Comments on "Optimal Supply of Public and Private Liquidity" by Marina Azzimonti and Pierre Yared

Discussion by Saki Bigio

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1 Introduction

"Optimal Supply of Public and Private Liquidity" by Marina Azzimonti and Pierre Yared, henceforth AY, is a study on the optimal provision of public debt, a classic subject in public finance.[1] There is tradition that studies this subject by endowing a fictitious Government with a set of tax limited instruments and asks when and which taxes should be to raised to finance expenditures.[2] This paper is part of a different tradition though. In this alternative tradition, the modeler introduces a market imperfection and ask how the supply of public debt changes a market outcomes. I think of Bewley [1983], Lucas [1980], Woodford [1990], Holmström and Tirole [1998], Kiyotaki and Moore [2008] and more recently of Angeletos et al. [2018], when I think of this separate tradition.

In AY, the reader reviews an old lesson and learns a new one. The review lesson is a Ricardian equivalence result that applies to the cross-section: when there is a consumption smoothing motive, even if private debt is shut down to borrowers, the Government can save on behalf borrowers. In fact, there is a lower bound on public debt that reproduces the efficient market allocation, one where all credit is available. The new lesson is that, even if it can, the Government may chose to provide less than the amount of public debt needed to implement the efficient-market allocation. The reason is that the efficient-market allocation may produce too much inequality. Restricting the supply of public debt away from the efficient amount increases equality, but comes at the expense of economic efficiency.

The model in AY is rich in features. Some features are essential for the lesson and but some others are not. The first features is that the Government has a limited set of instruments in the sense that transfers are not individual specific. This is an essential part of the model because, otherwise, the Government could implement any feasible allocation. The second feature is a complex form of market imperfection. The credit market is anonymous, and agents can renege on their payments. Third, default carries a physical cost. Thus, the market allocation that prevails with these frictions produces two inefficiencies. Inefficient default and insufficient variation. Both features are inefficiencies since there could be Pareto trades that improve welfare to both borrowers and savers. Furthermore, the fact that the debt market is anonymous produces a pecuniary externality in the sense that the allocation is not even constrained efficient.

In what follows, I will present a variation to the model. Under this variation, no resources are wasted because no resources are destroyed when there's default. The inefficiency follows only from imperfect consumption smoothing. I will describe why this departure has an economic reasoning behind. I will use this alternative model, to reproduce the arguments in AY, while keeping the same acts, and same morale of the story, but changing the setting a little.

2 Efficient Benchmark

2.1 The Environment

The model has two periods, $t = 0, 1$. There are two agents, the rich (R) and the poor (P); I index them with $i \in \{P, R\}$. The measure of each group is $1/2$. Although unnecessary, I introduce a time a discount factor β so that the Euler equations resemble those familiar to macroeconomists. Both agents have identical concave utility u.

[1] See Azzimonti and Yared [2018].

[2] I am thinking of the sequence Barro [1989],Lucas and Stokey [1983], and Aiyagari et al. [2002] when I think of this tradition.

The endowments of agent i at time t is given by y_t^i and take values:

$$y_0^R = 1 + \Delta \text{ and } y_1^R = 1,$$

for the rich and for the poor,

$$y_0^P = 1 - \Delta \text{ and } y_1^P = 1.$$

The rich have a greater endowment at time 0, because $\Delta > 0$, but by $t = 1$ endowments are equal.

The consumption of agent i at time t is denoted by c_t^i and are tied through the budget constraints:

$$c_0^i + qc_1^i = y_0^i + qy_1^i$$

where q is the time 0 price of time 1 goods. At time t resources satisfy $\sum_i \frac{1}{2} c_t^i = \sum_i \frac{1}{2} y_t^i$.

Equal Weight Welfare. An egalitarian Welfare criterion assigns equal weights on the utility of all agents. Thus, allocations maps into a welfare via:

$$W\left(\left\{c_t^i\right\}\right) = u\left(c_0^P\right) + \beta u\left(c_1^P\right) + u\left(c_0^R\right) + \beta u\left(c_1^R\right).$$

Maximizing this criterion is the objective of the Government, here and in AY. Naturally, the planner allocation would set consumption equal to 1 at all periods. If agents consume their endowments, we would have the Autarky allocation.

We can plot welfare as a function of the allocation given the consumption of the poor $\{c_0^P, c_1^P\}$—the consumption of the rich is obtained from the resource constraint. Figure 1 places c_0^P in the x-axis and c_1^P in the y-axis—the x-y plane is the Edgeworth box. The vertical axis is the level of welfare given an allocation. This figure will guide the arguments in this discussion. For now, observe that the blue curve is the different levels of welfare of the egalitarian planner given allocations along the Pareto frontier. The Pareto frontier is the 45-degree projection in the x-y plane. The planner's optimal allocation is the black diamond a the highest point of the welfare at the Pareto frontier. If it could, the planner would set allocations in that point. The red square located where the poor consume little in the first period, is the Autarky allocation. It delivers a much lower welfare than the planner allocation. Next, I describe the efficient market allocation.

2.2 The Efficient Market Allocation

Standard textbook analysis shows us that this economy admits a representative agent. Thus, $q = \beta$ since the endowment always equals 1. Furthermore, there's perfect consumption smoothing for all agents. The question is then, what are those levels of consumption and the level of private credit? Take the consumption of agent i. With perfect smoothing $c_0^i = c_1^i$. Replacing constant consumption in the budget constraint at the correct price delivers:

$$c_0^R = c_1^R = 1 + \frac{\Delta}{1+\beta} \text{ and } c_0^P = c_1^R = 1 - \frac{\Delta}{1+\beta}.$$

We can then obtain the borrowing of the poor agents which equals:

$$l_0^p = \frac{\Delta}{1+\beta},$$

where l^p is the amount owed by the poor at $t = 1$. This market allocation is, off course, efficient. This is the Efficient Market Allocation. Back in Figure 1, the efficient market allocation is located at the Pareto frontier. It is indicated by the yellow diamond. Observe that although efficient, the welfare produced is lower than the welfare at the egalitarian planner's allocation.

2.3 A Ricardian Equivalence in the Cross Section

We now introduce a government that chooses a stock of debt B_0 at time zero, and a sequence of lump-sum taxes, $\{T_0, T_1\}$. The bond is held by the poor and rich and thus $B_0 = \frac{1}{2} b_0^R + \frac{1}{2} b_0^P$. I denote b_0^i the holdings of the public bond by group i. There are no government expenditures so:

$$-T_0 = qB_0 \text{ and } T_1 = B_0.$$

Combining yields $-T_0 = qT_1$.

Let's guess and verify that the allocation is invariant to the Government's policy. If that's the case, the market price of public or private debt is again $q = \beta$. Thus, again, the time-zero budget constraint for any agent is,

$$c_0^i + \beta c_0^i \quad = \quad y_0^i - T_0 + \beta\left(y_1^i - T_1\right).$$

If we replace the conjectured price into the Government's budget balance intertemporal budget constraint, we obtain $-T_0 = \beta T_1$. Thus, the budget constraint of agent i satisfies:

$$c_0^i + \beta c_0^i \quad = \quad y_0^i + \beta y_1^i,$$

just as in the previous example. The conjecture is verified. The sense in which this is a Ricardian equivalence result is that the level of Government debt doesn't matter to affect the cross-sectional allocation of goods. Back in Figure 1, when markets are efficient, given the restriction on that taxes are equal for all the population, there is no way that the planner can improve it's welfare function away from the yellow diamond that corresponds to the efficient market allocation.

A natural question at this point is how much private debt is issued given an amount of public debt? The answer is, in fact, indeterminate. However, we can determine bounds for private debt that are consistent with the efficient market allocation, once we introduce public debt.

Maximal Private Liquidity. We can obtain an equilibrium with the highest level of private debt. This is one where public debt is is perfectly offset by taxes. Assume that:

$$-T_0 = qb_0^R = qb_0^P \text{ and } T_1 = b_0^R = b_0^P.$$

In this equilibrium, the Government issues debt at time zero to fund lump-sum transfers. However, both rich and poor react by using their lump-sum transfers to purchase public debt. At time $t = 1$, they use their public debt holdings to pay for their lump-sum taxes. The operation is obviously neutral. The policy is perfectly neutralized. Hence, private debt issuances are exactly those of the efficient equilibrium without Government debt. There's an opposite extreme, where the rich hold a larger amount of public debt, and public debt substitutes private debt.

Minimal Private Liquidity. The equilibrium with the lowest amount of private debt that delivers the efficient allocation depends on the level of public debt. There are two segments of interest:

- If $B \leq \frac{\Delta}{1+\beta}$, rich buy all public debt $\left\{b_0^R = 2B, b_0^P = 0\right\}$ and furthermore,

$$l^p = \frac{\Delta}{1+\beta} - B.$$

- If $B \geq \frac{\Delta}{1+\beta}$, then, we can have then $l^p = 0$. The holding of public debt are:

$$b_0^R = B + \frac{\Delta}{1+\beta} \text{ and } b_0^P = B - \frac{\Delta}{1+\beta}.$$

Thus, when the public supply of debt is above the level of private debt (without any Government debt), $\frac{\Delta}{1+\beta}$, the efficient allocation does not require any private debt issuances. In essence, the Government saves on behalf of the poor.

2.4 Taking Stock

Consider a ban on private debt. The Ricardian equivalence result above shows that if the stock of Government debt equals amount of private needed to produced the efficient market equilibrium, $\frac{\Delta}{1+\beta}$. The allocation will be reproduced by having the

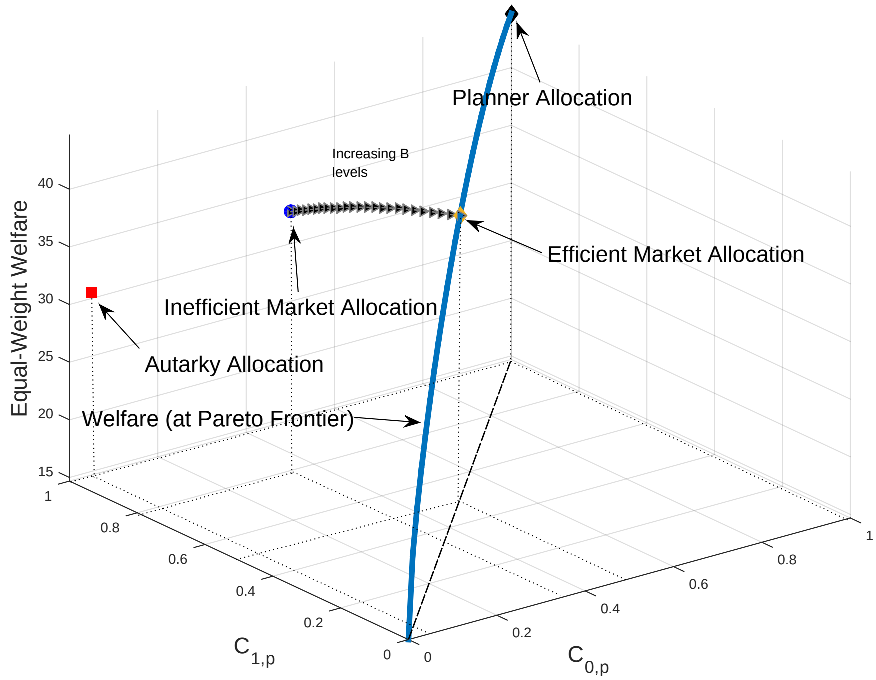


Figure 1: Welfare as a Function of Allocations. The parametric example is built with a CRRA utility with coefficient $\gamma = 0.5$; the discount factor is $\beta = 0.9$; the endowment discrepancy is $\Delta = 0.95$; the coefficient of private information is $\rho = 2.5$. The welfare function is exponentiated for graphical purposes.

rich hold all the public debt. Thus, it is safe to conclude that if we impose any frictions that affect private debt issuances, the Government can replace the issuances of the poor and deliver the efficient market allocation. This is the review lesson.

Consider then a Welfare function for the Government with equal Pareto weights. The new lesson in the AY piece is that, even if it can, the Government will chose to provide less than the amount of public debt that reproduces efficiency. I will delve into the details in the next section, but we can anticipate the reasoning: the reason for providing less public debt than needed for the efficient market allocation is that efficiency produces too much inequality. The Government may wish to restrict the supply of bonds, exploiting the market inefficiencies, but in a way that produces a more equal allocation.[3] This is an interesting result. It means that the Government can exploit market inefficiencies to redistribute wealth. If there was no market inefficiency, there wouldn't be a way to produce redistribution by the Government.

Debt with Anonymity, a Critique. The AY paper adopts a specific form of market imperfection. Agent's can default on debt, default produces a costs in resources, and markets are anonymous. I see this market structure as the main weakness of the paper. First, costly default introduces an additional source of inefficiency—in addition inefficient consumption variation. This loss of resources is not needed to tell the story. It makes the message less transparent and the arguments more difficult to produce. Second, an most important, I see defaultable debt under anonymous markets as a problematic assumption. My argument is that if a bond market is anonymous, and if default costs are bounded below by endowment, nothing should prevent a Ponzi scheme. The argument is simple: get infinite instantaneous utility by taking infinite debt, and then default. As long as utility is still finite in a default state, borrowers will want to take infinite debt. Yes you bear a cost in the future after default, but you enjoy infinite utility today. Of course, a Ponzi scheme would occur in equilibrium, so the credit market would unravel. This is why models with anonymity and default typically introduce a collateral constraints—borrowing constraints conflict with anonymity.

[3] A technical way to phrase this is as follows. Negishi's Theorem states that for any competitive equilibrium, there's a pair of Pareto weights that reproduce that allocation in the planner's problem. When one agent is poorer than the other, in present value terms, the Pareto weights.

In AY, Ponzi schemes don't occur because default costs are unbounded. Hence, there is no AY inconsistency: if the poor take large amounts of wealth, with a positive probability they will hit an Innada condition. However, I believe that unbounded costs of default are unrealistic.

There are two routes two circumvent this issue. One is to abandon anonymity and introduce debt prices that depend on the individual's default probability. An alternative is to introduce collateral assets and a corresponding friction. In the next section, I will reconstruct the main argument of AY, with a market structure that resembles collateralized debt.

3 An Alternative Model

Private Credit, Private Information, and without Public Debt. I now consider an economy where just as in an Arrow-Debreu world, we allow that at $t=0$ agents can borrow against their $t=1$ endowment. However, I introduce two departures from Arrow-Debreu. The first departure is that now, the endowment of the poor has many pieces pieces. I index each piece by some index $\omega \in [0,1]$. Each piece is associated with a different amount of $t=1$ endowment. The amount associated with each piece is $\lambda(\omega)$—technically each piece is measure 0, but think of $\lambda(\cdot)$ as a function that approximates a quantity in a discrete division. The total endowment adds to the same number we have in the previous section:

$$y_1^P = \int_0^1 \lambda(\omega)\, d\omega.$$

Without loss of generality, I impose the assumption that$\lambda(\omega)$ be increasing in ω putting an order—in fact, we can take a non-monotone function and re-order the index into a different index such that λ is monotone under that index. Hence, the assumption is without loss of generality. The second departures is that now, ω is private information.[4]

The budget constraints are given by:

$$c_0^R = y_0^R - p^B d \text{ and } c_0^R = y_1^R + d$$

for the rich and

$$c_0^P = y_0^P - p^S \underline{\omega} \text{ and } c_1^P = y_1^P + \int_{\underline{\omega}}^1 \lambda(\omega)\, d\omega,$$

for the poor. In the budget of the rich household, d are purchased units of time 1 consumption. Also, p^B is the price of those units, so effectively, it's the inverse of the gross risk-free rate. The price for the units of the poor, which is a pooling price, is p^S. All pieces are sold at the same price because of the private information assumption.

The quantity sold $\underline{\omega}$ coincides with a threshold piece sold. This is a cut-off rule and it makes sense. If the poor agent wants to obtain some funds today, it will always choose to sell worse qualities first. This is why $\underline{\omega}$ stands for the quantity sold of the $t=1$ endowment. It is also the reason why the endowment that remains with him at $t=1$ is $\int_{\underline{\omega}}^{\infty} \lambda(\omega)\, d\omega$.

The rich must buy consumption from the poor. The market clearing condition for t=1 goods is:

$$d = \int_0^{\underline{\omega}} \lambda(\omega)\, d\omega = \mathbb{E}\left[\lambda(\omega) | \omega < \underline{\omega}\right] \underline{\omega}.$$

Multiply both sides by p^B and the total sales of $t=1$ goods to the rich equal:

$$p^B \mathbb{E}\left[\lambda(\omega) | \omega < \underline{\omega}\right] \underline{\omega}.$$

Since total purchases of $t=1$ goods to the poor equal $p^S \underline{\omega}$, we obtain a zero-profit condition:

$$p^B \underbrace{\mathbb{E}\left[\lambda(\omega) | \omega < \underline{\omega}\right]}_{\text{expected conditional value}} = p^S. \tag{1}$$

The first-order condition (with respect to d) for the rich is:

$$p^B u'\left(c_0^R\right) = \beta u'\left(c_1^R\right). \tag{2}$$

[4] In Bigio [2015] I worked with a similar formulation to divide the capital stock of an agent into a continuum pieces. There, the demand for capital sales was to obtain liquidity to exploit an investment opportunity. Here's the motive for sales of endowments is to smooth consumption.

Applying Leibniz's rule, the first-order condition (with respect to $\underline{\omega}$) for the poor is:

$$p^S u'\left(c_0^P\right) = \beta u'\left(c_1^P\right) \cdot \underbrace{\lambda\left(\underline{\omega}\right)}_{\text{threshold value}}. \tag{3}$$

Combining (1), (2), and (3) we obtain an expression for the ratio of marginal utilities:

$$\frac{u'\left(c_1^R\right)}{u'\left(c_0^R\right)} = \frac{u'\left(c_1^P\right)}{u'\left(c_0^P\right)} \cdot \frac{\lambda\left(\underline{\omega}\right)}{\mathbb{E}\left[\lambda\left(\omega\right)|\omega < \underline{\omega}\right]}. \tag{4}$$

Notice that in the efficient equilibrium, the ratio of marginal utilities is equated. Here, it is not, and this measures the extent of the market inefficiency—lack of consumption smoothing. Also, observe that $\lambda\left(\underline{\omega}\right)/\mathbb{E}\left[\lambda\left(\omega\right)|\omega < \underline{\omega}\right]$, is always a number greater than 1, for $\underline{\omega} > 0$. Thus, an equilibrium with private information will always produce inefficiency.[5] As in a Lemons model, the ratio $\lambda\left(\underline{\omega}\right)/\mathbb{E}\left[\lambda\left(\omega\right)|\omega < \underline{\omega}\right]$ is a measurement of the extent of asymmetric information.

The allocations $\left\{c_0^p, c_1^p, c_0^r, c_1^r\right\}$ can be obtained implicitly through the budget constraints given $\left\{p^B, \underline{\omega}\right\}$. Replacing the (1) we have:

$$c_0^P = y_0^P + p^B \int_0^{\underline{\omega}} \lambda\left(\omega\right) d\omega \text{ and } c_0^R = y_1^R + p^B \int_0^{\underline{\omega}} \lambda\left(\omega\right) d\omega.$$

and

$$c_1^P = \int_{\underline{\omega}}^1 \lambda\left(\omega\right) d\omega \text{ and } c_1^R = y_1^P + \int_0^{\underline{\omega}} \lambda\left(\omega\right) d\omega.$$

We have two unknowns to solve for $\left\{p^B, \underline{\omega}\right\}$ before we obtain the equilibrium allocation. We need two equations. We can use (2) and (4) to obtain the system of two equations and two unknowns. Let me label the resulting allocation as the inefficient market allocation that results from this friction. This inefficient market equilibrium outcome is plotted in Figure 1. It corresponds to the blue circle. Welfare is better than under autarky, but lower than under the efficient market allocation. The next section introduces public debt again, but now to the model with private information.

Private Credit, Private Information, and without Public Debt. Let's now introduce Government debt into the model with credit under private information. It is easy to verify that the first-order conditions for d and $\underline{\omega}$ won't change. The first-order condition for Government debt is the same as for private debt since the rich are indifferent between private and public debt. Thus, we will again use (2) and (4) to solve for $\left\{p^B, \underline{\omega}\right\}$.

What changes once we introduce public debt are the equilibrium allocations which now satisfy:

$$c_0^P = y_0^P + p^B B + p^B \int_0^{\underline{\omega}} \lambda\left(\omega\right) d\omega \text{ and } c_0^R = y_1^R - p^B B + p^B \int_0^{\underline{\omega}} \lambda\left(\omega\right) d\omega.$$

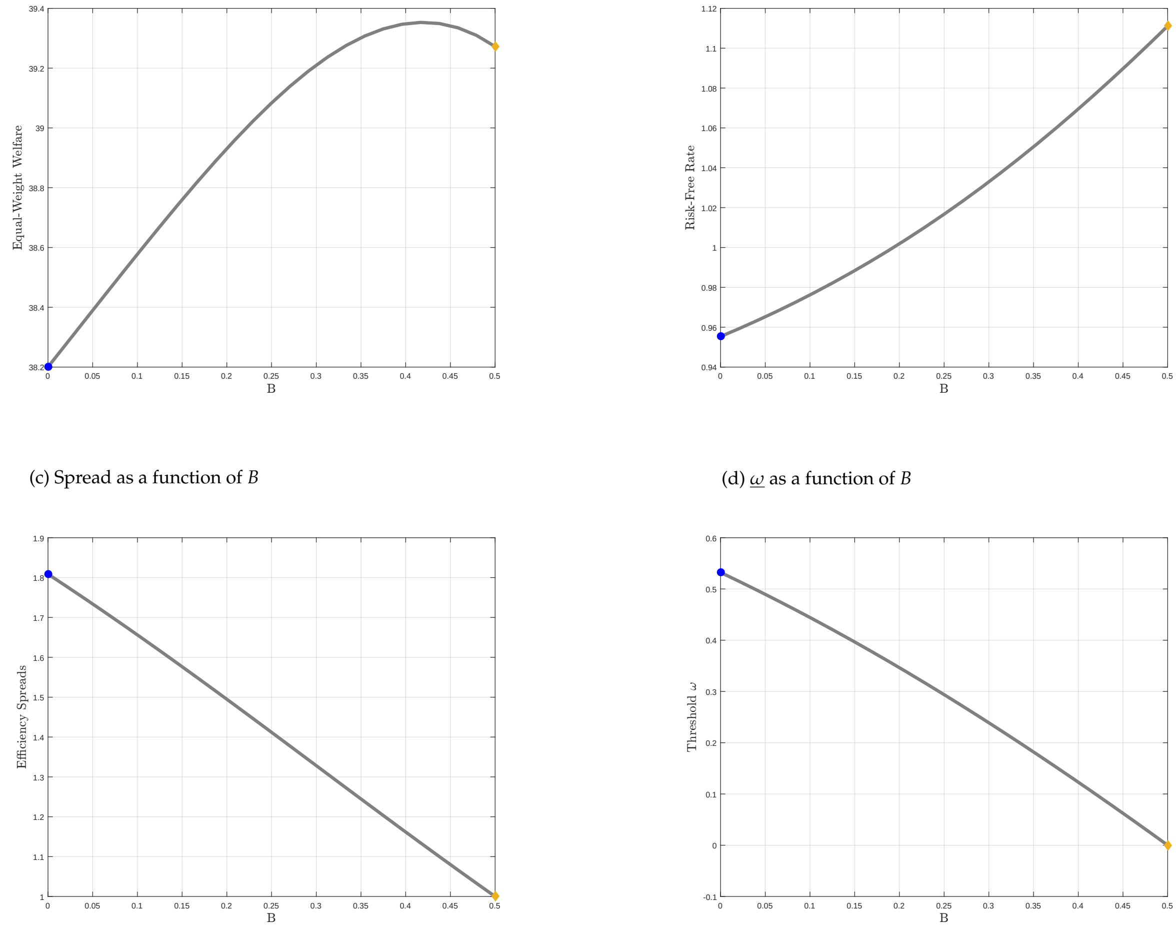
and

$$c_1^P = -B + \int_{\underline{\omega}}^1 \lambda\left(\omega\right) d\omega \text{ and } c_1^R = y_1^P + B + \int_0^{\underline{\omega}} \lambda\left(\omega\right) d\omega.$$

To obtain these allocations I used some immediate results: (a) that only the rich buy Government debt, (b) that Government debt has also a price of p^b, (c) and the Government budget constraints described earlier. The system is again a system of two equations and two unknown. Obviously, now, the equilibrium is paterametirized by by B, which we treat as an exogenous parameter. By the Ricardian proposition, we know that when $B = \frac{\Delta}{1+\beta}$, we obtain the efficient market equilibrium. When $B = 0$, we are at the inefficient market equilibrium. Thus, B indexes a curve that represents a bridge from the inefficient to the efficient market allocation. This bridge is the set of points Figure 1 that connect the efficient to the inefficient market allocation. The lesson from AY is that there is a point in that bridge with $B < \frac{\Delta}{1+\beta}$ where the welfare function is higher that under the efficient market allocation.

Back to Azzimonti and Yared. We can trace some equilibrium objects along the bridge from the inefficient to the efficient market allocation, by tracing equlibrium values along the curve where we vary B. This is what I do in the Figure 2. The main message in AY is read from Panel (a). That panel plots the level of welfare as we vary B. Observe that welfare for the egalitarian planner is not maximized at the efficient allocation. Rather, there's a lower level of public debt where the planner maximizes welfare. A mathematical argument, set aside model differences, is presented in AY. The intuition can be read from the rest of the

[5]Note that at $\omega \in \{0,1\}$ the first-order conditions are no longer valid. So I am assuming an interior solution which is not always the case.



(a) Welfare as a function of B

(b) Risk-Free Rate as a function of B

(c) Spread as a function of B

(d) $\underline{\omega}$ as a function of B

Figure 2: Equilibrium Outcomes with Market Inefficiency as Function B. The parametric example is built with a CRRA utility with coefficient $\gamma = 0.5$; the discount factor is $\beta = 0.9$; the endowment discrepancy is $\Delta = 0.95$; the coefficient of private information is $\rho = 2.5$. The welfare function is exponentiated for graphical purposes. B ranges from 0 to the value needed to attain the efficient market allocation.

panels in the figure. Panel (b) shows how at the efficient level of public debt, the interest is the highest. The first-order condition of the rich, (2), produces an increasing interest-rate path as we approach the efficient allocation, because the endowment of the rich decreases with time. As we provide more smoothing, the interest rate that prevails for the rich should rise. Higher interest rates distribute wealth in favor of the rich, since they are the natural lenders. In terms of the poor, the planner trades off more smoothing against less present value wealth, as it moves it's level of public debt. This is the reason why the level of public debt should not go all the way to the efficient allocation.

Panels (c) and (d) show a final by-product of the model. When the level of public debt is low, the information spread $\lambda\left(\underline{\omega}\right)/\mathbb{E}\left[\lambda\left(\omega\right)|\omega < \underline{\omega}\right]$ is very high. This is because the poor have a high demand for issuing private debt to smooth their consumption. As more public debt is introduced, the information cost drops because less private liquidity is introduced in equilibrium. This partial crowding out is plotted in Panel (d) where I present the threshold quality sold, as a function of B. This is another manifestation of the partial crowding out that appears in AY.

Collateralized Debt. Selling time-1 at time 0 endowment is a form of credit, but it's silent about default and collateral. In Bigio [2015] I present a model where assets can be used as collateral under private information. A loan specifies a loan size and a face value of debt. Agents can default if they chose too. That a model with collateralized debt. In fact, we can think of the $t = 1$ endowment sale as collateral. The price p^S can be thought of as a loan size. Seized collateral in this case is not different than an asset sale. In general, there are other equilibria without default on all assets, but that feature requires a market structure. The point I want to make here is that selling endowment under private information is not that different from collateralized debt.

4 AY in Perspective

I learned a new economic lesson from AY. A common fresh-water criticism to models like this, is the so-called chicken-government criticism. In this context, the chicken-government means that there's an asymmetry between what privates and the Government can do. Whereas private debt can't be enforced, the Government can enforce the levy of taxes. I think the chicken-government criticism is wrong. I think it's perfectly reasonable to have a Government good at enforcing taxes, but bad producing court rules.

Once we learn the lesson, another question is to implement it. This volume is dedicated to Charles Irving Plosser, an economist that transitioned from theory to policy making. I can't help wonder if we can extract some practical rule from AY to implement it's recommendation in the real world.

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