Supply, Demand, and Specialized Production

Discussion of Hamilton

by Saki Bigio (UCLA)
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Introduction

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* This paper
  * perhaps more ideas than the optimal amount
  * still, a paper I would want to have written

Discussion

* Key Idea:
  * adjustment to sectoral demand shocks
  * world with special inputs:
    * sunk, specialized, and market power
    * adjustment will be inefficient: lead to underutilization
* My Discussion:
  * present simplified model
  * what assumptions are needed
  * connect with some classical and recent literature
* Paper has more to say: growth, product entry, inequality
  * explain why model naturally connects to these ideas...
  * but focus on the core

Hamilton-ish Model: Environment

* representative household
    * complete market: no risk, no inequality
* j sectors
* one unit of labor
    * n_j supply of sector j work
    * allocated in advance

Hamilton-ish Model - Production

* n_j pre-determined

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* Production of j: subject to a capacity constraint

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* j workers form a coalition
    * choose price p_j
    * maximizes revenues R_j

$$w_j = R_j / n_j$$

    * important: u_j endogenous under-capacity

Static Block: Sectoral Consumption Choice

* Consumption choice:

$$C = \max_{\{c_j\}} \left(\sum_{j \in \mathcal{J}} \left(\alpha_{j,t}^{1/\sigma} \left(c_j + \bar{y}_j \right)^{1-1/\sigma} \right) \right)^{\frac{\sigma}{\sigma-1}}$$

subject to:

$$\sum_{j \in \mathcal{J}} p_j \cdot c_j = e \equiv \text{expenditures.}$$

Static Block: Transformed problem

* Define $x_j \equiv c_j + \bar{y}_j$ so that:

$$C = \max_{\{x_j \geq \bar{y}_j\}} \left(\sum_{j \in \mathcal{J}} \left(\alpha_j^{1/\sigma} \left(x_j\right)^{1-1/\sigma} \right) \right)^{\frac{\sigma}{\sigma-1}}$$

subject to:

$$\sum_{j \in \mathcal{J}} p_j \cdot x_j = \bar{e} \equiv \sum_{j \in \mathcal{J}} R_j + \underbrace{p_j \bar{y}_j}_{\text{threat}}$$

Static Block: Product Demand

* Usual CES solution:

$$x_j = \left(\frac{p_j}{P}\right)^{-\sigma} \alpha_j C$$

Static Block: Product Demand

* Usual CES solution:

$$x_j = \left(\frac{p_j}{P}\right)^{-\sigma} \alpha_j C$$

* Apply definition:

$$c_j = \left(\frac{p_j}{P}\right)^{-\sigma} \alpha_j C - \underbrace{\bar{y}_j}_{\text{threat}}.$$

Union Problem

* Revenue:

$$R_j = \max_{p_j} p_j \cdot c_j \left(p_j \right)$$

subject to capacity constraint:

$$p_j > \bar{p}_j$$

Union Problem

* Replacing demand function:

$$R_j = \max_{p_j} p_j \underbrace{\left(\frac{p_j}{P}\right)^{-\sigma} \alpha_j C - p_j \bar{y}_j}_{c_j(p_j)}$$

subject to:

$$p_j > \bar{p}_j$$

where $\bar{p}_j$

$$\underbrace{n_j}_{\text{maximal output}} = \left(\frac{\bar{p}_j}{P}\right)^{-\sigma} \alpha_j C - \bar{y}_j$$

Solution

* FOC:

$$\left(1-\sigma\right)\left(\frac{p_j}{P}\right)^{-\sigma}\alpha_j C - \bar{y}_j = 0$$

* Critical: bliss point
  * why? marginal cost is zero
* Critical: $\sigma < 1$ and
  * why? otherwise no output

After some algebra...

- Sectoral output:

$$y_j = \min\left\{ \underbrace{n_j}_{\text{binding}}, \underbrace{\frac{\sigma}{(1-\sigma)}\bar{y}_j}_{\text{non binding}} \right\}$$

- Revenues:

$$R_j = \min\left\{ \underbrace{}_{\text{binding}}, \underbrace{}_{\text{non binding}} \right\} (\alpha_{j,t} C)^{1/\sigma}$$

- Binding: Walrasian wage
- Non-biding: excess supply monopoly pricing

Symmetric Case

- Aggregate output:

$$C = \min \left\{ \underbrace{n + \bar{y}}_{\text{binding}}, \underbrace{\frac{\bar{y}}{(1-\sigma)}}_{\text{non binding}} \right\}$$

- If n grows: output remains fixed!
- Possibly binding at steady-state:
  - under-utilization of factors
  - this is why variety is interesting? Bilbie, Ghironi, and Melitz

Aggregate Demand Shocks?

* Patience shock:

$$1 = \beta(t) \frac{U'(C_{t+1})}{U'(C_t)} \mathcal{R}_t$$

* Output is supply determined
  * Does not affect under utilization!
* Important: think of sectoral shocks

Dynamics: Sectoral Shocks

* Lucas-Prescott Island Model (Alvarez-Shimer)
  * worker ex-ante choice of j island
  * assumption: union cannot reject worker
    * Training, skill monopoly?
    * Firm-market power?
    * Connect with Caballero Hammour

Dynamics: Sectoral Shocks

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* Idea:
  * $\alpha \in \left\{\alpha^L, \alpha^H\right\}$ Markov switching intensity θ
  * work value function:

$$\rho v(n,\alpha) = \frac{R(\alpha, n; C)}{n} + \theta \underbrace{\mathbb{J}\left[v(n,\alpha')\,|\alpha\right]}_{\text{shock}}.$$

  * Unemployment island:
    * stuck for τ periods: then choose where to go

$$\rho \mathcal{V} = \underbrace{\exp(-\rho\tau)}_{\text{wait}} \max_{\alpha} \left\{v(n(\alpha), \alpha)\right\}.$$

Two Shock Example

* Two capacities: $\{n^l, n^h\}$
  * Unemployment Rate:

$$u = \tau \cdot \theta \cdot \left(n^h - n^l\right)$$

  * Equilibrium Condition

$$\underbrace{v\left(\alpha^L\right) = \mathcal{U}}_{\text{adjustment indiff}} = \exp\left(-\rho\tau\right) v\left(\alpha^H\right).$$

Solution

* Labor flows:

$$1 = n^h + \underbrace{\tau \cdot \theta \cdot \left(n^h - n^l\right)}_{\text{unemployment}} + n^l$$

and indifference:

Solution

- Labor flows:

$$1 = n^h + \underbrace{\tau \cdot \theta \cdot \left(n^h - n^l\right)}_{\text{unemployment}} + n^l$$

and indifference:

$$v\left(\alpha^l, n^l\right) = \exp\left(-\rho\tau\right) v\left(\alpha^h, n^h\right)$$

- Recall:

$$\rho v\left(\alpha, n\right) = \frac{R\left(\alpha, n; C\right)}{n} + \theta\left(v\left(\alpha', n'\right) - v\left(\alpha, n\right)\right)$$

Paper Pencil Case

* Assume $\theta = \rho$ and $\sigma = \bar{y} = 1/2$ and $\tau = 1$

Solution:

- Resource Constraint

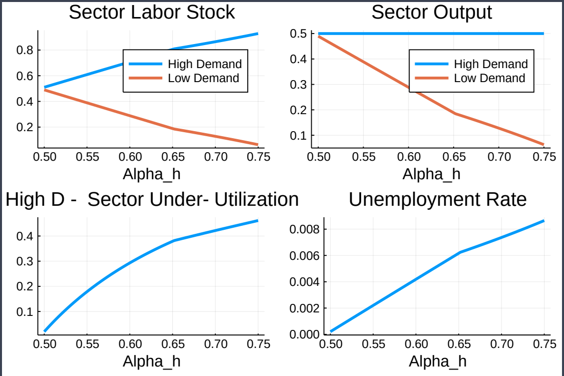
$$1 = n^h + \underbrace{\tau \cdot \theta \cdot \left(n^h - n^l\right)}_{\text{unemployment}} + n^l.$$

- Indifference Condition

$$\left(\frac{\exp\left(\rho\right) - 1/2}{1 - \exp\left(\rho\right)/2}\right)^2 \underbrace{\frac{1}{\bar{y} + n^l}}_{\text{non-binding}} = \underbrace{\frac{\bar{y}}{n^{h2}}}_{\text{binding}} \frac{\alpha^h}{\alpha^l}$$

- Possibly no solution

One Picture



Various Degrees of High Demand

Characterization:

* Equilibrium must feature:
    * (too little) unemployment | reallocation..
    * ...but under-utilization in high-demand sector

* frictionless economy: wage would absorb the cost
    * not here

* Dynamics: very interesting but left out!

Conclusion

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* Very nice paper!
* Lot to think about!