. The second line subtracts mass due to banks that had zombie loan ratio z and received a loan default shock, and adds mass from banks that after receiving a loan default shock have their zombie loan ratio jump to z. The third line computes changes in the density due to banks shifting boundaries. For $z=0$:

$$\begin{aligned}\dot{g}(0,S) &= -\frac{\partial}{\partial z}\left[\mu^{z}(0,S)\,g(0,S)\right] \\ &\quad -\sigma g(0,S)+\sigma\int_{0}^{\infty} g(z,L)\,dz \\ &\quad -P_{S\to L}\cdot g(0,S)+P_{L\to S}\cdot g(0,L)\end{aligned}$$

  which is different in the second line: we no longer have banks jumping to z after receiving a loan default shock (because $J^{z}>0$), but we add to the density to replace banks that were liquidated, to keep the mass of banks constant.

- in the liquidation boundary:

$$\begin{aligned}\dot{g}(z,L) &= -\frac{\partial}{\partial z}\left[\mu^{z}(z,L)\,g(z,L)\right] \\ &\quad -\sigma g(z,L) \\ &\quad +P_{S\to L}\cdot g(z,L)-P_{L\to S}\cdot g(z,L)\end{aligned}$$

for $z>0$. The first line represents changes in the density due to z drifting. The second line subtracts mass due to banks that had zombie loan ratio z and received a loan default shock. The third line computes changes in the density due to banks shifting boundaries.

The distribution of the state variable z and fundamental leverage λ is reported in Figure G.1. The grey area of the figure represents the liquidation region. The largest portion of the distribution lies in the shadow boundary.

G.2 Market returns.

Let $S(z)$ be the valuation of a risk-neutral investor of the equity of a bank, which we use to construct market returns. It satisfies the following HJB equation:

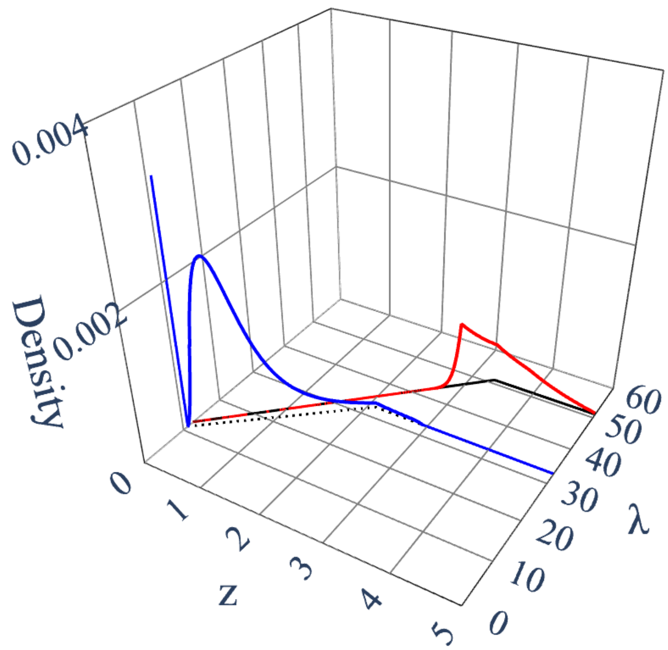
$$\rho^{I}S(z,W)=c(z)\,W+S_{Z}(Z,W)\,\mu^{Z}W+S_{W}(Z,W)\,\mu^{W}W+\sigma\left[S\left(Z+J^{Z},W+J^{W}\right)-S(Z,W)\right]$$

where $S(z)=0$ if $z>\Gamma(z)$. Just like $V(Z,W)$, $S(Z,W)$ is homogeneous in W and we can therefore express it as $S(Z,W)=s(z)\,W$, with $s(z)$ satisfying:

$$\rho^{I}s(z)=c(z)+s_{z}(z)\,\mu^{z}+s(z)\,\mu^{W}+\sigma\left[s\left(z+J^{z}\right)\left(1-\varepsilon\lambda(z)\right)-s(z)\right]$$

where $s(z)=0$ if $z>\Gamma(z)$.

Figure G.1: Model Stationary Distribution of Banks Across the z and λ State Space



Notes: This figure presents a two dimensional plot of the stationary distribution of banks across the (λ, z). The black dashed line traces out the shadow boundary $\Lambda(z)$ and the solid black line the liquidation boundary $\Gamma(z)$. The blue and red lines are the density of banks conditional on choosing the shadow and liquidation boundaries, respectively. The density conditional on choosing the liquidation boundary has been multiplied by 20 for visualization purposes, as it is otherwise not visible.

Using $s(z)$, we can construct market returns $r_t(z)$ as:

$$r_t(z) \equiv \frac{\int_{t-1}^{t} c_\tau(z) W_\tau d\tau + s_t(z) W_t}{s_{t-1}(z) W_{t-1}} \tag{70}$$

where t indexes quarters.

G.3 Formulas used to compute moments.

We list here formulas that we derived to compute moments from the model.

At the bank level, we compute the bank failure rate as $\sigma \int_0^\infty g(z, L) dz$; book loans as $(\lambda + z) W$; the chargeoff rate as $\alpha (1 - \lambda / (z + \lambda))$; Tobin's Q as $s/(1+z)$; market equity as sW; liabilities as $(\lambda - 1) W$; market leverage as $(\lambda - 1)/s$; book loans over book equity as $(\lambda + z)/(1+z)$; and distance to default as $\frac{1}{0.06} \frac{s+\lambda-1}{\lambda-1}$, where 0.06 stands for the volatility of market equity, which is common across banks.

At the aggregate level, the mean growth rate of equity is $\int_0^\infty \left[\mu^W(\Lambda(z)) - \sigma\varepsilon\Lambda(z)\right] g(z, S) dz + \int_0^\infty \left[\mu^W(\Gamma(z)) - \sigma\right] g(z, L) dz$. Note that, upon receiving a loan default shock, banks in the shadow boundary lose a fraction $\varepsilon\Lambda(z)$ of their equity and banks in the liquidation boundary lose 100% of their equity.

To aggregate variables to a quarterly frequency, we set time steps $dt = 1/30$ and for every 30 time steps we use the last value for stocks and the mean for flows.

G.3.1 Aggregate Loans

To compute the law of motion of loans L note that, in the shadow boundary, the drift of leverage is:

$$\lambda = \begin{cases} \frac{\Xi+(\Xi-1)z}{1+(\Xi-1)\varepsilon} & \text{if } z \leq z^s \\ \\ \frac{\kappa}{1+(\kappa-1)\varepsilon} & \text{if } z > z^s \end{cases} \Rightarrow \mu^{\lambda}_{Shd}(z) = \begin{cases} \frac{(\Xi-1)}{1+(\Xi-1)\varepsilon}\mu^z(z,S) & \text{if } z \leq z^s \\ \\ 0 & \text{if } z > z^s \end{cases}$$

and the jump in loans is:

$$J^L = \lambda' W' - L = \Lambda(z+J^z)(W-\varepsilon L) - L \Rightarrow \frac{J^L}{L} = \frac{\Lambda(z+J^z)}{\lambda}(1-\varepsilon\lambda) - 1$$

With these two terms, we can compute the expected loan growth rate of a bank in the shadow boundary as:

$$\frac{1}{dt}E\left[\frac{dL}{L}\right] = \frac{\mu^{\lambda}_{Shd}(z)}{\lambda} + \mu^W(\Lambda(z)) + \sigma\frac{J^L}{L}$$

Likewise, for a bank in the liquidation boundary:

$$\lambda = \begin{cases} \Xi + (\Xi-1)z & \text{if } z \leq z^m \\ \\ \kappa & \text{if } z > z^m \end{cases} \Rightarrow \mu^{\lambda}_{Liq}(z) = \begin{cases} (\Xi-1)\mu^z(z,L) & \text{if } z \leq z^m \\ \\ 0 & \text{if } z > z^m \end{cases}$$

$$J^L = 0 - L \Rightarrow \frac{J^L}{L} = -1$$

$$\frac{dL}{L} = \left(\frac{\mu^{\lambda}_{Liq}(z)}{\lambda} + \mu^W(\Gamma(z))\right)dt - 1dN$$

and hence:

$$\frac{1}{dt}E\left[\frac{dL}{L}\right] = \frac{\mu^{\lambda}_{Liq}(z)}{\lambda} + \mu^W(\Gamma(z)) - \sigma$$

We then compute the mean growth rate of loans as:

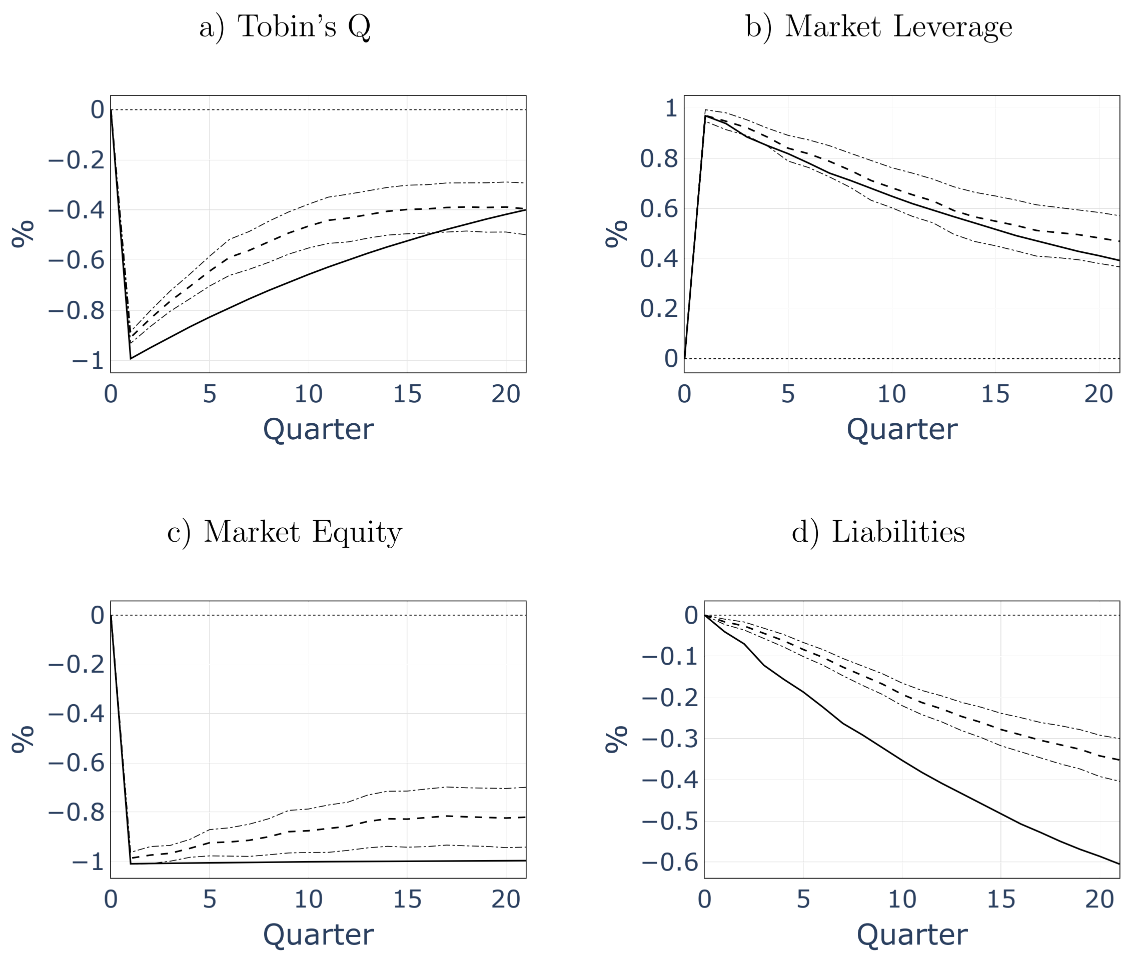
$$\int_0^{\infty}\left[\frac{\mu^{\lambda}_{Shd}(z)}{\lambda} + \mu^W(\Lambda(z)) + \sigma\frac{J^L}{L}\right]g(z,S)\,dz + \int_0^{\infty}\left[\frac{\mu^{\lambda}_{Liq}(z)}{\lambda} + \mu^W(\Gamma(z)) - \sigma\right]g(z,L)\,dz.$$

G.4 The Role of the IES for the Quantitative Fit.

In Section 4.2 of the main text, we argue that the delayed loan loss recognition mechanism is driving the slow adjustment of banks to net-worth shocks. Since the bankers' preference imply an intertemporal smoothing incentive whenever $\theta > 0$, one might worry that instead what drives the slow adjustment is θ. We investigate the role of the IES ($1/\theta$) for the quantitative results by solving the model for the same parameter configuration as in the baseline model (see Section 4), except setting $\theta = 2$, and reestimate the IRF on the model generated data. This calibration implies significantly lower intertemporal smoothing incentives. The results are in Figure G.2.

The blue line presents the impulse response functions of the model for the case when $\theta = 2$. Relative to Figure 10, the IRF still shows substantially slow adjustment to a negative net-worth shock.

Figure G.2: Data IRFs versus Model IRFs with $\theta = 2$



Notes: The figures present the impulse response functions of model simulated data (blue) for the benchmark calibration, except with the IES $1/\theta$ increased to $1/2$, and compares it to the data (gray line represents the point estimates and the shaded area the 95% confidence interval). We show the impulse response function of Tobin's Q (Panel a), market leverage (Panel b), market equity (Panel c), and liabilities (Panel d).

The higher IES makes liabilities adjust faster in response to a loan default shock, so the fit of the IRF for liabilites relative to the data worsens, and the one for market leverage improves, but the delayed adjustment feature remains.