Endogenous Liquidity and the Business Cycle

By Saki Bigio*

I study an economy where asymmetric information about the quality of capital endogenously determines liquidity. Liquid funds are key to relaxing financial constraints on investment and employment. These funds are obtained by selling capital or using it as collateral. Liquidity is determined by balancing the costs of obtaining liquidity under asymmetric information against the benefits of relaxing financial constraints. Aggregate fluctuations follow increases in the dispersion of capital quality, which raise the cost of obtaining liquidity. An estimated version of the model can generate patterns for quantities and credit conditions similar to the Great Recession.

I. Introduction

The recent financial crisis was the deepest recession of the post-war era. The recession began with an abrupt collapse in many asset markets. A common view is that this collapse followed a surge in uncertainty about the quality of collateral assets. The consequent shortfall in liquidity may have spread to the real economy because liquidity is essential to finance payroll and investment.

This paper presents a theory where liquidity-driven recessions follow from surges in the dispersion of collateral quality. I use this theory to quantify the potential damage to the real economy caused by this class of dispersion shocks. The theory builds on the interaction of two financial frictions: *limited enforcement* and *asymmetric information*. Limited contract enforcement prevents transactions if future payments cannot be guaranteed. This constraint can be relaxed if an agent does not promise future payments, but instead makes payments immediately with liquid assets. However, agents hold capital that is illiquid unless it is sold or used as collateral. Asymmetric information about the quality of capital translates into a cost to obtain liquidity. The paper characterizes the decision to obtain liquidity under asymmetric information in order to relax enforcement constraints through a marginal condition. This marginal condition equates the marginal cost of sell-

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ing —or collateralizing— assets under asymmetric information to the marginal benefit of relaxing enforcement constraints. An increase in the dispersion of capital quality, which obscures the quality of capital, shifts this trade-off toward less liquidity.

This interaction between limited enforcement and asymmetric information takes place within an otherwise real business cycle model. Entrepreneurs require labor and investment inputs to produce consumption and capital. They face limited enforcement because they may default on their payroll or promises to repay investment inputs. The source of asymmetric information is the depreciation —which can be thought of as quality— of their capital. The paper studies two contracting environments. In the first environment, entrepreneurs can obtain liquidity by selling capital. In the second environment, which is a general case of the first, they can obtain liquidity by pledging capital as collateral. In either environment, recessions occur after the dispersion of capital quality increases. These shocks raise the cost of obtaining liquidity which further translates into lower employment, output, and investment. A salient feature of those liquidity-driven recessions is that they occur even though the production possibility frontier or the distribution of wealth does not change.

The paper then evaluates whether, through the endogenous liquidity mechanism, increases in dispersion can be meaningful sources of business cycles. To do so, I calibrate the model to match historical business cycle facts. This quantitative analysis reveals that increases in capital quality dispersion can generate economic fluctuations consistent with several business cycle features. [1] The model explains sizeable liquidity-driven recessions. These recessions operate primarily through fluctuations in hours worked together with increases in labor productivity. These features were characteristic of the Great Recession (see Ohanian, 2010) and seem predominant in the business-cycle decomposition of Chari et al. (2007). These features cannot be generated through negative total factor productivity (TFP) shocks. [2] The model produces two opposing forces that drive a low correlation between Tobin's Q and investment (see Gomes, 2001). As in standard Q-theory, TFP shocks induce a positive correlation but dispersion shocks reverse this correlation by inducing higher funding costs. These same forces also induce a negative correlation between aggregate investment and labor productivity. Other studies such as Justiniano et al. (2010) argue that financial factors are responsible for this co-movement. [3] When liquidity is obtained by selling capital, the model is also consistent with counter-cyclical capital reallocation as documented by Eisfeldt and Rampini (2006). When liquidity is obtained via the use of collateral, the model delivers countercyclical interest rate spreads and loan charge-off rates (see Gilchrist and Zakrajek, 2012) together with procyclical lending at extensive and intensive margins (see Covas and Den Haan, 2011). [4] Finally, the model can explain drops in risk-free rates together with increases in interest rate spreads during recessions.

The model features financial frictions that distort both employment and invest-

ment. Both are necessary features to generate consistent business cycle patterns. The enforcement constraint on payroll is key to generating sizeable recessions. This feature of the model distinguishes it from most models with financial frictions whose primary focus is on frictions that distort capital accumulation. It is commonplace to find that, on their own, investment frictions cannot generate strong output responses.[1] Instead, here there is a strong transmission of liquidity shocks through labor demand which has empirical support in recent work by Chodorow-Reich (2013) and Fort et al. (2013). Although the enforcement problem in labor is sufficient to deliver strong output responses, the quantitative analysis shows that the enforcement problem in investment is key to generate pro-cyclical investment. The reason is that while dispersion shocks cause a labor demand contraction, wages and hours drop in a combination that increases entrepreneurial profits. Without the investment friction, entrepreneurs invest more during recessions in response to their increased wealth.

As a case study, the paper also analyzes the extent to which dispersion shocks could have generated the data patterns of the Great Recession. For this, I deduce a sequence of dispersion shocks from a subset of the equilibrium conditions and use this sequence to contrast the model's predictions for output, consumption, investment, labor productivity, and hours with those of the data. I also use the version with collateralized debt to study the model's predictions about four credit market indicators: aggregate liquidity, loan sizes, interest rates, and loan charge-off rates. The model is successful in generating paths similar to the data attributing the early stage of the recession to a TFP decline and the latter stage to an unprecedented surge in dispersion. Moreover, the model also generates similar qualitative patterns for all credit market variables, although interest rates and charge-off rates are twice as high as in the data. In that application, I also study the behavior of credit market indicators for a version of the model with exogenous real wages and demand-determined hours. The fit to interest rates and charge-off rates improves once I feed that version of the model with real wages from the data. This last result is in line with other studies that find that real wage rigidities improve the quantitative performance of this class of models —e.g., Ajello (2012).

The paper builds on several insights found earlier. Eisfeldt (2004) studies a general equilibrium model where agents sell assets under private information to obtain funding for new projects and smooth their consumption. Kiyotaki and Moore (2008) (henceforth KM) lays out a business cycle model where liquidity varies exogenously and tightens the enforcement constraint on investment studied here. This paper combines elements of those models. The paper also shares insights with some recent studies. Kurlat (2013) independently develops a model where entrepreneurs receive private information about the survival of some of their capital units. As in KM or this paper, entrepreneurs in Kurlat (2013) sell

[1] The reason for this is that large fluctuations in investment have a minor impact on capital, which is ultimately what determines potential output.

their capital to fund investment projects. Like here, asymmetric information induces a shadow cost to obtain those funds. Entrepreneurs have heterogeneous incentives to bear that cost because they differ in their investment efficiencies. Shocks to the distribution of those efficiencies have direct effects, but also lead to selection effects that amplify the original shocks through contractions in liquidity. Here, the incentives to sell capital under asymmetric information are given by the marginal benefit of relaxing financial constraints.

Another related paper is Jermann and Quadrini (2012) (henceforth JQ). Like this paper, JQ stresses that financial frictions have important implications for output when they operate through labor demand. In JQ, entrepreneurs face shocks to an enforcement coefficient that limits their debt holdings. Both papers share the feature that entrepreneurs obtain liquid funds to finance their current operations.[2] The key distinction is that fluctuations here are caused by shocks that aggravate adverse selection. Finally, this model shares common elements with Christiano et al. (2014). That paper studies a business-cycle model with costly state verification about the returns to investment projects. The sources of fluctuations are increases in the dispersion of project returns. Like here, more dispersion coupled with costly state verification leads to lower investment. Christiano et al. (2014) perform a business-cycle decomposition and find that dispersion shocks are important sources of business cycles.

The relationship between liquidity fluctuations and asymmetric information studied here imposes time-series restrictions. Models where liquidity varies exogenously —e.g., KM or del Negro et al. (2010)— do not have an obvious counterpart to credit market conditions such as interest rate spreads, default rates, or loan sizes. Another feature is that adverse selection is aggravated when the returns to investment are low. This induces amplification of TFP shocks and a low correlation between Tobin's Q and investment. Finally, asymmetric information connects the literature on financial frictions with the literature on uncertainty shocks. Recently, Bloom (2009) provides evidence that the dispersion of profits and revenues increases during recessions. As noted by Christiano et al. (2014), this correlation does not establish a causal relationship between dispersion shocks and credit market conditions. However, these countercyclical measures of dispersion are suggestive of a common phenomenon.

This paper develops techniques to overcome several computational difficulties. The model features an interaction between asymmetric information and limited enforcement within a dynamic general equilibrium model with aggregate shocks. The paper shows how to solve for the full dynamics without keeping track of trade histories. I also show how to obtain global solutions to the model allowing for collateralized debt with default. This provides a rich description of loan sizes, interest rates, default rates, and liquidity throughout the business cycle.

The rest of the paper is organized as follows. Section II describes a static model

[2]Both papers share the insights from the literature on working capital constraints that follows from Christiano and Eichenbaum (1992).

of a firm that needs to raise liquid funds by selling capital under asymmetric information to relax enforcement constraints. This exercise describes the key tradeoffs in the determination of liquidity and how this affects labor demand and output. That section also describes a similar problem that distorts investment. Section III shows the relationship between selling capital under asymmetric information and using capital as collateral under asymmetric information. Section IV presents the dynamic model. Section V provides further characterizations using the solutions to the problems of Section II. Section VI presents some quantitative exercises and Section VII concludes. A detailed discussion of the data used and proofs omitted from the text are contained in the online Appendix.

II. Forces at Play

This section presents two static models. They illustrate the key forces behind the dynamic model studied later. Both models are subcomponents of the dynamic model that follows; hence, their analysis serves as an intermediate step.

A. Endogenous Liquidity, Output, and Hours

Consider a static economy in partial equilibrium. The economy is populated by workers that only supply labor, financial firms that buy and sell capital, and entrepreneurs. An entrepreneur maximizes the value of his firm which is the sum of current profits and the value of his capital. The entrepreneur holds k units of capital.

Production. Production is carried out via k, combined with labor, l, using a Cobb-Douglas technology $F(k,l) \equiv k^{\alpha}l^{(1-\alpha)}$ to produce output. The entrepreneur's profits are $AF(k,l) - wl$. The entrepreneur hires workers from an elastic supply schedule $w = l^{\nu}$. Wages are given.

Limited enforcement in labor contracts. Before production, an entrepreneur hires an amount of labor promising to pay wl. It is possible that the entrepreneur reneges on this promise and defaults on his payroll. In that case, workers are able to seize a fraction θ^L of production and the entrepreneur diverts $(1-\theta^L)$ for himself.

To relax this problem, the entrepreneur can pay a fraction $(1-\sigma)$ of the wage bill upfront. Of course, he has to obtain working capital to make this payment before production. He obtains this working capital by selling some capital units. Sold capital units are only reallocated after production. Thus, capital serves two purposes: it is used to obtain liquidity and as a production input. Due to asymmetric information, selling capital translates into a cost to obtain liquidity.

Heterogeneous Capital. The capital stock held by the entrepreneur is comprised of a continuum of pieces. Pieces are identified by their quality $\omega \in [0,1]$. Qualities determine the depreciation of each unit. In particular, there is an increasing, bounded, and continuous function $\lambda(\omega) : [0,1] \to R_+$ that determines the efficiency units that will remain from a given piece of quality ω. The distribution of

ω in that continuum is given by some $f_\phi(\omega)$ with cumulative distribution function (CDF) denoted by F_ϕ. For now, ϕ is a parameter and the unconditional expected value of $\lambda(\omega)$ is $\bar{\lambda}$.

Pieces can be sold separately. I use the indicator function $\iota(\omega) : [0,1] \to \{0,1\}$ to indicate the decision to sell a unit of quality ω.[3] That is, given $\iota(\omega)$, the entrepreneur sells

$$k \int_0^1 \lambda(\omega)\, \iota(\omega)\, f_\phi(\omega)\, d\omega$$

efficiency units and the capital that remains with him is:

$$k \int_0^1 \lambda(\omega)\, [1 - \iota(\omega)]\, f_\phi(\omega)\, d\omega.$$

Information. When a given piece is sold, ω cannot be observed by a buyer. This implies that only the entrepreneur knows the efficiency units that will remain from that particular unit. The buyers of those units are the financial intermediaries. Intermediaries observe the quantity of units being bought, $k \int_0^1 \iota(\omega) f_\phi(\omega)\, d\omega$. However, since they do not observe ω, they do not know how many efficiency units will remain from this portfolio, $k \int_0^1 \lambda(\omega)\, \iota(\omega) f_\phi(\omega) d\omega$.

Markets. The labor market is competitive. I impose the following:

Assumption 1. *Financial firms are competitive and the capital market is anonymous and non-exclusive.*

Competition ensures financial firms earn zero profits. Anonymity and non-exclusivity guarantees that the market for capital features a pooling price. Without anonymity and exclusivity, financial firms could offer menus of prices and quantities. For example, they can recover the full information outcomes if they offer a price schedule proportional to the cumulative distribution of f_ϕ.

The liquidity obtained by selling capital is $pk \int_0^1 \iota(\omega) f_\phi(\omega) d\omega$. Define the liquidity per unit of capital as $x = p \int_0^1 \iota(\omega) f_\phi(\omega) d\omega$. I assume that financial firms sell efficiency units at an exogenous price q.[4] A zero-profit condition for financial firms requires them to equate the value of efficiency units bought to the amount of liquidity given to the entrepreneur. Thus, in equilibrium,

$$pk \int_0^1 \iota(\omega) f_\phi(\omega) d\omega = qk \int_0^1 \lambda(\omega)\, \iota(\omega) f_\phi(\omega) d\omega.$$

This expression yields a relationship between the price under asymmetric information and the perfect information price of efficiency units q:

$$p = q \mathbb{E}_\phi [\lambda(\omega) \,|\iota(\omega) = 1]$$

[3] Qualities have zero measure so the focus on all-or-nothing sales is without loss of generality.
[4] This price is an equilibrium object in the dynamic model.

where $\mathbb{E}_\phi$ is the conditional expectation under f_ϕ. This relationship states that the pooling price equals the value of the expected quality sold. Formally, the entrepreneur's problem is defined as follows:

Problem 1 (Producer)**.** *The entrepreneur solves:*

$$W^p(k;p,q,w) = \max_{\sigma,\iota(\omega),l} \left[Ak^\alpha l^{1-\alpha} - \sigma wl\right] + (xk - (1-\sigma)wl) + q\int_0^1 (1-\iota(\omega))\lambda(\omega)kf_\phi(\omega)\,d\omega$$

subject to:

$$Ak^\alpha l^{1-\alpha} - \sigma wl \geq \left(1-\theta^L\right)Ak^\alpha l^{1-\alpha} \tag{1}$$

$$(1-\sigma)wl \leq xk \tag{2}$$

$$x = p\int_0^1 \iota(\omega)\,d\omega. \tag{3}$$

Recall that σ is the fraction of the wage bill paid after production. The first constraint in this problem, (1), is an incentive compatibility constraint. It states that the output that remains with the entrepreneur after he pays the $\sigma-$fraction of the wage bill must exceed the amount of funds he can divert. Rational workers require this incentive compatibility because they could otherwise provide work to other entrepreneurs at the market wage. The second constraint, (2), is a working capital constraint and it says that the fraction of the wage bill paid in advance, $(1-\sigma)wl$, cannot exceed the liquid funds on hand.

To solve this problem, I employ a version of the envelope theorem and exploit that this problem is homogeneous in capital. The strategy consists of breaking the problem into two subproblems. The first subproblem is an optimal labor choice subject to the enforcement and working capital constraints given an amount of liquidity. The value of this problem yields an indirect profit function of liquidity. The second subproblem determines the qualities sold by use of this indirect profit function.

Hence, let's hold $\iota(\omega)$ —and therefore x— at its optimal value. Once x is determined, the entrepreneur's objective is to choose employment subject to the enforcement constraint (1) and the working capital constraint (3). I solve this problem for $k = 1$ because the objective and constraints are linear in k.

Problem 2 (Optimal Labor)**.** *Given x, the entrepreneur solves*

$$r(x;w) = \max_{l,\sigma} \left[Al^{1-\alpha} - wl\right]$$

subject to

$$Al^{1-\alpha} - \sigma wl \geq \left(1-\theta^L\right)Al^{1-\alpha}$$

and

$$(1-\sigma)\, wl \leq x.$$

The optimal employment decision is given by:

Proposition 1 (Optimal Labor)**.** *The solution to Problem 2 is* $l^*(x) = \min\{l^{cons}(x), l^{unc}\}$ *where* $l^{cons}(x) = \max\left\{l : \theta^L A l^{1-\alpha} + x = wl\right\}$ *and* l^{unc} *is the unconstrained labor choice. Constraints are always slack if* $\theta^L \geq (1-\alpha)$*. If* $\theta^L < (1-\alpha)$*, then* $x > 0$ *is needed to achieve the unconstrained labor amount.*

This proposition states that if liquidity is below a certain level, the entrepreneur must hire less labor than the unconstrained amount. When this is the case, the enforcement and the working capital constraints bind. The entrepreneur is bound to choose employment so that his wage bill equals his liquid funds plus the pledgeable fraction of income. An immediate corollary of Proposition 1 is that if the pledgeable amount of output is less than the efficient labor share, $\theta^L < (1-\alpha)$, efficient employment requires a positive amount of liquid funds. The condition is intuitive: θ^L is the fraction of output that can be fully pledged to workers and since $(1-\alpha)$ is the efficient labor share of output, liquid funds must fill the gap. I return to this observation when I argue that the enforcement constraint will always be active.

Using the envelope theorem, $\iota(\omega)$ can be solved using the indirect profit of liquidity $r(x; w)$.

Lemma 1 (Producer's Problem II)**.** *Problem 1 is equivalent to:*

$$W^p(k; p, q, w) = \max_{\iota(\omega)\geq 0} r(x; w)\, k + xk + qk \int \lambda(\omega)\,(1-\iota(\omega))\, f_\phi(\omega)\, d\omega$$

$$x = p \int_0^1 \iota(\omega) d\omega$$

where $r(x; w)$ *is the value of Problem 2.*

Lemma 1 shows that the decision to sell ω can be analyzed without reference to the employment decision, and this can be analyzed indirectly through the value of liquidity $r(x; w)$. With this simplification, I solve for the optimal selling decision $\iota(\omega)$ and obtain an equilibrium expression for p.

Proposition 2 (Producer's Equilibrium Liquidity)**.** *An equilibrium is characterized by a threshold quality* ω^**. All qualities under* ω^* *are sold. Equilibrium liquidity* x *and the pooling price* p *are given by:*

$$x = pF_\phi(\omega^*) \;\; \text{and} \;\; p = q\mathbb{E}_\phi\left[\lambda(\omega)\,|\omega \leq \omega^*\right].$$

In addition, ω^* *belongs to one of the following cases: [1] Interior solution:* $\omega^* \in (0, 1)$ *and solves*

$$(1 + r_x(x))\, \mathbb{E}_\phi\left[\lambda(\omega)\,|\omega \leq \omega^*\right] = \lambda(\omega^*). \tag{4}$$

[2] Fully liquid: $\omega^* = 1$ *if* $r_x\left(q\mathbb{E}_\phi\left[\lambda\left(\omega\right)\right]\right) \geq 0$. *[3] Market Shutdown:* $\omega^* = \emptyset$ *with* $p = 0$.

Proposition 2 establishes that all equilibria are characterized by a threshold quality ω^* such that all qualities below this one are sold. Equation (4) resembles the equilibrium condition in Akerlof (1970)'s classic lemons example where a marginal quality valuated by a seller equals the expected quality valuated by the buyer. However, there is a key distinction. Whereas in Akerlof (1970) valuations by buyers and sellers are exogenously given, here those valuations depend on the shadow value of an extra unit of liquidity.

The value of additional liquidity to the entrepreneur is $\left(1 + r_x\left(x\right)\right)$. To see this, recall that the entrepreneur obtains p liquid funds by selling a given unit. Those liquid funds are used to pay for the entrepreneur's payroll in advance. Those funds return to the entrepreneur when he sells his output but they also carry the benefit of allowing him to hire additional workers which yields a value of $r_x\left(x\right)$. Hence, the overall, marginal benefit of a given quality of capital is $p\left(1 + r\left(x\right)\right)$. Naturally, costs and benefits must be equal at the margin. When the entrepreneur sells the threshold unit $\lambda\left(\omega^*\right)$, he loses these efficiency units. Those units are worth $q\lambda\left(\omega^*\right)$. Substituting the market-clearing condition and clearing q from both sides gives us the corresponding expression for the interior solutions for ω^*:

$$\underbrace{\left(1 + r_x\left(x\right)\right)}_{\text{Marginal Value of Liquidity}} = \underbrace{\frac{\lambda\left(\omega^*\right)}{\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\omega \leq \omega^*\right]}}_{\text{Marginal Cost of Liquidity}}. \tag{5}$$

This marginal condition is the heart of the model.

Comparative Statics. I assume the following about f_ϕ :

Assumption 2. f_ϕ *satisfies that* $\frac{\lambda(\omega^*)}{\mathbb{E}_\phi[\lambda(\omega)|\omega \leq \omega^*]}$ *is increasing in* ω^*.

This assumption guarantees uniqueness:

Proposition 3 (Interior Solutions)**.** *Under Assumption 2 and* $\lambda\left(0\right) > 0$*, there always exists a single positive* ω^* *in Proposition 2.*

Consider a family of distributions $\{f_\phi\}$ indexed by ϕ. This family has some structure that provides an interpretation to ϕ:

Assumption 3. *The set* $\{f_\phi\}$ *satisfies:*

1) *Mean preserving:* $\int \lambda\left(\omega\right) f_\phi\left(\omega\right) d\omega = \bar{\lambda}$ *for any* $\phi \in \Phi$.
2) *Monotone adverse selection:* $\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\omega \leq \omega^*\right]$ *is weakly decreasing in* ϕ *for any* ω^*.

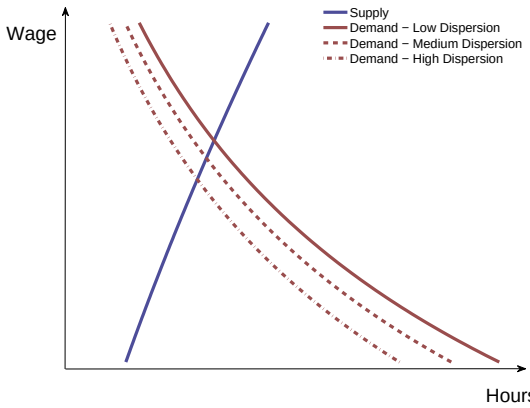


FIGURE 1. LABOR SUPPLY AND LABOR DEMAND AS FUNCTIONS OF ϕ.

The first condition states that for any ϕ, the mean of f_ϕ is always $\bar{\lambda}$. The implication of this condition is that the aggregate amount of capital does not change with ϕ. The second condition provides an order to Φ because it implies that adverse selection is more severe for higher ϕ. Since the second property can often be obtained by an increase in the variance of f_ϕ, from now on, I refer to an increase in ϕ as an increase in dispersion.

For any value of ϕ, equation (5) must hold in equilibrium. Consider then an increase in ϕ. Since by assumption, $\mathbb{E}_\phi[\lambda(\omega)|\omega \leq \omega^*]$ falls with ϕ for any ω^*, the marginal benefit of liquidity, $(1 + r_x(x))$, must increase and the threshold quality ω^* must fall to restore equilibrium. The intuition is that for any given ω^*, after an increase in ϕ, financial firms will pay a lower price because they expect a reduction in the average quality sold. If the entrepreneur does not choose a lower cut-off quality, he will face a marginal loss because losing $\lambda(\omega^*)$ is not worth the new pooling price. The entrepreneur therefore reduces ω^* to the point where the shadow value of relaxing his enforcement constraint compensates for the loss of the new marginal quality. This means that increases in ϕ cause a reduction in the equilibrium amount of liquidity. By Proposition 1, this translates into a contraction in labor demand.

The values of ϕ change over time in the dynamic model so the comparative statics analysis about f_ϕ clarifies the endogenous liquidity mechanism that will be present there. Figure 1 plots the labor-supply schedule against three labor-demand curves that correspond to different values of ϕ. For any wage level, an increase in ϕ reduces the labor demand since the cost of obtaining liquidity becomes higher. The solid lines in Figure 2 exhibit how ϕ determines all the aggregate outcomes for the static economy. The figure illustrates how ϕ induces worse adverse selection and consequent declines in ω^*, p, and x. In turn, hours fall in response to the reduction in liquidity. The contraction in hours explains the

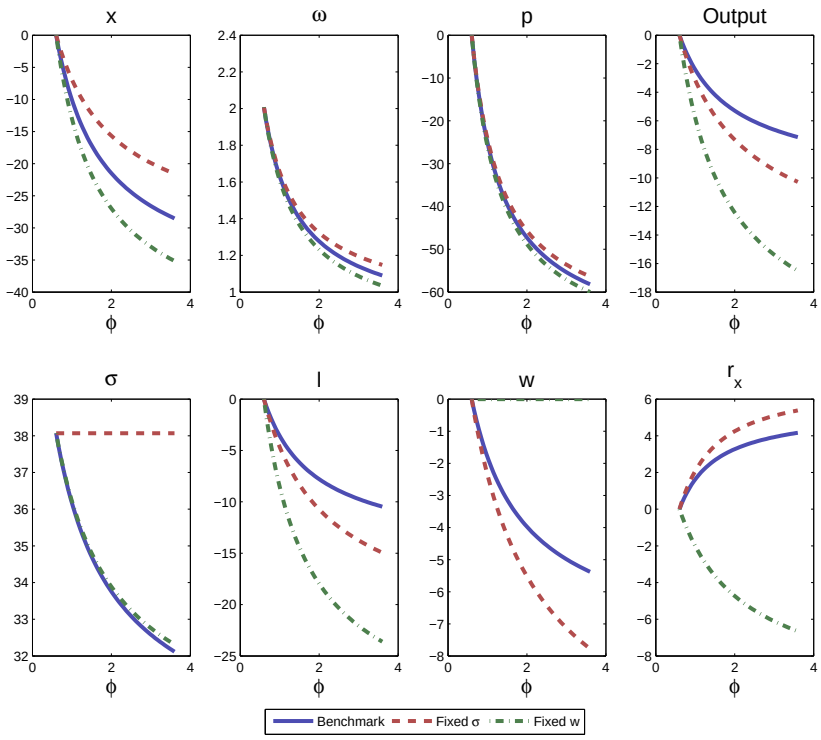


FIGURE 2. COMPARATIVE STATICS ABOUT ϕ FOR DIFFERENT MODEL SPECIFICATIONS.

contraction in output. Moreover, wages fall as labor moves downward along the supply schedule. A final observation is that the entrepreneur's profits increase with ϕ. The general effect of ϕ on profits is ambiguous because the induced movements in hours and wages have opposite effects on profits.

Homotheticity. An important corollary to Proposition 2 is that the entrepreneur's problem is linear in k. This result is key in order to solve the dynamic model and to establish an observational equivalence result with collateralized debt.

Proposition 4 (Value of the Firm)**.** *$W^p(k;p,q,w) = \tilde{W}^p(q,w)k$ where:*

$$\tilde{W}^p(q,w) \equiv r\left(x;w\right) + q\bar{\lambda}. \tag{6}$$

Here, $r\left(x;w\right)$ is the solution to Problem 1 and x, p, and ω^* are given by Proposition 2.

In the Proposition, $\tilde{W}^p(q,w)$ is the sum of per-unit-of-capital profits given x and the marginal value of the entrepreneur's capital stock. The entrepreneur's financial wealth is $xk + qk\int_{\omega^*}^{1}\lambda\left(\omega\right)f_\phi\left(\omega\right)d\omega$, but the zero-profit condition for intermediaries implies $x = q\int_0^{\omega^*}\lambda\left(\omega\right)f_\phi\left(\omega\right)d\omega$. When added, this terms yield $q\bar{\lambda}$.

Discussion

Limited Enforcement of Labor Contracts. The option to default on labor contracts imposes a constraint on the entrepreneur's use of hours that depends on his liquid funds. This form of limited enforcement has a similar effect to exogenous working capital constraints that require the entire wage bill to be paid up front. Exogenous working capital constraints, first introduced by Christiano and Eichenbaum (1992), relate labor demand to borrowing costs. Quantitative work by Christiano et al. (2005) or Jermann and Quadrini (2012) shows that working capital constraints may be important to explain certain business cycle facts.

Exogenous working capital constraints correspond to a limiting case where $\theta^L = 0$. For values of $\theta^L > 0$, the fraction of the wage bill paid up front, $(1-\sigma)$, is not a constant. This distinction has some implications. Under decreasing returns to labor, average labor costs are increasing in the production scale. When the fraction of output that can be pledged is constant, but average costs are increasing, the entrepreneur needs to secure a greater portion of payroll as production increases. The implication is that liquidity per unit of output is increasing in the production scale. The quantitative analysis shows that liquidity over GDP is procyclical and this is consistent with a time-varying, working capital constraint. The dashed curves in Figure 2 repeat the partial equilibrium exercise of the solid curve by varying ϕ under a fixed working capital constraint —when σ is a constant. Overall, a constant working capital constraint amplifies the impact of ϕ.

Wage Rigidity. The model can be easily adapted to incorporate real wage rigidities. The dot-dashed curves in Figure 2 plot the corresponding movements in aggregate variables to changes in ϕ when real wages are constant —and hours are demand determined. The figure shows that wage rigidity leads to a stronger response to ϕ. The reason for this amplification is that wage rigidity opens a feedback effect. Under rigid wages, marginal profits are flatter in hours and this tightens the entrepreneur's enforcement constraint. Thus, more liquidity is needed to employ the same amount of hours. In turn, this higher liquidity need is not met because flatter marginal profits reduce the incentives to obtain liquidity. I draw on this observation when I discuss the quantitative performance of the model.

B. *Endogenous Liquidity and Investment*

This section studies how the endogenous liquidity mechanism may distort investment when an entrepreneur who —as in KM— produces capital needs liquidity to purchase investment inputs. This entrepreneur faces a similar enforcement problem to the one studied previously. I call this entrepreneur the i-entrepreneur to distinguish him from the p-entrepreneur of the previous section.

Production of investment goods. The i-entrepreneur has a constant returns-to-scale technology that transforms a unit of consumption into a unit of capital.

Limited enforcement in investment claims. The i-entrepreneur can sell claims against his investment projects in exchange for consumption goods. Following KM, an i-entrepreneur can divert a fraction $(1-\theta^I)$ of his projects for personal use. This possibility imposes a constraint on his issuance of claims.

Information. This entrepreneur uses capital only to obtain liquid funds. The i-entrepreneur has the same private information about ω as before. In contrast, there is no asymmetric information about investment projects. As before, intermediaries buy capital under asymmetric information, resell capital under full disclosure at an exogenous price q, and earn zero profits.

An i-entrepreneur's problem is similar to the p-entrepreneur's problem except for the differences in technologies: he maximizes his end-of-period wealth. To finance production, he obtains inputs either by selling capital under asymmetric information or issuing claims:

Problem 3 (Investor)**.** *The i-entrepreneur solves:*

$$W^i(k;p,q) = \max_{k^b,i^d,i^s,\iota(\omega)} i - i^s + k^b + \int_0^1 (1-\iota(\omega))\,\lambda(\omega)\,k f_\phi(\omega)\,d\omega$$

subject to:

$$i = i^d + q i^s$$

$$i - i^s \geq \left(1-\theta^I\right) i \tag{7}$$

$$qk^b + i^d \leq xk \tag{8}$$

$$x = p \int_0^1 \iota(\omega) f_\phi(\omega) d\omega.$$

The i-entrepreneur's liquid funds, xk, are also obtained selling capital $\int_0^1 \iota^s(\omega) f_\phi(\omega) d\omega$ at a price p. These funds are used to buy k^b at price q or to buy i^d investment inputs directly—equation (8). Additional investment inputs are obtained by issuing i^s claims against his output at the market price q. Since his production function is linear, his output is $i = i^d + qi^s$. Thus, i^d plays a similar role as the portion of the wage bill paid upfront by the p-entrepreneur. Finally, condition (7) prevents the entrepreneur from diverting funds. I follow the same steps as for p-entrepreneurs and solve for the i-entrepreneur's financial decision given x —and $\iota(\omega)$— first:

Proposition 5 (Optimal Financing)**.** *When $q > 1$, any solution to Problem 3 requires $i^s = \theta^I i$, $k^b = 0$ and $i^d = xk$. When $q = 1$, the solution for i^s, i^d and k^b is indeterminate. If $q < 1$, $k^b = xk$ and $i^d = i^s = 0$.*

The interesting case occurs when $q > 1$. When $q > 1$, the entrepreneur issues as many claims as possible because he exploits an arbitrage —capital costs one consumption unit but sells for $q > 1$ units. Thus, for any investment scale, the i-entrepreneur only finances the $\left(1 - \theta^I q\right)$ fraction but keeps the $\left(1 - \theta^I\right)$ fraction of output. Therefore, his effective internal cost is $q^R = \frac{\left(1-\theta^I q\right)}{\left(1-\theta^I\right)}$. Proposition 6, the analogue of Proposition 2, describes the endogenous liquidity for i-entrepreneurs:

Proposition 6 (Investors Equilibrium Liquidity)**.** *An equilibrium is characterized by a threshold quality ω^i such that all qualities under ω^i are sold by the i-entrepreneur. The equilibrium liquidity and price for i-entrepreneurs are given by:*

$$x^i = p^i F\left(\omega^i\right) \text{ and } p^i = q\mathbb{E}_\phi\left[\lambda(\omega) | \omega \leq \omega^i\right].$$

In addition ω^i is either: [1] Interior solution: $\omega^i \in (0,1)$ and solves

$$\frac{q}{q^R}\mathbb{E}_\phi\left[\lambda(\omega) | \omega \leq \omega^i\right] = \lambda\left(\omega^i\right), \tag{9}$$

[2] Fully liquid: $\omega^i = 1$ if $\frac{q}{q^R} \geq \lambda(1)/\bar{\lambda}$. [3] Market Shutdown: $\omega^i = \emptyset$ with $p^i = 0$.

As with p-entrepreneurs, Proposition 6 states that the solution to the i-entrepreneur's problem is also characterized by a threshold quality. However, in this case, the exogenous valuations in the lemons problem are replaced by Tobin's Q, the ratio of the market price of capital, q, over the replacement cost q^R. Thus, this entrepreneur equates the marginal cost of liquidity to the marginal benefit of

obtaining liquidity —his arbitrage opportunity:

$$\underbrace{\frac{q}{q^R}}_{\substack{\text{Marginal Value of Liquidity}\\ \text{(Tobin's Q)}}} = \underbrace{\frac{\lambda\left(\omega^i\right)}{\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\omega \leq \omega^i\right]}}_{\text{Marginal Cost of Liquidity}}$$

As with the p-entrepreneur, ϕ increases the cost of liquidity. The consequent reduction in liquidity leads to an investment contraction.

Homotheticity. A final result is that linearity of policy functions also holds for the i-entrepreneur's problem:

Proposition 7 (Value of the Firm)**.** *$W^i(k;p,q,w) = \tilde{W}^i(q)k$ where*

$$\tilde{W}^i(q) \equiv \left[\frac{q}{q^R}\int_0^{\omega^i}\lambda\left(\omega\right)kf_\phi\left(\omega\right)d\omega + \int_{\omega^i}^1 \lambda\left(\omega\right)kf_\phi\left(\omega\right)d\omega\right] \tag{10}$$

where ω^i is given by Proposition 6.

For investors, virtual wealth per unit of capital, $W^i\left(X\right)$, takes a different form. This quantity is the sum of his liquid funds times the internal cost of capital plus his unsold units.

III. Collateralized Debt

This section extends the analysis to allow the use of capital as collateral. In practice, productive capital is more commonly used as collateral than for outright sales. In the model, collateralization is also a more efficient form of finance. Notice that in the lemons problem studied above, high-quality capital is not sold because the funds obtained are too low compared to the value of those units. With collateralization, an entrepreneur may be willing to pledge some high-quality units in exchange for the same funds. This is because an entrepreneur only has to pay the interest —instead of the full-information price— to retrieve those high-quality units into his capital stock after he uses his liquidity. This section shows that enriching the contract space along this dimension improves adverse selection but does not alter the essence of the problem. An observational equivalence result shows how to solve equilibria with collateralized debt (CD) and default.

Environment with collateralized debt. The physical environment remains as in Section II. The only distinction is the presence of CD contracts. A CD contract is as follows: The entrepreneur pledges a specific unit of capital as collateral. The contract then specifies a loan size, p^S, and a face value for debt, p^F. The implicit gross interest rate is $R \equiv \frac{p^F}{p^S}$. Thus, with a CD contract, the entrepreneur obtains p^S in IOUs per unit of collateral. The collateral is returned if the entrepreneur pays back p^F after production. If the entrepreneur defaults, the intermediary

seizes the collateral. Seized collateral is sold immediately at a price q and there are no additional default costs.[5]

Markets. I maintain the assumption that the financial market is anonymous and non-exclusive. Under this assumption, the identity of the entrepreneur remains unknown and an entrepreneur can issue CD contracts with many intermediaries. Although there is anonymity about ownership, intermediaries can identify whether a collateral unit has been already pledged in another contract. In particular, I assume there is a collateral registry that prevents the use of the same collateral in multiple contracts. The quality of collateral remains private information, of course. As in the previous section, I focus on contracts where intermediaries earn zero profits and there is full commitment on the side of financial firms. For simplicity, I analyze the decision to collateralize capital under a single contract, $\left(p^S, p^F\right)$. For the rest of this section, I only solve the p-entrepreneur's problem because outcomes are isomorphic for i-entrepreneurs.

Let the indicator $\iota(\omega) : [0,1] \to \{0,1\}$ summarize the decision to use ω as collateral. Given the terms of the CD contract, the entrepreneur obtains $x = p^S \int_0^1 \iota(\omega) f_\phi(\omega) d\omega$ funds per unit of capital stock k. As before, the entrepreneur uses these funds to finance payroll. At the end of the period, the entrepreneur makes an additional financial decision. For every CD contract, he has to decide either to pay the face value of his debt or default and lose his collateral. Let $I(\omega) : [0,1] \to \{0,1\}$ be the indicator for the decision to default on a CD of collateral ω. Total payments to financial intermediaries are $k \int_0^1 p^F (1 - I(\omega)) \iota(\omega) d\omega$ and the value of the capital that remains with the entrepreneur is:

$$qk \int_0^1 \left(\underbrace{(1 - I(\omega)) \iota(\omega)}_{\text{1 if } \omega \text{ in CD without default}} + \underbrace{(1 - \iota(\omega))}_{\text{1 if } \omega \text{ not used as collateral}} \right) \lambda(\omega) f(\omega) d\omega. \tag{11}$$

This remaining capital stock is the sum of two components: The first term inside the parenthesis indicates units used as collateral in contracts that are honored —$\iota(\omega) = 1$ and $I(\omega) = 0$. The second term inside the parenthesis indicates the units that are not used as collateral —$(1 - \iota(\omega)) = 1$. The whole term is zero for qualities that feature default. The value of the remaining capital is priced at q.

The p-entrepreneur's decisions to use collateral and default are based on the calculations above. Recall that the p-entrepreneur's decisions to obtain liquidity using outright sales in Section II can be solved using the indirect profit from liquidity, $r(x; w)$, without reference to his liquidity use. The same principle of optimality also applies for CD contracts. The only additional complication is the decision to default. The analogue to the problem in Lemma 1 is:

[5] This is similar to the contracts in DeMarzo and Duffie (1999). The main difference is that DeMarzo and Duffie (1999) study a security design problem where a borrower and a lender pre-commit to a specific contract that resolves ex-post frictions.

Problem 4 (Producer with CD)**.** *The p-entrepreneur maximizes:*

$$W^p(k;p^S,p^F,q,w) = \max_{I(\omega),\iota(\omega)} r(x;w)\,k + xk - k\int_0^1 p^F\left(1-I(\omega)\right)\iota(\omega)\,d\omega$$

$$qk\int_0^1 \left(1-I(\omega)\right)\iota(\omega)\left(\lambda(\omega)\right) + \left(1-\iota(\omega)\right)\lambda(\omega)\,f(\omega)\,d\omega \tag{12}$$

subject to:

$$x = p^S\int_0^1 \iota(\omega)\,f(\omega)\,d\omega.$$

In this problem, $r(x;w)$ is again the value of liquidity —the value of Problem 2. The entrepreneur maximizes revenues, $r(x;w)\,k + xk$, minus payments to intermediaries, plus the value of his remaining capital stock.

Financial Intermediary Profits. A financial intermediary earns $\left(p^F - p^S\right)$ if a CD contract is honored. If that contract features a default, intermediaries only recover $q\lambda(\omega)$. In either case, intermediaries issue p^S in IOUs. Hence, given price $\left\{p^S,p^F\right\}$ and the entrepreneurs' policies, $\left\{\iota(\omega),I(\omega)\right\}$, average profits for intermediaries are:[6]

(13)

$$\Pi\left(p^F,p^S,\iota(\omega),I(\omega)\right) = \int_0^1 \left[\left(\underbrace{\left(1-I(\omega)\right)p^F}_{\text{non-defaulted debt}} + \underbrace{I(\omega)\,q\lambda(\omega)}_{\text{default recovery}}\right) - \underbrace{p^S}_{\text{loans}}\right]\iota(\omega)\,f(\omega)\,d\omega.$$

Equilibrium with CD. A static equilibrium for the CD market is a pair of prices $\left\{p^S,p^F\right\}$ and policy functions $\left\{I(\omega),\iota(\omega)\right\}$ such that: (1) $\left\{I(\omega),\iota(\omega)\right\}$ are solutions to Problem 4 given prices; (2) intermediaries earn zero profits, i.e., $\Pi\left(p^F,p^S,\iota(\omega),I(\omega)\right)=0$. These equilibria are summarized by a system of three equations and four unknowns:

Proposition 8 (CD Equilibria)**.** *An equilibrium with a single CD contract is characterized by a pair of prices* $\left\{p^S,p^F\right\}$ *and a pair of threshold qualities* $\left\{\omega^p,\bar{\omega}^p\right\}$*. These satisfy the following conditions:*

$$p^S\int_0^{\bar{\omega}^p} f(\omega)\,d\omega = \int_0^{\omega^p} q\lambda(\omega)\,f(\omega)\,d\omega + p^F\int_{\omega^p}^{\bar{\omega}^p} f(\omega)\,d\omega \tag{14}$$

and

$$q\lambda\left(\omega^p\right) = p^F \tag{15}$$

[6]This expression sums profits across all qualities used as collateral —hence, $\iota(\omega)$ outside the bracket in the integral. The term inside the parenthesis indicates the revenue earned on each CD contract. If $I(\omega)=1$ (default), the intermediary earns $q\lambda(\omega)$ and p^F otherwise. Total costs are p^S per contract.

and

$$r_x(x^*) = \left(p^F - p^S\right)/p^S. \tag{16}$$

Qualities satisfy $\omega^p \leq \bar{\omega}^p$. *The equilibrium liquidity is* $x^* = p^S \int_0^{\bar{\omega}^p} f(\omega)\, d\omega$, $\iota(\omega)$ *equals 1 for* $\omega < \bar{\omega}^p$ *and* $I(\omega)$ *equals 1 for* $\omega < \omega^p$.

Proposition 8 characterizes the entire set of possible competitive CD contracts. The proof is relegated to the Appendix, but its idea is simple. In contrast to outright sales, which are characterized by only one threshold quality, there are now two critical thresholds $\{\omega^p, \bar{\omega}^p\}$. All $\omega \in [0, \bar{\omega}^p]$ are used as collateral and all $\omega \in [0, \omega^p]$ feature default. It is natural to observe defaults only among low qualities because if this were not the case, the entrepreneur could always swap a high quality unit that features a default for a low quality that does not. By doing this, he could maintain the same payments to the intermediary, but improve the average quality of his capital stock.

Equation (14) is then the zero profit condition for intermediaries expressed in terms of $\{\omega^p, \bar{\omega}^p\}$. Equation (15) determines ω^p as the quality that makes the entrepreneur indifferent between default and not. Since there are potential defaults, the loan size must be smaller than the face value of debt so that intermediaries do not generate losses. Thus, $p^F - p^S \geq 0$. Consequently, pledging high-quality collateral translates into a financial loss of $p^F - p^S$. This marginal loss, in turn, determines the overall use of collateral because the threshold $\bar{\omega}^p$ is the quality for which additional liquidity $r_x(x^*)\, p^S$ compensates the financial loss of obtaining liquidity $p^F - p^S$. This is the interpretation of equation (16), the analogue of the marginal condition (5) for outright sales.

I discuss the properties of CD contracts in the Appendix. That discussion shows that outright sales are a special case of the CD contracts studied here. The discussion also shows that dispersion also lowers liquidity under CD contracts. Hence, the effects of ϕ under both contracting environments are very similar.

Observational Equivalence. Finally, there is an important observational equivalence. If the zero-profit condition for the intermediary is substituted into the entrepreneur's budget constraint, the value of the entrepreneur's problem is:

$$W^p(k; p^S, p^F, q, w) = \left(r(x) + q\bar{\lambda}\right) k.$$

This is the same value obtained in Proposition 4. This implies that as long as the sales contract of Section II and the CD contracts of this section yield the same amount of liquidity, wealth —and therefore employment— will be the same. An observational equivalence result follows. Fix a given ϕ. For any allocation under outright sales, for another shock ϕ' that yields the same amount of liquidity under CD contracts, the allocations in both environments must be the same. Thus, if ϕ is unobservable, both contracting environments are indistinguishable from aggregate data on liquidity and allocations. This observation also provides an algorithm to

compute equilibria with CD contracts. Moreover, the dynamic model studied in the following section admits aggregation so I will solve the dynamic model using outright sales first —which is simpler— and then obtain the shocks ϕ' that deliver the same allocations when allowing for CD contracts. I use this equivalence result to derive the model's implied terms for CD contracts through the Great Recession.

IV. Dynamic Model

A. Environment

Time is discrete and the horizon infinite. There are two goods: a perishable consumption good (the *numeraire*) and capital. Every period there are two aggregate shocks: a TFP shock $A_t \in \mathbb{A}$ and a shock $\phi_t \in \{\phi_1, \phi_2, ..., \phi_N\} \equiv \Phi$ that selects a member among the family of capital quality distributions $\{f_\phi\}$. A Markov process for (A_t, ϕ_t) evolves according to a transition probability Π.

B. Demography and Preferences

The economy is populated by workers, financial firms, and entrepreneurs as in the static counterparts. All populations are normalized to unity.

Workers. Workers choose consumption and labor. Their period utility is given by $U^w(c, l^w)$ where l^w is their labor supply and c consumption. Workers don't save so they satisfy a static budget constraint: $c_t = w_t l_t^w$ where w_t is the wage. As in Section II, $U^w(\cdot, \cdot)$ leads to a constant Frisch elasticity of ν^{-1}.

Financial Firms. Financial firms purchase capital under asymmetric information and resell capital under full disclosure. They satisfy the same conditions and offer outright sales contracts as in Section II.

Entrepreneurs. An entrepreneur is identified by some $z \in [0, 1]$. Every period, entrepreneurs are randomly assigned one of two possible types: investors and producers. I refer to these types as i-entrepreneurs and p-entrepreneurs because they face the same problems as in Section II. The probability of becoming an i-entrepreneur is equal to π.[7] The entrepreneur's preferences are evaluated by:

$$\mathbb{E}\left[\sum_{t\geq 0} \beta^t U(c_t)\right]$$

where $U(c) \equiv \frac{c^{1-\gamma}}{1-\gamma}$ and c_t is the entrepreneur's consumption at date t.

C. Technology

Technology of p-entrepreneurs. A p-entrepreneur produces consumption goods with the same technology of Section II. Again, he has the technology to divert θ^L

[7] There is a mass π of i-entrepreneurs and $1-\pi$ of p-entrepreneurs every period. With random types, the wealth distribution does not have to be tracked over time.

of his output for personal benefit.

Technology of i-entrepreneurs. The i-entrepreneur has access to the same constant returns-to-scale technology that transforms consumption goods into capital as in Section II. In his case, he can issue investment claims and divert θ^I. Thus, the economy operates like a two-sector economy with sectors producing according to the technologies of the static models presented before.

Capital. At the beginning of every period, capital is divisible into a continuum of pieces. Each piece is identified with a quality ω. Then, $\lambda(\omega)$ determines the corresponding efficiency units that remain from a quality ω. Thus, ω and λ are the same objects defined in Section II.

The distribution among qualities assigned to each piece changes randomly over time. In particular, the distribution ω is determined by the density f_ϕ which, in turn, depends on ϕ_t. This distribution is the same for all entrepreneurs although it is time-varying. Therefore, the measure of units of quality ω out of a capital stock k is $k(\omega) = k f_\phi(\omega)$. Between periods, each piece is transformed into future efficiency units by scaling qualities by $\lambda(\omega)$. Thus, $\lambda(\omega) k(\omega)$ efficiency units remain from the $\omega-$qualities. Once capital units are scaled by efficiency, they form homogeneous capital that can be merged or divided to form larger or smaller pieces. Thus, by the end of the period, the capital stock that remains from k is,

$$\tilde{k} = \int_0^1 \lambda(\omega) k(\omega) d\omega = k \int_0^1 \lambda(\omega) f_\phi(\omega) d\omega. \tag{17}$$

In the next period, every capital stock is divided the same way and the process is repeated indefinitely. Using the earlier notation, $\iota^s(\omega)$ indicates the decision to sell units of quality ω. In equilibrium, financial firms purchase the units sold by entrepreneurs. An entrepreneur transfers $k \int_0^1 \iota^s(\omega) f_\phi(\omega) d\omega$ to the financial sector so the efficiency units that remain with the entrepreneur are $k \int_0^1 \lambda(\omega)(1 - \iota^s(\omega)) f_\phi(\omega) d\omega$. Including investments and purchases of capital, the entrepreneur's capital stock evolves according to:

$$k' = i - i^s + k^b + k \int_0^1 \lambda(\omega)(1 - \iota^s(\omega)) f_\phi(\omega) d\omega, \tag{18}$$

where $i - i^s$ is the net-of-claims internal investment and k^b are purchases of capital from intermediaries. I impose the same assumptions on $\{f_\phi\}$ as before. The implication is that the production possibility frontier is invariant to ϕ.

D. Markets, Information and Timing

Information. Aggregate capital, $K_t \in \mathbb{K} \equiv \left[0, \bar{K}\right]$, is the only endogenous aggregate state variable. The aggregate state of the economy is summarized by the vector $X_t = \{A_t, \phi_t, K_t\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$. At the beginning of each period,

X_t and the entrepreneurs' types are common knowledge. Thus, financial firms offer two pooling prices, one for each activity. Recall that ω is only known to the entrepreneur. Thus, financial firms observe the amount of capital transferred to them, $k\int_0^1 \iota^s(\omega)f_\phi(\omega)d\omega$, but not the quantity that will remain from that purchase, $k\int_0^1 \lambda(\omega)\,\iota^s(\omega)f_\phi(\omega)d\omega$. Hence, the choice of $\iota^s(\omega)$ affects only the distribution of $t+1$ capital between entrepreneurs and intermediaries. Since in the following period $f_{\phi'}$ affects every entrepreneur no matter how they obtain k_{t+1}, entrepreneurs only care about the total amount of capital that remains with them and not its composition. This modeling device is essential to solve the model without keeping track of the history of trades.

Timing. At the beginning of each period, information is revealed. Then, p-entrepreneurs choose $\iota^s(\cdot)$ to obtain liquid funds. Entrepreneurs transfer these funds as an upfront payment to workers. After production, p-entrepreneurs pay the remaining wage bill. With the rest of their output, p-entrepreneurs consume or purchase capital from intermediaries. In exchange for consumption inputs, i-entrepreneurs then sell existing capital and claims against their investment projects to financial firms. All claims are finally settled after the production of capital. This sequence of events is consistent with the physical requirement that consumption is produced before capital. For the rest of the paper, I treat these actions as simultaneous.

Notation: For the remainder of the paper, I append terms like $^j(k,X)$ to indicate the policy function of an entrepreneur of type j in state (k,X). I use $\iota^j(\omega,k,X)$ to refer to the decision to sell a quality ω. I denote by $\mathbb{E}_\phi$ the expectations over the quality distribution f_ϕ and $\mathbb{E}$ the expectations about future states.

E. Entrepreneur Problems and Equilibria

I begin with the description of the p-entrepreneur's problem:

Problem 5 (Producer's Problem)**.** *The p-entrepreneur solves:*

$$V^p(k,X)=\max_{c\geq 0,k^b\geq 0,\iota(\omega),l,\sigma\in[0,1]} U(c)+\beta\mathbb{E}\left[V^j(k',X')|X\right],\ j\in\{i,p\}$$

subject to:

$$c+q(X)\,k^b=AF(k,l)-\sigma wl+xk-(1-\sigma)wl \tag{19}$$

$$k'=k^b+k\int\lambda(\omega)\,(1-\iota(\omega))\,f_\phi(\omega)\,d\omega \tag{20}$$

$$AF(k,l)-\sigma wl\geq\left(1-\theta^L\right)AF(k,l) \tag{21}$$

$$x=p^p(X)\int\iota^s(\omega)f_\phi(\omega)\,d\omega \tag{22}$$

$$(1-\sigma)\,wl\leq xk \tag{23}$$

The first of the five constraints is the budget constraint. The right-hand side of the budget constraint is the entrepreneur's profits minus the amount of liquid funds he holds after paying for the σ fraction of the wage bill. The entrepreneur uses these funds to consume c, and to purchase k^b at the full-information price $q(X)$. The second constraint corresponds to the evolution of the entrepreneur's capital stock with the restriction that p-entrepreneurs cannot produce capital or issue claims. The remaining constraints are the same as those of Section II.

An i-entrepreneur's problem is:

Problem 6 (Investor's Problem)**.** *The i-entrepreneur solves:*

$$V^i(k,X) = \max_{c\geq 0, i, i^s\geq 0, k^b\geq 0, \iota(\omega)\geq 0} U(c) + \beta\mathbb{E}\left[V^j\left(k', X'\right)|X\right], j\in\{i,p\}$$

subject to:

$$c + k' = \tilde{k} \tag{24}$$

$$\tilde{k} = k^b + i - i^s + k\int \lambda(\omega)(1-\iota^s(\omega))f_\phi(\omega)d\omega \tag{25}$$

$$i - i^s \geq (1-\theta^I)i \tag{26}$$

$$q(X)k^b + i^d \leq xk \tag{27}$$

$$i = q(X)i^s + i^d \tag{28}$$

$$x = p^i(X)\int \iota^s(\omega)f_\phi(\omega)d\omega \tag{29}$$

The right-hand side of the i-entrepreneur's budget constraint is the entrepreneur's capital stock after production. He builds this capital stock by producing directly or buying capital. He finances this investment selling capital under asymmetric information and issuing i^s claims to investment at $q(X)$. The constraints in this problem have the same interpretation as in Problem 3. Since capital is reversible, post-production capital is used to consume c or stored for use in subsequent periods.

Financial firms. Financial firms purchase capital units of different qualities from both entrepreneur types at corresponding pooling prices p^p and p^i. They also purchase claims to investment projects at the full-information price $q(X)$. All their capital is resold by the end of the period. I guess and then verify that the decision to sell a unit ω is a function only of the entrepreneur's type and the aggregate state X, but independent of his wealth. Hence, we have the same zero-expected-profit conditions as before:

$$p^p(X) = q(X)\,\mathbb{E}_\phi\left[\lambda(\omega)\,|\iota^{s,p}(\omega,X) = 1\right] \tag{30}$$

and

$$p^i(X) = q(X)\,\mathbb{E}_\phi\left[\lambda(\omega)\,|\iota^{s,i}(\omega, X) = 1\right]. \tag{31}$$

The measure over capital holdings and entrepreneur types at a given period is denoted by $\Gamma(k, j)$ for $j \in \{i, p\}$. By independence,

$$\int k\Gamma(dk, i) = \pi K \quad \text{and} \int k\Gamma(dk, p) = (1-\pi)K. \tag{32}$$

The total aggregate demand for capital and supply of investment claims are:

$$D(X) \equiv \underbrace{\int k^{b,p}(k, X)\,\Gamma(dk, p)}_{\text{Capital demand of p-types}} + \underbrace{\int k^{b,i}(k, X)\,\Gamma(dk, i)}_{\text{Capital demand of i-types}} \text{ and } I^s(X) \equiv \underbrace{\int i^s(k, X)\,\Gamma(dk, i)}_{\text{Supply of new units by i-types}}.$$

Sales of capital by both groups are:

$$\begin{aligned} S(X) \equiv & \underbrace{\int k\left[\int_0^1 \iota^{s,i}(k, X, \omega)\lambda(\omega)\,f_\phi(\omega)\,d\omega\right]\Gamma(dk, i)}_{\text{Efficiency units supplied by i-entrepreneurs}} ... \\ & + \underbrace{\int k\left[\int_0^1 \iota^{s,p}(k, X, \omega)\lambda(\omega)\,f_\phi(\omega)\,d\omega\right]\Gamma(dk, p)}_{\text{Efficiency units supplied by p-entrepreneurs}}. \end{aligned}$$

Clearing of the capital market is given by $D(X) = I^s(X) + S(X)$. Labor market clearing requires: $\int l(k, X)\,\Gamma(dk, p) = l^w(X)$. Finally, one can define aggregate liquidity relative to physical capital as $x(X) \equiv \left(x^i(X)\,\pi + x^p(X)\,(1-\pi)\right)$.

Definition 1 (Recursive Competitive Equilibrium)**.** *A recursive competitive equilibrium is (1) a set of price functions,* $q(X)$, $p^i(X)$, $p^p(X)$, $w(X)$, *(2) a set of policy functions,* $\left\{c^j(k, X), k^{b,j}(k, X), \iota^{s,j}(\omega, k, X)\right\}_{j=p,i}$, $c^w(X)$, $l^w(X)$, $i(k, X)$, $i^s(k, X)$, $l(k, X)$, $\sigma(k, X)$, *(3) a pair of value functions,* $\left\{V^j(k, X)\right\}_{j=p,i}$, *and (4) a law of motion for the aggregate state* X *such that for any distribution of capital holdings* Γ *satisfying (32), the following hold: (1) Taking price functions as given, the policy functions solve the entrepreneurs' and worker's problem and* V^j *is the value of the j-entrepreneur's problem. (2)* $p^p(X)$ *and* $p^i(X)$*satisfy the zero-profit conditions (30) and (31). (3) The labor market clears. (4) The capital market clears. (5) Capital evolves according to* $K' = \int i(k, X)\,\Gamma(dk, i) + \bar{\lambda}K$. *(6) The law of motion for the aggregate state is consistent with the individual's policy functions and the transition function* Π.

V. Characterization

Producer's dynamic problem. I begin with the solution to the p-entrepreneur's problem. The strategy is to break the problem into two subproblems. In the first subproblem, the entrepreneur maximizes the value of his wealth statically by choosing liquidity, and employment. Then, the decision to consume or increase his capital stock is collapsed into a standard consumption-savings problem with linear-stochastic returns that depend on the value of the first subproblem. To see this, note that once k^b is substituted from the capital accumulation equation, (20), into the p-entrepreneur's budget constraint, (19), we obtain:

$$c + q(X)\,k' = AF(k,l) - wl + xk + q(X)\,k \int_0^1 \lambda(\omega)\,(1 - \iota^s(\omega))\,f_\phi(\omega)\,d\omega.$$

The choice of $\iota^s(\omega), l$, and σ only affects the right-hand side of this budget constraint, not the objective function in the p-entrepreneur's problem. The rest of the entrepreneur's constraints only affect the choice of $\iota(\omega), l$, and σ, but not the consumption or savings decision. Hence, the entrepreneur's problem can be broken into two. In the first, he chooses $\iota^s(\omega), l$, and σ to maximize the right-hand side of his budget constraint satisfying the enforcement, liquidity, and working capital constraints. Then, he solves for the c and k^b that maximize his wealth. The first subproblem corresponds to Problem 1 in Section II.

The solutions to $l(X)$ and $\sigma(X)$ are given by Proposition 1 and the equilibrium qualities sold are given by Proposition 2. Thus, a recursive competitive equilibrium is characterized by a threshold quality function $\omega^p(X)$ below which all qualities are sold by p-entrepreneurs in state X. Liquidity $x^p(X)$ is determined by this solution. Once we replace these choices in the p-entrepreneur's problem, we collapse his consumption-savings decisions to a problem where wealth depends on his liquidity-labor choice:

Problem 7 (Producer's Reduced Problem)**.**

$$V^p(k,X) = \max_{c\geq 0, k'\geq 0} U(c) + \beta \mathbb{E}\left[V^j(k',X')|X\right],\ j \in \{i,p\} \tag{33}$$

$$\textit{subject to: } c + q(X)\,k' = W^p(X)\,k \tag{34}$$

$$\textit{where: } W^p(X) \equiv r(x^p(X), X) + q(X)\,\bar{\lambda}. \tag{35}$$

Here, $W^p(X)$ is the entrepreneur's virtual wealth per unit of capital described in Proposition 4.

Investor's dynamic problem. The investor's problem can be solved similarly. Hence, a recursive equilibrium is also characterized by a threshold function $\omega^i(X)$.

This threshold and his financing decisions are characterized by Proposition 6. His problem collapses to:

Problem 8 (Investor's Reduced Problem)**.**

$$V^i(k,X) = \max_{c\geq 0, k'\geq 0} U(c) + \beta \mathbb{E}\left[V^j(k',X')|X\right],\ j\in\{i,p\}$$

$$\text{subject to: } c + k' = W^i(X)\,k$$

(36)

$$\text{where: } W^i(X) \equiv \frac{q(X)}{q^R(X)}\int_{\omega\leq\omega^i(X)} \lambda(\omega)\, f_\phi(\omega)\, d\omega + \int_{\omega>\omega^i(X)} \lambda(\omega)\, f_\phi(\omega)\, d\omega.$$

In this case, the i-entrepreneur's virtual wealth per unit of capital $W^i(X)$ takes the form described in Proposition 7.

Optimal consumption-savings decisions. Problems 7 and 8 are standard consumption-savings problems with homogeneous preferences and linear returns. It is straightforward to show that policy functions are linear in wealth. Therefore, Gorman's aggregation result applies and we have the necessary conditions for aggregation. This result guarantees the internal consistency of the definition of competitive recursive equilibrium without reference to wealth-quality distributions. The optimal consumption-savings decisions are given by:

Proposition 9 (Optimal Policies)**.** *The policy functions for p-entrepreneurs are* $c^p(k,X) = (1-\varsigma^p(X))\,W^p(X)\,k$ *and* $k'^{,p}(k,X) = \frac{\varsigma^p(X)W^p(X)}{q(X)}k$*. For i-entrepreneurs these are* $c^i(k,X) = \left(1-\varsigma^i(X)\right)W^i(X)\,k$ *and* $k'^{,i}(k,X) = \varsigma^i(X)\,W^i(X)\,k$*.*

The functions $\varsigma^p(X)$ and $\varsigma^i(X)$ are marginal propensities to save for p-entrepreneurs and i-entrepreneurs and solve a system of non-linear functional equations. When $\gamma = 1$, this becomes the log-utility case where $\varsigma^p = \varsigma^i = \beta$.

Full information price of capital. The last objects to characterize are $q(X)$ and aggregate investment. One can rearrange the i-entrepreneur's capital accumulation equation, substitute the policy functions in Proposition 9 and integrate across individuals to obtain their net-of-claims aggregate demand for investment:

$$I(X) - I^s(X) = \left[\varsigma^i(X)\,W^i(X) - \int_{\omega>\omega^i(X)} \lambda(\omega)\, f_\phi(\omega)\, d\omega\right]\pi K. \tag{37}$$

Similar steps lead to an expression for the aggregate demand for capital by p-entrepreneurs:

$$D(X) = \left[\frac{\varsigma^p(X)\,W^p(X)}{q(X)} - \int_{\omega>\omega^p(X)} \lambda(\omega)\, f_\phi(\omega)\, d\omega\right](1-\pi)\,K. \tag{38}$$

Total sales of used capital under asymmetric information are obtained by aggre-

gating over the capital sales of both types:

(39)
$$S(X) = \underbrace{\left[\int_{\omega \leq \omega^p(X)} \lambda(\omega) f_\phi(\omega) d\omega\right](1-\pi)K}_{\text{Capital sales by p-types}} + \underbrace{\left[\int_{\omega \leq \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega\right]\pi K}_{\text{Capital sales by i-types}}.$$

Capital market clearing requires $D(X) = S(X) + I^s(X)$. In addition, all investors must satisfy their constraints given by inequality (7). By linearity, an aggregate version of this condition holds if and only if all the individual constraints hold. Thus, any equilibrium must be characterized by $q(X)$ such that $D(X) = S(X) + I^s(X)$ and $\theta^I I(X) \leq I^s(X)$ hold. The solution to $q(X)$ and the set of equilibrium conditions are found in the Appendix.

Inefficiency. A distinguishing feature of this environment is that active enforcement constraints are key to support transactions under asymmetric information. This implies that if positive liquidity is required to support efficient allocations, then enforcement constraints must always be strictly binding:

Proposition 10. *Employment is sub-efficient* $(l^w)^\nu < A_t F_l(l^w, K_t)$ *if and only if* $\theta^L < (1-\alpha)$. *Investment is sub-efficient in the sense that* $I_t > 0$ *implies* $q_t > 1$.

VI. Quantitative Analysis

This section analyzes the qualitative and quantitative business-cycle patterns generated by shocks to the distribution of asset qualities. For this purpose, I construct a series for ϕ_t and use this series to generate artificial data from the model. I then contrast this data with actual financial and economic activity data from the US. The goal is to provide a quantitative sense of the strength of the endogenous liquidity mechanism.

A. Calibration, Estimation, and Measurement of ϕ_t

There are two parameter sets. The first set is standard in the real business-cycle literature. The second relates to the financial frictions in the model so I have no benchmark for its calibration. Instead, I use a two-step procedure to estimate this set. In the first step, I construct an inferred series for ϕ_t using a subset of the equilibrium conditions of the model and arbitrary parameters. In the second step, I insert the constructed series into the other equilibrium conditions to construct model-implied moments. I then use these model-implied moments to estimate some parameters through the generalized method of moments (GMM).

Notation. I adopt the following notation. I denote by $\hat{d}_t$ an observed empirical counterpart of a model variable d, by ϑ the vector of parameters that I estimate and by $\hat{\Theta}_t$ the data vector —at period t— that I use to estimate parameters. I denote by $\hat{d}_t|\vartheta, \hat{\Theta}_t$ an unobserved empirical counterpart of a variable d deduced from the model's equilibrium conditions given parameter values and data.

Data. I use several quarterly macroeconomic time series ranging from 1983:IV-2013:II.[8] I use a subset of this data to construct the series for ϕ_t. The rest of the data is used for the estimation and to evaluate the model's performance. In total, I use seven time series corresponding to the series of aggregate output, consumption, investment, the capital stock, total hours, aggregate liquidity, and TFP. The data for output, investment, and consumption is obtained from the National Income and Product Accounts (NIPA) corresponding to the Gross Domestic Product, Gross Private Non-Residential Fixed Investment and the Personal Consumption Expenditures. The series of capital stock is obtained from Fernald (2012) who applies the perpetual inventory method to various forms of capital. The data on hours corresponds to the series of Hours of All Persons Working in the Nonfarm Business Sector (NFBS)from the US Bureau of Labor Statistics (BLS). The time series for liquidity represents total external finance, which is the sum of Credit Market Instruments and Net Worth of Nonfinancial Noncorporate Businesses and Nonfinancial Corporate Businesses as in Covas and Den Haan (2011). The source of this data is the Flow of Funds (FoF) constructed by the Board of Governors of the Federal Reserve System. However, since there is no single ideal data counterpart for liquidity, in Section VI.C I also compare the model with the data series for Commercial and Industrial Loans (C&I) obtained from banks' call reports (banks) and individual issuances of syndicated loans (syndication). Finally, for TFP I use the series constructed by Fernald (2012).

The data used for the construction of $\hat{\phi}_t|\vartheta, \hat{\Theta}_t$ is the output-to-capital share $\hat{Y}_t/\hat{K}_t$, the investment-to-capital share, $\hat{I}_t/\hat{K}_t$, and hours, $\hat{l}_t$. Thus, the data vector is $\hat{\Theta}_t \equiv \left\{\hat{Y}_t/\hat{K}_t, \hat{I}_t/\hat{K}_t, \hat{l}_t\right\}$. All the data in the paper is used and reported in real terms and detrended. I detrend the data combining the Hodrick-Prescott (HP) filter with a linear trend for the Great Recession. This resolves some common issues found when running the HP filter on Great Recession data.[9] A detailed description is contained in the Appendix.

Calibrated Parameters. A period is a quarter. I use log utility: for any choice of $\{\gamma, \beta\}$, one can find a corresponding value for β such that marginal propensities to consume under log preferences are approximately the same as with the original pair of parameters. Thus, I set $\beta = 0.97$ and $\gamma = 1$ to approximate policy functions corresponding to CRRA preferences with a coefficient of relative risk aversion of 2 and a discount factor of 0.991. Log utility is a useful benchmark because the stochastic process for $\{A_t, \phi_t\}$ does not affect intertemporal decisions so the impulse response analysis in the next section does not depend on the actual

[8] I use this sample period because the time series I use for liquidity were very volatile prior to this period. The same sample period is used in JQ or Christiano et al. (2014).

[9] For output, the HP-filtered series shows a decline prior to the Great Recession. This leads to a positive cycle component during the beginning of the recession. Moreover, the magnitude of the recession seems small relative to potential output. See Comin and Gertler (2006) for a lengthier discussion about similar problems.

process for $\{A_t, \phi_t\}$. I calibrate $U^w(c,l)$ to obtain a static labor supply with a Frisch elasticity of 2; this elasticity falls within the range used in macro models.[10] The value of $\bar{\lambda}$ is set to obtain an annualized depreciation rate of 10% and the fraction of investors, π, is set to 0.1 to match plant level investment frequencies found in Cooper et al. (1999).[11] I use the cycle component of Fernald's TFP series as the counterpart $\hat{A}_t$.

Estimated Parameters. I assume $\hat{A}_t$ follows a log-AR(1) process with mean, autoregressive coefficient, and standard deviation of innovations denoted by $\{\mu_A, \rho_A, \sigma_A\}$. I estimate this process via maximum likelihood (ML) to obtain estimates for $\{\mu_A, \rho_A, \sigma_A\}$. The rest of the set of estimated parameters are: [1] the family distributions $\{f_\phi\}$, [2] the coefficient of capital in the production function, α,[12] [3] the enforcement parameters θ^L and θ^I, [4] Φ, the set of possible values of ϕ_t, and [5] the transition matrix Π. For $\{f_\phi\}$, I assume a log-normal parametric form.[13] Under this parametric form, ϕ represents the standard deviation of $f_\phi(\omega)$ where the mean under f_ϕ has to equal $\bar{\lambda}$. The set of parameters I estimate using GMM is $\vartheta \equiv \left\{\alpha, \theta^I, \theta^L\right\}$.

In models such as JQ, financial shocks can be constructed as a residual from a single equilibrium condition. This model does not have a single equation from which to infer ϕ_t. Instead, I use the two-stage procedure to estimate ϑ and jointly obtain a measurement for ϕ_t. Once I perform this estimation, I compute Φ and Π using the frequencies of values of the constructed time series for $\hat{\phi}_t$ and $\hat{A}_t$.

The two-step procedure to estimate ϑ and obtain the measurement of ϕ_t follows Burnside et al. (1993). In the first step, I construct a time series $\hat{\phi}_t|\vartheta, \hat{\Theta}_t$ given arbitrary values of ϑ. To construct $\hat{\phi}_t|\vartheta, \hat{\Theta}_t$, I use a subset of the equilibrium conditions of the model. This subset is the set of equations that relate the corresponding investment and labor decisions to the liquidity holdings of each entrepreneur, the marginal conditions that determine their liquidity holdings, and their capital sales. These set of equations constitute a system of six equations and nine unknowns. However, three of those unknowns correspond to the variables in $\hat{\Theta}_t$. If I treat each data point as a parameter, I then only have six unknowns which I solve for each t. These solutions are the empirical counterparts of $\left\{\hat{\phi}_t, \hat{q}_t, \hat{\omega}_t^i, \hat{\omega}_t^p, \hat{x}_t^i, \hat{x}_t^p\right\}$ given values for ϑ and $\hat{\Theta}_t$.[14]

[10] I use $U^w(c,l) \equiv \frac{c^{1-\gamma}}{1-\gamma} - \frac{l^{1+\zeta}}{1+\zeta}$. With the assumption that workers do not save, this specification yields a static demand schedule where ν is a function of γ and ζ.

[11] The data suggests that around 20%-40% of plants augment a considerable part of their physical capital stock in a given year. These figures vary depending on plant age. Setting π to 0.1, the arrival of investment opportunities is such that 30% of firms invest in a year.

[12] In models where the labor market is distorted, the labor share is no longer equal to $(1-\alpha)$. Hence, I cannot calibrate α by setting it equal to the labor share.

[13] The choice of log normals is immaterial. I have performed a robustness check for the choice of $\{f_\phi\}$. I calculated the impulse response analysis for families of Beta, Gamma and exponential distributions. Only minor changes in the quantitative results are found. A log normal family is chosen because it is used in many continuous time models with stochastic volatility and dispersion.

[14] Appendix VIII.C describe exact conditions and the algorithm to obtain $\hat{\phi}_t|\vartheta, \hat{\Theta}_t$.

In the second step, I insert $\hat{\phi}_t|\vartheta, \hat{\Theta}_t$ and $\hat{A}_t$ back into the remaining equilibrium conditions of the model. With this, I obtain additional series for the rest of the variables in the model and compute moment conditions from these series. Clearly, these moment conditions will be indexed by the value of ϑ that delivered the particular estimate of $\hat{\phi}_t|\vartheta, \hat{\Theta}_t$. Following Burnside et al. (1993), I estimate ϑ by GMM using the two-step estimator in Newey and West (1987).[15] I use both first and second moment conditions. I use the average residual between the marginal product of labor (MPL) and the worker's marginal rate of substitution (MRS) as a first moment. This moment corresponds to the average labor wedge in Shimer (2009) which is reported to be about 0.4. Because I do not want to attribute all the residual to the financial friction in the model, I target a value of 0.15 which is also consistent with a 25% average labor tax.[16] The distance between the MPL and the MRS provides information about the relationship between $\{\theta^L, \alpha\}$. As an additional first moment, I target the fraction of investment that is externally financed. This fraction is 35.2% and is obtained from the estimates in Ajello (2012). In the model, this fraction corresponds to $q_t\theta^I$ and therefore provides information about θ^I. The values of ϑ also affect the construction of $\hat{\phi}_t$ and, consequently, the correlations and relative volatility of hours and investment with respect to output. Thus, I use these correlations and relative standard deviations as additional second moments. These correlations and relative deviations are computed from the sub-sample that precedes the Great Recession and are reported in Table 2 together with other data moments used for the evaluation of the model.

The estimates and calibration values are found in Table 1. The value of θ^I is low in comparison to values calibrated in del Negro et al. (2010) and Ajello (2012). As explained in Section II, this model needs a low value for θ^I to deliver the right investment-output comovement because liquidity shocks shift wealth from workers toward the agents that invest, entrepreneurs. The estimate for α is lower than the usual 1/3 because the labor friction requires this to obtain a labor-income share of 2/3. The value of θ^L implies that p-entrepreneurs must obtain funds for roughly 2/3 of their payroll. Finally, I perform a J-test to evaluate the validity of the overidentification restrictions: at the 95% confidence interval, the test cannot reject the null hypothesis that the orthogonality conditions of the model hold —the J-statistic yields a value of 2.78 and there are three degrees of freedom.

After I obtain estimates for ϑ, I set Φ as a uniform grid over the range of values of $\hat{\phi}_t|\vartheta, \hat{\Theta}_t$. I set Π to be consistent with the empirical frequencies of $\left\{\hat{A}_t, \hat{\phi}_t\right\}$ on that grid.[17]

[15] I search for values in $0 \leq \theta^L \leq \alpha \leq 1/3$, and $0 \leq \theta^I \leq 1 - \pi$. These restrictions follow from Proposition 10 and guarantee that liquidity is needed in equilibrium.

[16] For the average labor tax I take the sum of the average individual and payroll tax rates across all income brackets. See Table A.3 in Piketty and Saez (2006).

[17] The correlation between $\hat{A}_t$ and $\hat{\phi}_t|\alpha, \theta^L, \theta^I, \hat{\Theta}_t$ is not significant. Thus, Π is built from two inde-

Table 1—Calibration Summary.

Parameter	Value	Notes
Preferences		
γ	1	2.5% risk-free rate and CRRA of 2
β	0.97	2.5% risk-free rate and CRRA of 2
ν	1/2	Frisch elasticity of 2
Technology		
π	0.1	Investment freq. in Cooper et al. (1999)
λ	0.9781	10% annual depreciation
α	0.31 (0.24,0.37)	GMM estimate
θ^I	0.09 (0.08,0.10)	GMM estimate
θ^L	0.36 (0.31,0.41)	GMM estimate
TFP		
μ_A	0	Normalized Constant
ρ_A	0.78 (0.71,0.85)	ML estimate
σ_A^2	0.008 (0.0074,0.0086)	ML estimate

Notes: Numbers in parenthesis correspond to the 95% confidence intervals. The confidence interval for the values ϑ is computed using the asymptotic distribution of the parameter estimates derived in Hansen (1982).

Measured Series. Figure 3 reports some series obtained corresponding to the last 30 years. The top panels plot the measured series for $\hat{\phi}_t$ and $\hat{A}_t$. Dispersion shocks $\hat{\phi}_t$ take low values for most of the sample but feature two short-lasting and medium-sized spikes following the recessions of the early nineties and mid two thousands. At the beginning of the Great Recession, dispersion is close to its historical level. However, towards the midst of the crisis, $\hat{\phi}_t$ shows a dramatic increase which persists even after the recession is over and reverts back to historical values only by 2012. The bottom panels plot the model's implied output and liquidity series —the weighted sum over both entrepreneurs— against their data counterparts. Figure 3 shows a good fit to the output series. Moreover, the model does a good job fitting the path for liquidity in the data —which is not used in the construction $\hat{\phi}_t$— for the entire sample. Section VI.C describes the fit of the model to data from the Great Recession in more detail. Before that, I discuss the model's properties.

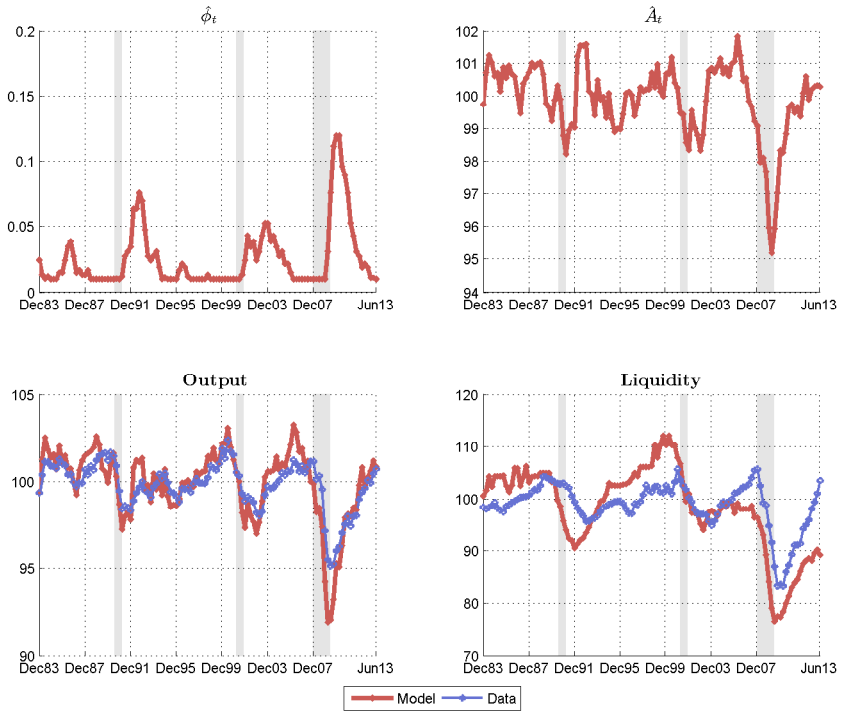
B. *Stationary Equilibrium Properties*

Computation. This section studies the stationary equilibrium of the model. Since the model is non-linear, I use global methods to compute this equilibrium. All the exercises use a grid of six elements for both A and Φ and 120 for aggregate capital. A larger grid size does not affect results.

Business Cycle and Financial Statistics. Table 2 compares the model-generated moments with the data moments and the moments of the canonical Real Business Cycle (RBC) model in King and Rebelo (1999). Naturally, the correlation

pendent processes.

FIGURE 3. MEASURED $\hat{\phi}_t$ AND $\hat{A}_t$ AND MODEL FIT TO OUTPUT AND LIQUIDITY.



Note: The series for $\hat{A}_t$, Liquidity, and Output are reported in trend deviations normalized to 100. The series for $\hat{\phi}_t$ is reported in levels.

Table 2—Data, Model and RBC Statistics.

Correlation with Y:	A_t	l_t	c_t	I_t	x_t
Data	0.79	0.88	0.76	0.83	0.48
	(0.03)	(0.02)	(0.08)	(0.05)	(0.13)
Model	0.91	0.97	0.96	0.94	0.5
RBC	1.00	0.98	0.94	0.99	-
Std. Relative to Y:	A_t	l_t	c_t	I_t	x_t
Data	0.53	1.17	0.77	4.52	2.26
	(0.13)	(0.065)	(0.041)	(0.16)	(0.51)
Model	0.42	0.73	0.90	1.7	2.4
RBC	0.68	0.48	0.44	2.95	-

Note: Numbers in parenthesis indicate standard errors computed through the Delta method.

between output and TFP is lower here than in the RBC model —and closer to the data— because dispersion shocks are an additional source of fluctuations that the RBC lacks. Dispersion shocks lower the correlation between hours and output slightly and bring the model closer to the data. Similarly, the correlation between investment and output is also lower and closer to the data. The model can deliver lower correlations than the RBC because productivity may move in the opposite direction than hours and investment when liquidity moves in the opposite direction. Another feature of the model is that consumption and output have a higher correlation than in the RBC model which is also why consumption is more volatile than in the RBC model —and closer to the data. However, the volatility of investment here is lower than in the RBC model. The higher volatility of consumption and the lower volatility of investment follows from the assumption that workers are hand-to-mouth which, as explained earlier, causes an increase in entrepreneurial wealth that partially offsets the volatility of investment after dispersion shocks. Section II suggests that wage rigidity may improve the performance of the model by removing that countervailing force. Finally, the correlation and relative volatility of liquidity and output are also very close to the data —the correlation between liquidity and output equals 0.45, a figure consistent with cross-sectional evidence in Table 2 in Covas and Den Haan (2011).

Impulse Response. The impulse response analysis of a one-time shock to ϕ_t is useful to single out the effects of dispersion shocks. Although in the calibration ϕ_t features persistence, a one-time shock provides a measure of the magnitude and persistence of the responses through the internal propagation of the model. Figure 4 reports the responses of several variables when ϕ_t increases from its unconditional mean to a value that brings output down as in the Great Recession.[18]

[18] All impulse responses are computed by constructing 20,000 Monte Carlo simulations of the model. For this, I take a random draw from the invariant distribution of X and then simulate the model forward. For every simulation, I extract the path of shocks, generate an alternative path that differs only in the first period dispersion shock, and then construct a new simulation of the model from the alternative path of shocks. For each model variable, the reported impulse response is the average, across simulations pairs, of the difference between the sample paths in each simulation pair.

In response to the shock, the value of liquid funds contracts immediately for both i- and p-entrepreneurs —by 10.5% and 20%, respectively. These responses are induced by the adverse selection that raises the implicit cost of obtaining liquidity. On impact, hours and wages fall by 8% and 4%, respectively, given the contraction of labor demand. Output falls by 6.0% due to the reduction in hours. With less liquidity, investment falls 11.0%. Less investment translates into lower future capital which drives the dynamics of the system in subsequent periods. The effect on most variables almost vanishes after one period because the impact on the capital stock is small. The plots at the bottom present the responses of p^i and p^p relative to q —which shows the increase in the cost of obtaining liquidity. Although the model does not feature a riskless bond, an implied risk-free rate can be obtained from the aggregate consumption of entrepreneurs.[19] The implied risk-free rate features a negative response of 50 basis points. The patterns in this impulse response are consistent with facts [1]-[4] in the Introduction.

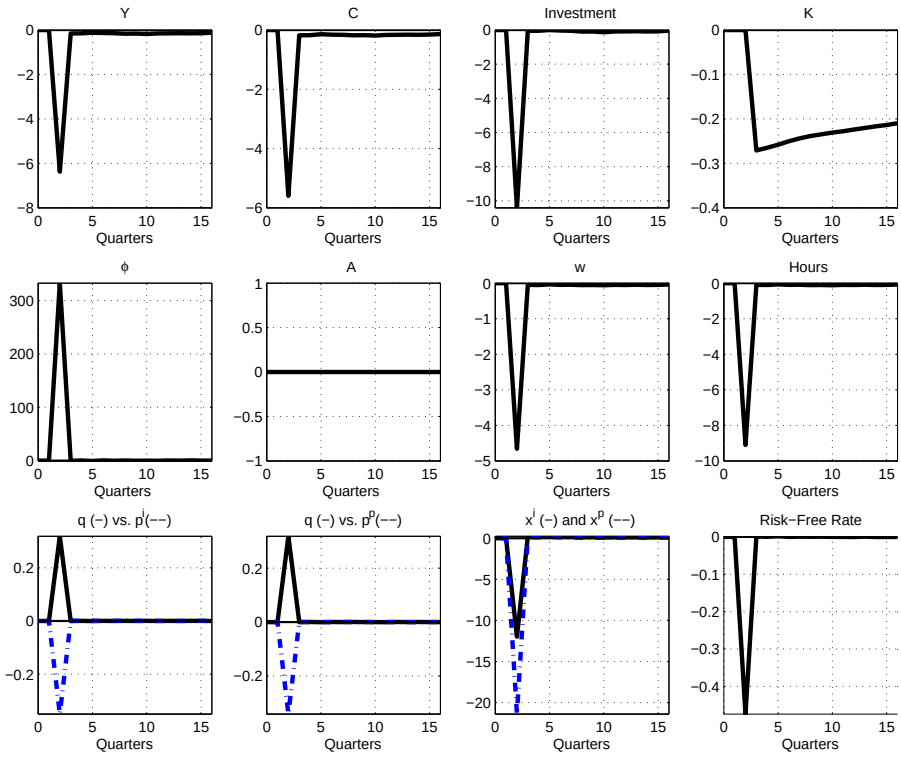
Labor-supply elasticity. A key parameter to deliver a large output response is the labor-supply elasticity. The top panels of Figure 5 display impulse responses for different values of labor-supply elasticity. Section II explains how reductions in liquidity affect labor demand and, in equilibrium, this is met with a reduction in hours and wages. The relative response of either margin depends on the labor-supply elasticity. Naturally, the response of hours —and consequently output, consumption, and investment— is stronger for higher Frisch elasticities.

Which frictions matter? Both, the enforcement constraints on investment and labor are needed to generate the right comovement between output, consumption and investment after a dispersion shock. The bottom panels of Figure 5 present the impulse responses —when both frictions are active— and the responses when only one of the frictions is active. The exercise shows that the enforcement constraint on labor is essential in order to generate a strong output response. Without the labor friction, the shock only affects output through its effects on capital which, in turn, moves little. The model also needs the investment friction: without this friction, investment reacts positively to a dispersion shock through the increase in the entrepreneur's wealth.

TFP Amplification. Equations 4 and 5 are the marginal conditions that characterize the endogenous amount of liquidity. The model features an amplification mechanism of TFP shocks captured through those equations: a TFP shock raises the wealth of p-entrepreneurs and their demand for capital. In turn, a greater demand for capital translates into a higher value for q which leads to higher liquidity for both entrepreneurs. Although this amplification mechanism is present, a quantitative investigation reveals that it is not a strong amplification mechanism.

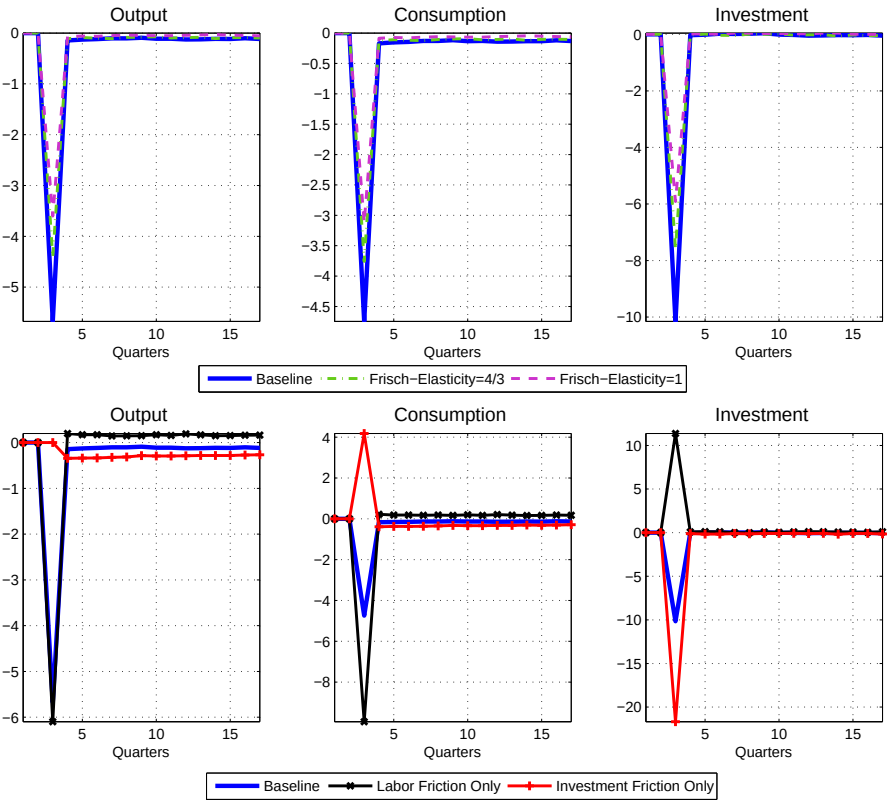
[19] If entrepreneurs are allowed to buy and sell type insurance, the model features a representative entrepreneur. I use this representative entrepreneur to obtain a price for a riskless bond in zero-net supply. The pattern for the response in the risk-free rate to dispersion shocks depends on the participation of workers in asset markets.

FIGURE 4. IMPULSE RESPONSE TO ϕ.



Note: Except for the risk-free rate, all impulse responses are reported in per cent deviations from the unconditional mean under the invariant distribution of the model. The response of the risk-free rate is reported in levels.

FIGURE 5. IMPULSE RESPONSE TO ϕ UNDER DIFFERENT ENFORCEMENT PARAMETERS AND FRISCH ELASTICITIES.



Note: All impulse responses are reported in per cent deviations from the unconditional mean given a parameter combination.

C. *Endogenous Liquidity during the Great Recession*

The measurement of $\hat{\phi}_t$ shows that the model suggests an abnormal increase in dispersion to explain the Great Recession. This section investigates the model's fit to macroeconomic and credit-market data during that episode. Unless expressed in rates, I report detrended data and model variables as percent deviations from their corresponding values during 2007:III.[20]

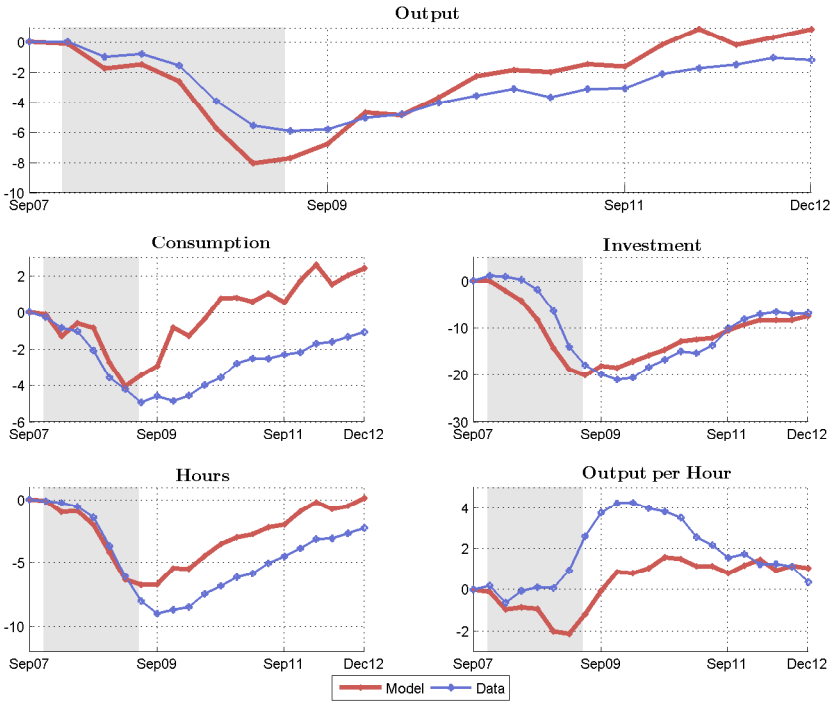
Real Quantities. Figure 6 describes the model's fit to macroeconomic variables. Figure 6 shows that the model closely tracks the magnitudes and patterns for consumption, investment, hours, and output-per-hour. The model does overstate the decline of output by 2% because output-per-hour is lower in the model. The model also shows an investment path close to the data although consumption recovers more quickly. According to the model, the first half of the recession was due to lower TFP. The economic decline from the second half onward is attributed to hours and investment which, in turn, fall due to the contraction in liquidity.

The model attributes the onset of the recession to TFP because it does not feature utilization. Thus, TFP here is in part capturing the decline in utilization noted in Fernald (2012). In turn, $\hat{\phi}_t$ increases during the middle of the Great Recession and persists even after the recessions ends. This is because in the data output recovers, but investment and hours remain depressed: the lack of liquidity distorts employment and investment so the model attributes this pattern to high values of $\hat{\phi}_t$.

Figure 3 already shows that the implied path for liquidity is similar to the path of external funding in the data, a comovement consistent with the microeconomic evidence in Chodorow-Reich (2013) and Fort et al. (2013). Next, I compare the model's predictions for credit market variables with the data to test whether dispersion shocks are also consistent with the observed credit-market patterns and their magnitudes.

Credit Conditions. To relate $\hat{\phi}_t$ to credit market conditions, I draw on the observational equivalence between sales of capital and collateralized debt developed in Section III. That section explains how to reconstruct an equilibrium with CD contracts that replicates equilibrium allocations under outright sales. I use the allocations from the dynamic model with sales given $\hat{\phi}_t$ to reverse engineer an alternative measure of dispersion shocks, $\hat{\phi}'_t$, consistent with the same allocations under CD. I then use the loan sizes, interest rates, and charge-offs —defaults rates times recovery amounts— of CD contracts to compare the model with the data. The data for C&I loans data and from loan syndication provide data on these variables. As noted earlier, there is a continuum of CD contracts consistent with a level of dispersion. Thus, for the rest of this section, I focus on contracts

[20] Recall that $\hat{\phi}_t$ is constructed from a subset of the equilibrium conditions and a combination of the data on economic activity. The series implied by the model are recovered after I introduce $\left\{\hat{\phi}_t, \hat{A}_t\right\}$ back into the model. Thus, the data that conforms Θ_t and the model series do not have to be identical by construction. Credit market data is not used at all to obtain $\hat{\phi}_t$.



Note: All series are reported in deviations from trend.

FIGURE 6. MODEL FIT TO GREAT RECESSION DATA FOR MACROECONOMIC VARIABLES.

that maximize aggregate liquidity given $\hat{\phi}_t'$ —the contracts for which all units are used as collateral.[21] To summarize the model's predictions into a single variable, I compute the CD contracts for i-entrepreneurs and p-entrepreneurs and report their averages weighted by π and $(1-\pi)$.

The main message of this section is that, without additional features, the model requires high implied interest rates to explain the decline in liquidity during the Great Recession. However, as explained in Section II, the marginal benefit of obtaining liquidity —the interest rates implied by the model— is less responsive under fixed real wages. That analysis shows that counterfactually low real wages can induce counterfactually high interest rates. Indeed, during the Great Recession, the real wages implied by the model fall more than in the data. This suggests that the large response of interest rates may be a result of that shortcoming of the model. To address this issue, I reconstruct the implied CD contracts for two versions of the model. In the first exercise, I obtain the CD contracts for the model as described so far. In the second exercise, I treat real wages as an exogenous process whose realizations are given by the data. In that exercise, hours are given by the demand for labor —the worker's first-order condition do not hold.[22]

Figure 7 contrasts the objects of the model-implied CD contracts with credit market data for the Great Recession. The model-implied CD contract objects are reported for both exercises, with the model-implied wages (MW) and with the realizations of wages from the data (DW). The top-left panel of Figure 7 reports the measures of liquidity in the model —for both MW and DW— and the data on firm external finance from the FoF. Both model series fit that liquidity measure well. The top-right panel shows two alternative measures of aggregate liquidity: the outstanding volume of C&I loans and the series for individual issuances of syndicated loans. When compared to the external finance series from the FoF, the bank data suggests a delayed decline in lending, although this decline reaches a similar magnitude at its trough. The volume of loan syndication is synchronized with the decline in the FoF but, as expected, its decline is more severe.[23]

The middle-left panel reports the two implied measures of dispersion when liquidity is obtained through CD. These measures are the CD counterparts of the dispersion shock in Figure 3 for both the MW and DW series. Both measures feature a similar path of dispersion as in Figure 3. The implied increase in dispersion is greater when wages are endogenously determined.

[21] I do not model this selection explicitly. However, Martin (2007) provides conditions such that equilibria in models with adverse selection are pooling and Pareto efficient. Here, the contracts that maximize aggregate liquidity —those for which $\bar{\omega} = 1$— are indeed, constrained Pareto efficient and pooling. These contracts are also consistent with the optimal security design in DeMarzo and Duffie (1999). Moreover, these are the contracts with the lowest default and interest rates, so they give the model the best fit.

[22] I use the series for Real Compensation Per Hour in the Nonfarm Business Sector reported by the Bureau of Labor Statistics. In both exercises, I can use the same procedure to back out ϕ_t because the procedure does not use equilibrium conditions from the labor supply. See the Appendix for more details.

[23] The series for syndicated loans corresponds to the volume of new issuances —this is the only series available—and not outstanding. Another difference is explained by Ivashina and Scharfstein (2010) who attribute the initial increase in C&I to previously agreed upon credit lines.

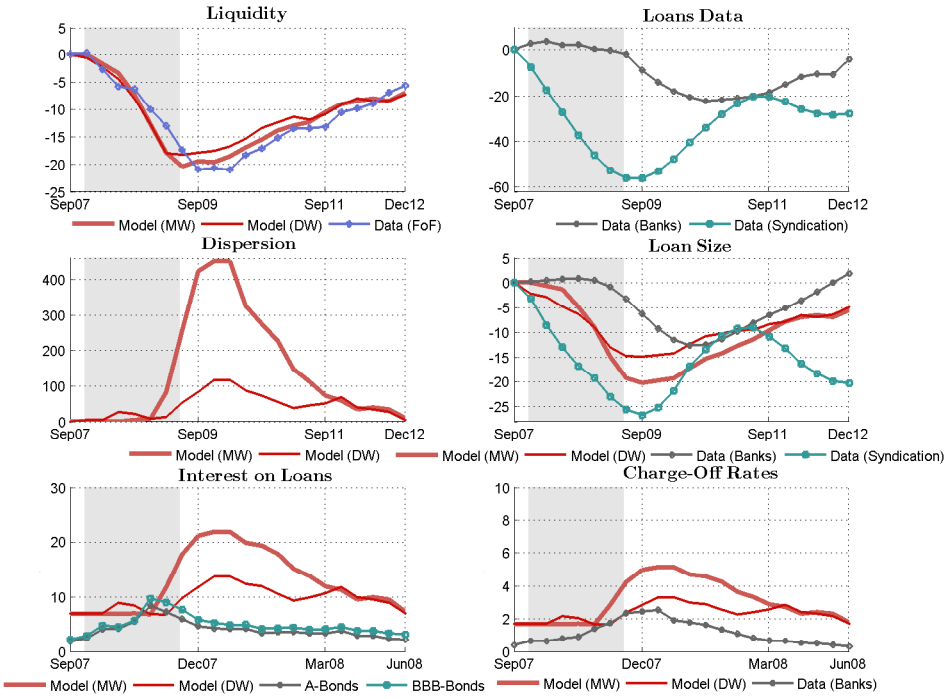
The middle-right panel reports the corresponding path for the average loan size, p^S, in the two model series and the two data series. Both the data and the model series show a decline in the average loan size through the recession. Under CD contracts, the increase in asset quality dispersion leads to a decline in loan size because the default thresholds increase and the value of collateral given a default threshold is, on average, lower. The decline in loan size under model-implied wages is slightly greater, although both model series lie between both data series.

In the model, financial firms respond to higher default thresholds by increasing the average interest rate. A consistent, sharp increase in interest-rate spreads is also found in the US corporate-bond market. The increased spreads in the model and the data are shown in the bottom-left panel —which correspond to the A and BBB BofA-Merrill Lynch US Corporate Bond indexes. Spreads in the model peak two quarters after spreads peak in the data. The delay in the model occurs because the model tracks the decline in hours and the trough in hours is posterior to the spike in spreads in the data. Moreover, the magnitudes of interest rates in the model with endogenous wages is two times higher than in the data, although these spreads are less pronounced when I use wage data. Business loan charge-offs from the data are shown in the bottom-right panel. Charge-offs in the model series are defined as the difference between p^F and the value of seized collateral $q\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\omega<\omega^d\right]$, also reported in the bottom-right panel. The timing of the model and data series are similar, although this could follow from delayed accounting in the data. Consistent with the behavior of interest rates, the magnitudes of charge-offs in the MW series are almost three times higher than in the data. With DW, charge-offs are only slightly higher than in the data.

Wage Rigidity and Model Fit. Why does the model perform better with wages taken from the data? In the model, hours and investment fall when the cost to obtain liquidity rises. This relationship is established through the marginal condition (16) that equates the interest rate on a CD contract to the increase in the marginal profits from obtaining additional liquidity. Given the calibration, a drop in hours of the magnitude observed in the data leads to a sharp increase in marginal profits from additional liquidity —approximately 10%— when wages are endogenous. This means that the model needs a large increase in interest rates which can only be explained by a large increase in charge-offs. When annualized, the differences in interest rates are twice higher than in the data.

The increase in marginal profits follows from two margins, the increase in the marginal product of labor and the decline in wages. The model improves its fit when I use the wage data because wages are less responsive in the data and this mitigates the second margin. Since with DW marginal profits do not increase as much during the Great Recession, the DW version requires a more modest increase in interest rates and charge-offs —see the discussion in Section II.

FIGURE 7. MODEL FIT TO GREAT RECESSION DATA FOR CREDIT VARIABLES.



Note: Except for interest on loans and charge-off rates, all series are reported in deviations from trend. Interest on loans and charge-off rates are reported in levels.

VII. Conclusions

This paper describes how asymmetric information about capital quality endogenously determines the amount of liquid funds when these are used to relax enforcement constraints. The paper shows how the dispersion of capital quality increases the cost of obtaining liquidity by selling capital or using capital as collateral. The increased costs of obtaining liquidity carry real effects through the exacerbation of financial frictions. One interpretation is that recessions are episodes where multiple economic forces cause disproportionate effects in the intrinsic value of different productive assets. Coupled with the endogenous liquidity mechanism, this leads to economic declines although the productive capacity of the economy did not change.

The main lessons are: [1] Endogenous liquidity is determined by a condition that equates the marginal benefit from relaxing financial constraints to the marginal cost of obtaining liquidity under asymmetric information. [2] To explain a large impact on output, the model requires limited enforcement in labor contracts and a high labor-supply elasticity. [3] A quantitative experiment shows that dispersion shocks can cause collapses in liquidity and other macroeconomic variables of the magnitudes and patterns observed during the Great Recession. However, the implied reduction in liquidity requires an excessively high cost to obtain liquidity. This deficiency is ameliorated using actual wage data.

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VIII. Equilibrium Conditions

Equilibrium Conditions. Aggregate labor demand is obtained aggregating across p-entrepreneurs via: $L^d(X) = l^*(x^p(X), X)(1-\pi)K$. Worker's consumption is $c = w(X) l^w(X)$. In equilibrium, the leisure-consumption tradeoff defines the aggregate labor supply: $w(X)^{\frac{1}{\nu}} = l^w(X)$ so $w(X) = (l^*(x^p(X), X)(1-\pi)K/\bar{\omega})^{\nu}$. Aggregate output is $Y(X) = r(x, X)(1-\pi)K + (l^*(x^p(X), X)(1-\pi)K)^{\nu+1}$. From Proposition 9, one can aggregate across entrepreneurs to obtain aggregate consumption and capital holdings:

$$\begin{aligned} C^p(X) &= (1-\varsigma^p(X))W^p(X)(1-\pi)K \text{ and } C^i(X) = \left(1-\varsigma^i(X)\right)W^i(X)\pi K, \\ K'^{,p}(X) &= \varsigma^p(X)W^p(X)(1-\pi)K/q(X) \text{ and } K'^{,i}(X) = \varsigma^i(X)W^i(X)\pi K/q^R(X). \end{aligned}$$

Aggregate capital evolves according to $K'(X) = K'^{,i}(X) + K'^{,p}(X)$.

Solving for $q(X)$. First note that $q(X) < 1$ can never be part of an equilibrium. If $q(X) < 1$, i-entrepreneurs would not supply investment claims because they would rather purchase capital than invest. Thus, if $q(X) < 1$ then $I(X) < 0$. However, if this is the case and capital is reversible, $q(X) = 1$ because the technical rate of transformation for all agents is 1. Hence, $q(X) \geq 1$.

Given prices and policy functions, $I(X) - I^s(X)$ can be solved for from (37) and $I^s(X) = D(X) - S(X)$. Given that $I^s(X)$ and $I(X) - I^s(X)$ are known, one can verify if $\theta^I I(X) \leq I^s(X)$. If this condition is satisfied, $q(X) = 1$. If not, $q(X)$ must be greater than 1 to satisfy incentive compatibility.

Proposition 5 ensures that when $q(X) > 1$, enforcement constraints bind so $I^s(X) = \theta^I I(X)$. Substituting this equality into (37) yields a supply schedule. In addition, the supply of capital $S(X)$ is increasing and demand $D(X)$ decreasing in $q(X)$. Thus, $q(X)$ is found by solving for the market-clearing condition when enforcement constraints are binding. Proposition 11 describes the solution to $q(X)$:

Proposition 11 (Market Clearing)**.** *The equilibrium full information price of capital is given by:*

$$q(X) = \begin{cases} q^o(X) & \text{if } q^o(X) > 1 \\ 1 & \text{if otherwise} \end{cases} \tag{40}$$

where $q^o(X)$ is a function of $(W^p(X), W^i(X), \varsigma^s(X), \varsigma^i(X))$.

The proof is presented in the online Appendix and is similar to the one found in Bigio (2009).

A. *Optimal Policies in Proposition 9*

Define the following:

$$\begin{aligned} R^{pp}\left(X',X\right) &\equiv \frac{W^p\left(X'\right)}{q\left(X\right)} \text{ and } R^{pi}\left(X',X\right) \equiv W^p\left(X'\right), \\ R^{ii}\left(X',X\right) &\equiv W^i\left(X'\right) \text{ and } R^{ip}\left(X',X\right) \equiv \frac{W^i\left(X'\right)}{q\left(X\right)}. \end{aligned}$$

These virtual returns are used to obtain $\varsigma^i(X)$ and $\varsigma^p(X)$:

Proposition 12 (Recursion)**.** *Marginal propensities to save, ς^i and ς^s satisfy:*

$$\left(1-\varsigma^i\left(X\right)\right)^{-1} = 1+\beta^{1/\gamma}\Omega^i\left(\left(1-\varsigma^p\left(X'\right)\right),\left(1-\varsigma^i\left(X'\right)\right)\right), \tag{41}$$

$$\left(1-\varsigma^p\left(X\right)\right)^{-1} = 1+\beta^{1/\gamma}\Omega^p\left(\left(1-\varsigma^p\left(X'\right)\right),\left(1-\varsigma^i\left(X'\right)\right)\right) \tag{42}$$

where

$$\begin{aligned} \Omega^i\left(a\left(X'\right),b\left(X'\right)\right) &\equiv \mathbb{E}\left[\left(1-\pi\right)\left(a\left(X'\right)\right)^{\gamma}R^{pi}\left(X'\right)^{1-\gamma}+\pi\left(b\left(X'\right)\right)^{\gamma}R^{ii}\left(X'\right)^{1-\gamma}\right]^{1/\gamma}, \\ \Omega^s\left(a\left(X'\right),b\left(X'\right)\right) &\equiv \mathbb{E}\left[\left(1-\pi\right)\left(a\left(X'\right)\right)^{\gamma}R^{pp}\left(X'\right)^{1-\gamma}+\pi\left(b\left(X'\right)\right)^{\gamma}R^{ip}\left(X'\right)^{1-\gamma}\right]^{1/\gamma}. \end{aligned}$$

In addition, $\varsigma^p,\varsigma^i\in(0,1)$ and equal (β,β) if $\gamma=1$.

B. Remaining Equilibrium Equations

An equilibrium is a fixed point of the functions $q(X),\omega^p(X),\omega^i(X),\varsigma^p(X)$ and $\varsigma^i(X)$. Once this fixed point is obtained, the rest of the equilibrium objects are obtained through the remaining equilibrium conditions. The following set of functional equations summarizes the equilibrium conditions. For presentation purposes, I present these in three blocks:

Capital Market Clearing Block:

$$q^R(X) = \frac{1-\theta q(X)}{1-\theta}$$

$$I(X)-I^s(X) = \left[\varsigma^i(X)W^i(X)-\int_{\omega>\omega^i(X)}\lambda(\omega)f_\phi(\omega)d\omega\right]\pi K$$

$$D(X) = \left[\frac{\varsigma^p(X)W^p(X)}{q(X)}-\int_{\omega>\omega^p(X)}\lambda(\omega)f_\phi(\omega)d\omega\right](1-\pi)K$$

$$S(X) = \underbrace{\left[\int_{\omega\leq\omega^p(X)}\lambda(\omega)f_\phi(\omega)d\omega\right](1-\pi)K}_{\text{Capital sales by p-types}} + \underbrace{\left[\int_{\omega\leq\omega^i(X)}\lambda(\omega)f_\phi(\omega)d\omega\right]\pi K}_{\text{Capital sales by i-types}}$$

$$D(X) = S(X) + I(X)$$

$$I^s(X)(1-\theta) \leq \theta(I(X) - I^s(X))$$

Marginal Propensities Block:

$$\begin{aligned} R^{pp}(X', X) &\equiv \frac{W^p(X')}{q(X)} \text{ and } R^{ip}(X', X) \equiv W^p(X') \\ R^{ii}(X', X) &\equiv W^i(X') \text{ and } R^{ip}(X', X) \equiv \frac{W^i(X')}{q(X)} \end{aligned}$$

$$\begin{aligned} \left(1 - \varsigma^i(X)\right)^{-1} &= 1 + \beta^{1/\gamma} \Omega^i\left(\left(1 - \varsigma^p(X')\right), \left(1 - \varsigma^i(X')\right)\right) \\ (1 - \varsigma^p(X))^{-1} &= 1 + \beta^{1/\gamma} \Omega^p\left(\left(1 - \varsigma^p(X')\right), \left(1 - \varsigma^i(X')\right)\right) \end{aligned}$$

$$\begin{aligned} \Omega^i(a(X'), b(X')) &\equiv \mathbb{E}\left[(1-\pi)\left(a(X')\right)^\gamma R^{pi}(X')^{1-\gamma} + \pi\left(b(X')\right)^\gamma R^{ii}(X')^{1-\gamma}\right]^{1/\gamma} \\ \Omega^p(a(X'), b(X')) &\equiv \mathbb{E}\left[(1-\pi)\left(a(X')\right)^\gamma R^{pp}(X')^{1-\gamma} + \pi\left(b(X')\right)^\gamma R^{ip}(X')^{1-\gamma}\right]^{1/\gamma} \end{aligned}$$

$$W^i(X) \equiv \frac{1}{q^R(X)}\left[q(X)\int_{\omega \leq \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega + q^R(X)\int_{\omega > \omega^i(X)} \lambda(\omega) f_\phi(\omega) d\omega\right]$$

$$W^p(X) \equiv r(x^p(X), X) + x^p(X) + q(X)\int_{\omega > \omega^p(X)} \lambda(\omega) f_\phi(\omega) d\omega$$

Liquidity Block:

$$\begin{aligned} \frac{q(X)}{q^R(X)} \mathbb{E}_\phi\left[\lambda(\omega) | \omega \leq \omega^i(X), X\right] &= \lambda\left(\omega^i(X)\right) \\ (1 + r_x(x^p, X)) \mathbb{E}_\phi\left[\lambda(\omega) | \omega \leq \omega^p(X), X\right] &= \lambda(\omega^p(X)) \end{aligned}$$

$$x^i(X) = q(X) \mathbb{E}_\phi\left[\lambda(\omega) | \omega \leq \omega^i(X), X\right]$$

$$\begin{aligned} x^p(X) &= q(X) \mathbb{E}_\phi\left[\lambda(\omega) | \omega \leq \omega^p(X), X\right] \\ w(X) &= (l^*(x^p, X) K)^\nu \\ l^*(x^p, X) &= \min\left\{\arg\max_l \theta^L A l^{1-\alpha} - wl = x^p, l^{unc}\right\} \end{aligned}$$

$$r\left(x^p, X\right) = Al^{*1-\alpha} - \left(l^*\left(x^p, X\right)K\right)^{\nu+1}$$

and $q\left(X\right)$ given by Proposition 11.

C. *Measurement of* $\hat{\phi}|\vartheta, \hat{\Theta}_t$

This section explains how I use a subset of the equilibrium conditions to obtain a time series for $\hat{\phi}_t|\vartheta, \hat{\Theta}_t$. The procedure also yields a series for $\hat{\omega}^p_t, \hat{\omega}^i_t, \hat{x}^p_t, \hat{x}^i_t$, and $\hat{q}_t$ given $\hat{\Theta}_t$ and arbitrary values for ϑ. The series for $\hat{\phi}_t|\vartheta, \hat{\Theta}_t$ is used to estimate ϑ, as explained in Section VI.

Proposition 10 shows that i-entrepreneurs are always constrained when $I_t \geq 0$. In the data, $I_t \geq 0$ always holds. Thus, I combine the conditions in Proposition 5 when constraints are binding to obtain[24]:

$$x^i k = \left(1 - q\theta^I\right) i. \tag{43}$$

In turn, Proposition 6 shows that:

$$x^i = q\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\omega \leq \omega^i\right] F_\phi\left(\omega^i\right). \tag{44}$$

Substituting (44) into (43) yields:

$$q\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\omega \leq \omega^i\right] F_\phi\left(\omega^i\right) = \left(1 - q\theta^I\right) i/k. \tag{45}$$

In turn, Proposition 6 also shows that,

$$q\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\omega \leq \omega^i\right] = q^R\lambda\left(\omega^i\right). \tag{46}$$

Replacing the left-hand side of (46) in (45) yields:

$$\lambda\left(\omega^i\right) F_\phi\left(\omega^i\right) = \frac{\left(1 - \theta^I\right) i}{k}. \tag{47}$$

Now, recall that the investment-to-capital ratio is the same across all i-entrepreneurs. Moreover, they hold the π fraction of the capital stock. Thus, we have that $i/k = \frac{1}{\pi}\frac{I}{K}$. Using this identity, and combining it with (47) yields an equation that relates $\hat{I}_t/\hat{K}_t$ in the data to $\hat{\omega}^i|\phi, \hat{I}_t/\hat{K}_t$:

$$\hat{I}_t/\hat{K}_t = \pi\lambda\left(\hat{\omega}^i\right) F_\phi\left(\hat{\omega}^i\right) / \left(1 - \theta^I\right). \tag{48}$$

This is an implicit equation in $\hat{\omega}^i$ that one must solve to obtain $\hat{\omega}^i|\phi, \hat{I}_t/\hat{K}_t$. I rearrange (46) and use the definition of q^R:

[24]The relationship follows from $i^d = xk$, $i^s = \theta^I i$, and $(i^d + qi^s) = i$.

$$\hat{q}|\phi,\hat{I}_t/\hat{K}_t = \lambda\left(\hat{\omega}^i\right)/\left(\left(1-\theta^I\right)\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\lambda<\lambda\left(\hat{\omega}^i\right)\right]+\theta^I\lambda\left(\hat{\omega}^i\right)\right). \tag{49}$$

So far the procedure yields values for $\hat{\omega}^i|\phi,\hat{I}_t/\hat{K}_t$ and $\hat{q}|\phi,\hat{I}_t/\hat{K}_t$, for an arbitrary ϕ.

An application of the implicit function theorem —see the proof of Proposition 2 in the online Appendix— shows that:

$$r_x = -\frac{\left(1-\alpha\right)Y - wL}{\left(1-\alpha\right)\theta^L Y - wL} \tag{50}$$

and hence,

$$r_x = -\left[\frac{\left(1-\alpha\right)-wL/Y}{\left(1-\alpha\right)\theta^L - wL/Y}\right] = -\left[\frac{\left(1-\alpha\right)-S}{\left(1-\alpha\right)\theta^L - S}\right]$$

where S is the labor share. I use this formula to obtain $r_x|\hat{S}_t$.

Equation (4) is used to obtain $\hat{\omega}^p|\phi,\hat{S}_t$ by implicitly solving for:

$$r_x|\hat{S}_t = \frac{\lambda^p\left(\hat{\omega}^p\right)}{\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\omega<\hat{\omega}^p\right]} - 1. \tag{51}$$

I substitute $\hat{\omega}^p|\phi,\hat{S}_t$ in for $\hat{\omega}^p$ in the definition of the p-entrepreneur's liquidity —see Proposition 2—to obtain:

$$\hat{x}^p|\phi,\hat{I}_t/\hat{K}_t,S_t = \hat{q}|\phi,\hat{I}_t/\hat{K}_t\cdot\mathbb{E}_\phi\left[\lambda\left(\omega\right)|\omega<\hat{\omega}^p|\phi,\hat{S}_t\right]\cdot F_\phi\left(\hat{\omega}^p|\phi,\hat{S}_t\right). \tag{52}$$

I now use the equilibrium conditions for p-entrepreneurs. Proposition 10 also shows that p-entrepreneurs are always constrained when $\theta^L<(1-\alpha)$, a parameter restriction that I impose. From (1), this implies:

$$\tilde{x}^p|\hat{Y}_t/\hat{K}_t,S_t = \left(\frac{\hat{Y}_t/\hat{K}_t}{(1-\pi)}\right)\left(\hat{S}_t-\theta^L\right).$$

I then solve for ϕ in the equation:

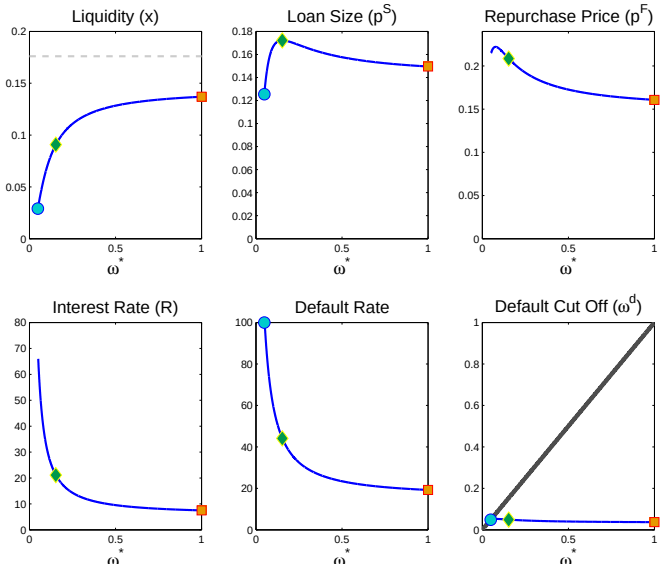
$$\tilde{x}^p|\hat{Y}_t/\hat{K}_t,\hat{S}_t = \hat{x}^p|\phi,\hat{I}_t/\hat{K}_t,\hat{S}_t.$$

The solution to this equation is the measurement $\hat{\phi}_t|\vartheta,\hat{\Theta}_t$. I reverse the process to obtain the measurements $\hat{\omega}^p_t,\hat{\omega}^i_t,\hat{x}^p_t,\hat{x}^i_t$ and $\hat{q}_t$. With deduced series, I can solve the rest of the model.

I use the series in $\hat{\Theta}_t$ to reconstruct $\hat{S}_t$, using the wages delivered by the model

given hours. The series is normalized to a value of 2/3. In Section VI.C, I use the real wage data to build an alternative series for $\hat{S}_t$. I normalize this series to obtain an average labor share of 2/3.

FIGURE 8. SET OF EQUILIBRIA CD CONTRACTS.



IX. Appendix (not for publication)

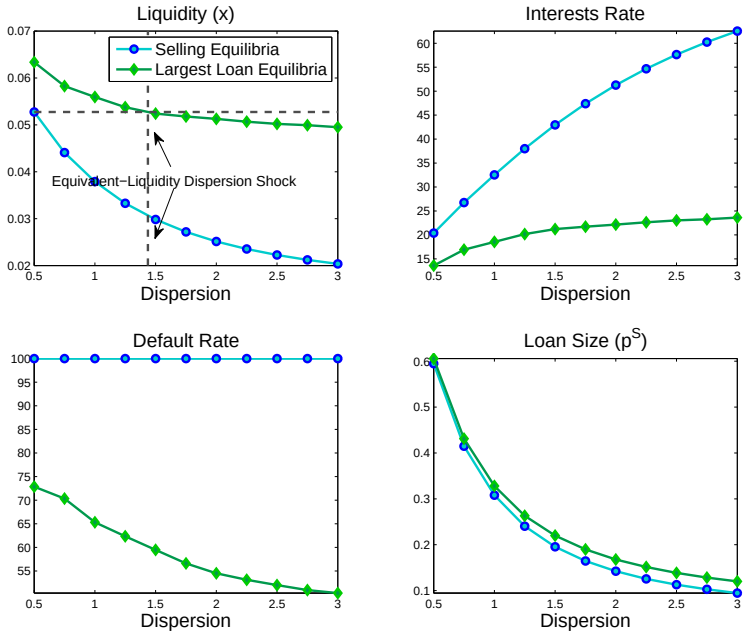
A. *Properties of CD contracts*

The set of competitive equilibrium CD has a continuum of contracts. For a particular example, Figure 8 depicts the entire set of equilibria. Each equilibrium is indexed by some ω^* corresponding to a participation threshold $\bar{\omega}^p$. The figure depicts the properties of the set. The upper panels display equilibrium liquidity and the implied interest rate for a participation cutoff ω^*. The bottom panels show the implied default rate, $F(\omega^p)/F(\bar{\omega}^p)$, and the loan size p^S for each equilibria. There are three equilibria of particular interest: the one for which, $\omega^p = \bar{\omega}^p$ —circle—, the equilibrium where $\bar{\omega}^p = 1$ —square— which corresponds to the optimal liquidity contract in DeMarzo and Duffie (1999), and the equilibrium with the largest loan size, p^S —diamond. It is worth discussing these properties.

Properties. The first property is that the CD for which $\bar{\omega}^p = \omega^p$, corresponds to the selling contracts of Section II. This is the case because, in equilibrium, defaulting or selling is the same. This is also the equilibrium with the lowest participation. Second, liquidity is increasing in the participation cutoff ω^*. The more collateralization, the higher the quality collateral pool and the lower the default rate. Third, because higher participation rates require greater incentives to participate, p^S may be decreasing in ω^*. As a consequence, p^S is possibly non-monotone in ω^*. In the quantitative section, I focus on the contract with the highest liquidity.

Observational Equivalence. Figure 9 follows the procedures to compute equilibria in Figure 8 and computes the highest liquidity contracts for different

FIGURE 9. OBSERVATIONAL EQUIVALENCE BETWEEN OUTRIGHT SALES AND CD CONTRACTS.



values of dispersion. In the top panel, one can observe that given an initial value of liquidity with sales, one can increase the dispersion in the equilibria with CD to obtain the same amount of liquidity. This figure illustrates the construction of observationally equivalent equilibria.

B. A Glance at Recursive Competitive Equilibria

Endogenous liquidity. Figure 10 presents four equilibrium objects in each panel. Within each panel, the four curves correspond to combinations A (high and low) and ϕ (high and low). The x-axis of each panel is the aggregate capital stock, the endogenous state.

The top panels describe the equilibrium liquid funds per unit of capital, x, for both entrepreneur types. Given a combination of TFP and dispersion shocks, liquidity per unit of capital decreases with the aggregate capital stock (although its total value increases) for both types. For p-entrepreneurs, this negative relationship follows from decreasing marginal profits in the aggregate capital stock. With lower marginal benefits from increasing liquidity, p-entrepreneurs have less incentives to sell capital under asymmetric information. Comparing the curves that correspond to low and high dispersion shocks, we observe that liquidity falls with dispersion. As explained in Section II, increases in the quality dispersion increases the shadow cost of selling capital under asymmetric information. In contrast, TFP has the opposite effect. These results are clear from equation (5)

which captures the tradeoffs in the choice of liquidity. An analogous pattern is found for i-entrepreneur's liquidity. The reason is that the demand for investment is weaker when the capital stock is greater or TFP is low.

Hours, consumption, investment, and output. As dispersion reduces the liquidity of producers, their effective demand for hours falls, causing a reduction in output. When TFP or the capital stock are high, hours and output are higher, as in any business cycle model. The figure also shows the negative effects of dispersion shocks on investment. With less liquidity available, the supply of investment claims shrinks. The reduction in the liquidity of p-entrepreneurs has ambiguous effects on their profits because this reduces the amount of labor hired but, wages also fall. This ambiguous wealth effect implies that the demand for capital may increase after liquidity shortages. Also, the ambiguous wealth effect could also increase consumption because of the increase in the cost of investment. For the calibration, the overall effect involves a strong reduction in investment, consumption, and hours together with an increase in the price of capital, q, as we should expect in a recession. The subsequent section discusses the ingredients that are needed for this result.

The analysis shows how the low correlation between Tobin's Q and investment is determined by two counterbalancing forces as in Lorenzoni and Walentin (2009). The first is TFP, which produces a positive correlation between Q and investment. The second is dispersion, which causes an increase in Tobin's Q together with a reduction in investment. This shows the connection among the six business cycle facts discussed in the Introduction.

C. *Proof of Proposition 1*

Rearranging the incentive compatibility constraints in the problem consists of solving:

$$\begin{aligned} r(x) &= \max_{l\geq 0,\sigma\in[0,1]} Al^{1-\alpha} - wl \\ &\quad \text{subject to} \\ \sigma wl &\leq \theta^L Al^{1-\alpha} \text{ and } (1-\sigma)\, wl \leq x. \end{aligned}$$

Denote the solutions to this problem by (l^*, σ^*). The unconstrained labor demand is $l^{unc} \equiv \left[\frac{A(1-\alpha)}{w}\right]^{\frac{1}{\alpha}}$. A simple manipulation of the constraints yields a pair of equations that characterize the constraint set:

$$l \leq \left[A\frac{\theta^L}{\sigma w}\right]^{\frac{1}{\alpha}} \equiv l^1(\sigma) \tag{53}$$

$$l \leq \frac{x}{(1-\sigma)\, w} \equiv l^2(\sigma) \tag{54}$$

$$\sigma \in [0,1].$$

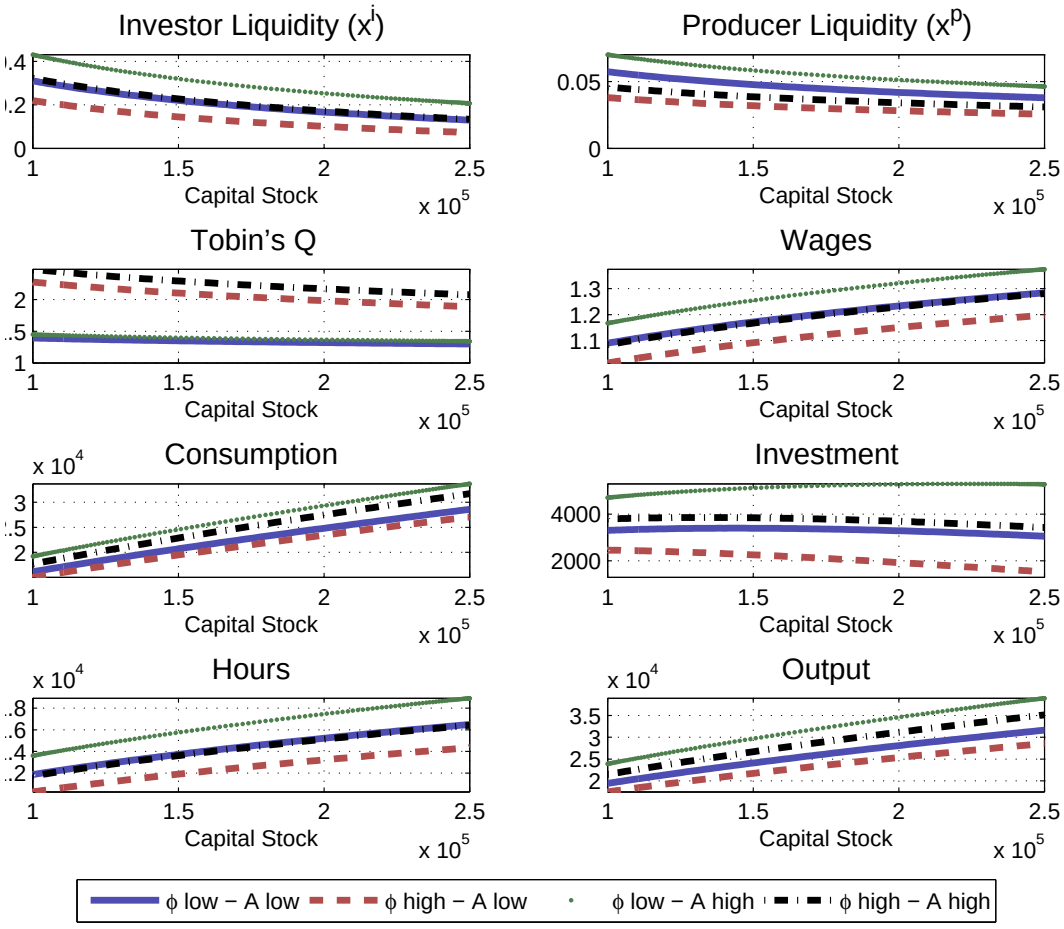


FIGURE 10. EQUILIBRIUM VARIABLES ACROSS STATE-SPACE.

As long as l^{unc} is not in the constraint set, at least one of the constraints will be active since the objective is increasing in l for $l \leq l^{unc}$. In particular, the tighter constraint will bind as long as $l \leq l^{unc}$. Thus, $l^* = \min\left\{l^1(\sigma^*), l^2(\sigma^*)\right\}$ if $\min\left\{l^1(\sigma^*), l^2(\sigma^*)\right\} \leq l^{unc}$ and $l^* = l^{unc}$ otherwise. Therefore, note that (53) and (54) impose a cap on l depending on the choice of σ. Hence, in order to solve for l^*, we need to know σ^* first. Observe that (53) is a decreasing function of σ. The following properties can be verified immediately:

(55)
$$\lim_{\sigma \to 0} l^1(\sigma) = \infty \text{ and } l^1(1) = \left(\frac{\theta^L}{(1-\alpha)}\right)^{\frac{1}{\alpha}} \left[\frac{A}{w}(1-\alpha)\right]^{\frac{1}{\alpha}} = \left(\frac{\theta^L}{(1-\alpha)}\right)^{\frac{1}{\alpha}} l^{unc}.$$

The second constraint curve (54) presents the opposite behavior. It is increasing and has the following limits,

$$l^2(0) = \frac{x}{\omega} \text{ and } \lim_{\sigma \to 1} l^2(\sigma) = \infty.$$

These properties imply that $l^1(\sigma)$ and $l^2(\sigma)$ will cross at most once if $x > 0$. Because the objective is independent of σ, the entrepreneur is free to choose σ that makes l the largest value possible. Since $l^1(\sigma)$ is decreasing and $l^2(\sigma)$ increasing, the optimal choice of σ^* solves $l^1(\sigma^*) = l^2(\sigma^*)$ to make l as large as possible. This implies that both constraints will bind if one of them binds. Adding them up, we find that $l^{cons}(x)$ is the largest solution to

$$\theta^L A l^{1-\alpha} - wl = -x. \tag{56}$$

This equation defines $l^{cons}(x)$ as the largest solution of this implicit function. If $x = 0$, this function has two zeros. Restricting the solution to the largest root prevents us from picking $l = 0$. Thus, if $x = 0$, then $\sigma = 1$ and l solves $wl = \theta^L A l^{1-\alpha}$. This is the largest l within the constraint set of the problem.

Thus, we have that,

$$l^*(x) = \min\left\{l^{cons}(x), l^{unc}\right\}.$$

Since $l^1(\sigma)$ is monotone decreasing, if $\theta^L \geq (1-\alpha)$, then, $l^1(1) \geq l^{unc}$, by (55). Because for $x > 0$, $l^1(\sigma)$ and $l^2(\sigma)$ cross at some $\sigma < 1$, then, $l^{cons} > l^{unc}$ and $l^* = l^{unc}$. Moreover, if $x = 0$, then the only possibility implied by the constraints of the problem is to set $\sigma = 1$. But since, $l^1(1) \geq l^{unc}$, then $l^* = l^{unc}$. Thus, we have shown that $\theta^L \geq (1-\alpha)$ is sufficient to guarantee that labor is efficient for any x. This proves the second claim in the proposition.

Assume now that $l^{unc} \leq \frac{x}{w}$. Then, the wage bill corresponding to the efficient employment can be guaranteed upfront by the entrepreneur. Obviously, $x \geq wl^{unc}$ is sufficient for optimal employment.

To pin down the necessary condition for the constraint to bind, observe that

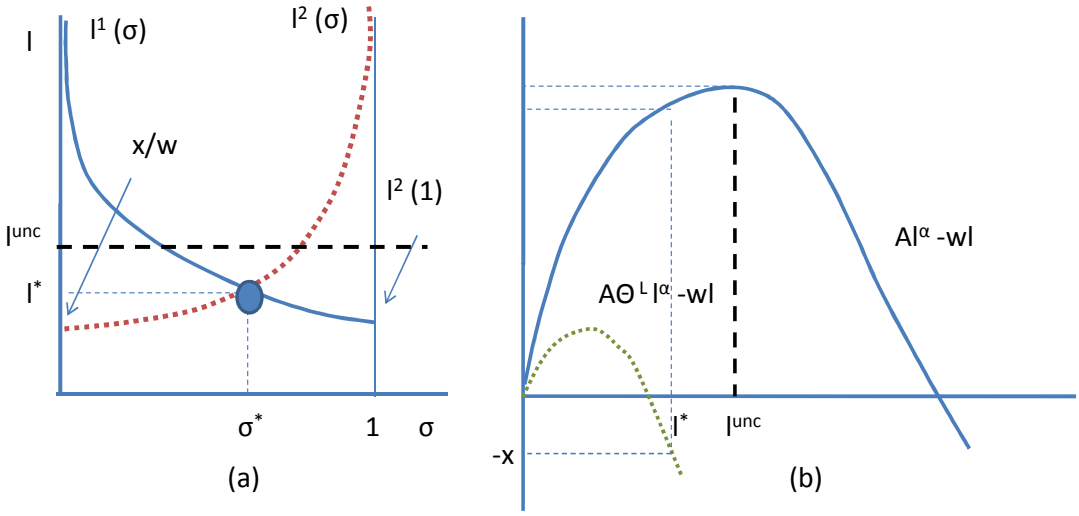


FIGURE 11. DERIVATION OF LABOR CONSTRAINT.

the profit function in (56) is concave with a positive interior maximum. Thus, at $l^{cons}(x)$, the left-hand side of (56) is decreasing. Therefore, if $l^{cons}(x) < l^{unc}$, then it should be the case that $\theta^L A (l^{unc})^{1-\alpha} - w l^{unc} < -x$. Substituting the formula for l^{unc} yields the necessary condition for the constraints to be binding:

$$x < w^{1-\frac{1}{\alpha}} [A(1-\alpha)]^{\frac{1}{\alpha}} \left(1 - \frac{\theta^L}{(1-\alpha)}\right).$$

This shows that if $\theta^L < (1-\alpha)$, the amount of liquidity needed to have efficient employment is positive.

Figure 11 provides a graphical description of the arguments in this proof. The left panel plots l^1 and l^2 as functions of σ. It is clear from the figure that the constraint set is largest at the point where both curves meet. If l^{unc} is larger than the point where both curves meet, then, the optima is constrained. A necessary condition for constraints to be binding is that l^{unc} is above $1^2(1)$, otherwise l^{unc} will lie above. A sufficient condition for constraints to be binding is described in the right panel. The dashed line represents the left hand side of (56) as a function of labor. The figure shows that when the function is evaluated at l^{unc}, and the result is below $-x$, then the constraints are binding.

D. *Proof of Lemma 1*

This Lemma is an application of the Principle of Optimality. By homogeneity, given a labor-capital ratio l/k, p-entrepreneur profits are linear in capital stock:

$$\left[A\left(l/k\right)^{1-\alpha} - w\left(l/k\right) + x\right] k. \tag{57}$$

Observe that once x is determined by the choice of $\iota\left(\omega\right)$, the incentive compatibility constraint (1) and the working capital constraint (3) can be expressed in terms of the labor-capital ratio only:

$$A\left(l/k\right)^{1-\alpha} - \sigma w\left(l/k\right) \geq \left(1-\theta^{L}\right) A\left(l/k\right)^{1-\alpha} \tag{58}$$

and

$$\left(1-\sigma\right) w\left(l/k\right) \leq x. \tag{59}$$

l and σ don't enter the entrepreneur's problem anywhere else. Thus, optimally, the entrepreneur will maximize expected profits per unit of capital in (57) subject to (58) and (59). This problem is identical to Problem 2. Thus, the value of profits for the entrepreneur considering the optimal labor-to-capital ratio is $r\left(x;w\right)k$.

Substituting this value into the objective of Problem 1 yields the following objective

$$W^{p}(k;p,q,w) = \max_{\iota(\omega)\geq 0} r\left(x;w\right)k + xk + qk\int \lambda\left(\omega\right)\left(1-\iota\left(\omega\right)\right) f_{\phi}\left(\omega\right) d\omega \tag{60}$$

subject to:

$$x = p\int \iota\left(\omega\right) d\omega$$

where $r\left(x;w\right)$ is the value of Problem 2 which shows. Lemma 1.

E. *Proof of Proposition 2*

The proof requires some preliminary computations. Note that the choice of ι determines x. In addition, Lemma 1 shows that the entrepreneur's profits are linear in the entrepreneur's capital stock. Thus, the following computations are normalized to the case when $k=1$.

Labor and liquidity. For any x such that $l^{*}\left(x\right) = l^{unc}$, the constraints (2) and (3) are not binding. Therefore, when x is sufficiently large to guarantee the efficient amount of labor per unit of capital, an additional unit of liquidity does not increase $r\left(x\right)$. For x below the amount that implements the efficient level of labor, both constraints are binding. Applying the Implicit Function Theorem to the pseudo-profit function (56) yields an expression for the marginal increase in

labor with a marginal increase in liquidity,

$$\frac{\partial l^{cons}}{\partial x} = -\frac{1}{(1-\alpha)\,\theta^L A l\,(x)^{-\alpha} - w}.$$

Note that the denominator satisfies,

$$(1-\alpha)\,\theta^L A l^{-\alpha} - w \leq \frac{\left[\theta^L A l^{1-\alpha} - wl\right]}{l} = \frac{-x}{l} < 0,$$

which verifies that $\frac{\partial l^{cons}}{\partial x} > 0$.

Marginal profit of labor. Let $\Pi(l) = Al^{1-\alpha} - wl$. The marginal product of labor is,

$$\Pi_l(l) = A(1-\alpha)\,l^{-\alpha} - w > 0 \text{ for any } l < l^{unc}.$$

Marginal profit of liquidity. Using the chain rule, we have an expression for the marginal profit obtained from an additional unit of liquidity.

$$r_x(x) = \Pi_l(l^*(x))\,l^{*\prime}(x) = -\frac{A(1-\alpha)\,l^*(x)^{-\alpha} - w}{(1-\alpha)\,\theta^L A l^*(x)^{-\alpha} - w},\ l^*(x) \in (l^{cons}(0), l^{unc})$$

and 0 otherwise.

Thus, liquidity has a marginal value for the entrepreneur whenever constraints are binding. Since $l^*(x)$ is the optimal labor choice, $\Pi(l^*(x)) = r(x)$, which explains the first equality $r_x(x) = \Pi_l(l^*(x))\,l^{*\prime}(x)$. Since $A(1-\alpha)\,l(x)^{-\alpha} - w$ approaches 0 as $l(x) \to l^{unc}$, $r_x(x) \to 0$, as x approaches its efficient level. Hence, $r_x(x)$ is continuous and $r(x)$ is everywhere differentiable. The marginal value of liquidity, $r_x(x)$, is decreasing in x ($r_{xx}(x) < 0$) since the numerator is decreasing and the denominator is increasing in x.

Equilibrium liquidity. To establish the result in Proposition 2, observe that as in the standard lemons problem in Akerlof (1970), if any capital unit of quality ω is sold in equilibrium, all the units of lower quality must be sold. Otherwise, the entrepreneur would be better off by substituting high-quality units and selling low-quality units instead. A formal argument requires dealing with jumps but the essence does not change.

Thus a cutoff rule defines a threshold quality ω^* for which all qualities below ω will be sold. Choosing the qualities to be sold is equivalent to choosing a threshold quality ω^* to sell. The entrepreneur chooses that threshold to maximize his objective function. Thus, ω^p solves:

$$\omega^p = \arg\max_{\omega^*} r(x)\,k + x + qk\int_{\omega^*}^{1} \lambda(\omega)\,f_\phi(\omega)\,d\omega$$

where

$$x = p^p \int_0^{\omega^*} \iota(\omega) f_\phi(\omega)\, d\omega.$$

The objective function is continuous and differentiable, as long as $f_\phi(\omega)$ is absolutely continuous. Thus, interior solutions are characterized by first order conditions. Substituting x, in $r(x)$ and taking derivatives yields the following first order condition:

$$(1 + r_x(x))\, p f_\phi(\omega^*) - q\lambda(\omega^*) f_\phi(\omega^*) \geq 0 \text{ with equality if } \omega^* \in (0,1). \tag{61}$$

Qualities where $f_\phi(\omega^*) = 0$ are saddle points of the objective function, so without loss of generality $f_\phi(\omega^*)$ is canceled from both sides. There are three possibilities for equilibria: $\omega^* = 1, \omega^* \in (0,1)$, or $\omega^* \neq \varnothing$, where the latter case is interpreted as no qualities are sold. Thus, substituting the zero-profit condition for financial intermediaries, $pF(\omega^*) = q\mathbb{E}_\phi[\lambda(\omega)|\omega \leq \omega^*] F(\omega^*)$, we obtain that 61 becomes

$$(1 + r_x(x))\, \mathbb{E}_\phi[\lambda(\omega)|\omega \leq \omega^*] > \lambda(\omega^*).$$

In equilibrium, ω^* must belong to one of the following cases:

Full liquidity. If $\omega^* = 1$, then it must be the case that

$$\left(1 + r_x\left(q\bar{\lambda}\right)\right)\bar{\lambda} \geq \lambda(1). \tag{62}$$

This condition is obtained by substituting $\omega^* = 1$ into 61. If this condition is violated, by continuity of r_x, the entrepreneur could find a lower threshold ω^* that maximizes the value of his wealth.

Interior solutions. For an interior solution $\omega^* \in [0,1)$, it must be the case that

$$(1 + r_x(x))\, \mathbb{E}_\phi[\lambda(\omega)|\omega \leq \omega^*] = \lambda(\omega^*) \tag{63}$$

for $x = q\mathbb{E}_\phi[\lambda(\omega)|\omega \leq \omega^*] F(\omega^*)$. Since $r_x(x)$ is continuous and decreasing, if the condition does not hold, the entrepreneur can be better of with a different cutoff.

Market Shutdowns. Finally, as in any lemons problem, there exists a trivial market shutdown equilibrium with $\omega^* = \emptyset$, and $p^p = 0$.

F. *Proof of Proposition 4*

Since, we can factor k from the objective in (60) to obtain

$$W^p(k; p, q, w) = k\left(\max_{\iota(\omega)\geq 0} r(x; w) + x + q\int \lambda(\omega)(1 - \iota(\omega)) f_\phi(\omega)\, d\omega\right). \tag{64}$$

For the optimal choice of $\iota(\omega)$, call it $\iota^*(\omega)$, zero profits for the intermediary

require:

$$p\int_0^1 \iota^*(\omega)\, f_\phi(\omega)d\omega = q\int_0^1 \lambda(\omega)\,\iota^*(\omega)\, f_\phi(\omega)d\omega.$$

Substituting this condition into (64) the objective yields:

$$\begin{aligned} W^p(k;p,q,w) &= k\left(r(x;w) + q\int_0^1 \lambda(\omega)\,\iota^*(\omega)\, f_\phi(\omega)d\omega + q\int \lambda(\omega)\,(1-\iota^*(\omega))\, f_\phi(\omega)\, d\omega\right) \\ &= k\left(r(x;w) + q\bar{\lambda}\right). \end{aligned}$$

This shows that $W^p(k;p,q,w)$ can be written as $W^p(k;p,q,w) = \tilde{W}^p(p,q,w)k$ if

$$\tilde{W}^p(p,q,w) \equiv r(x;w) + q\bar{\lambda}.$$

Here, $r(x;w)$ is the solution to Problem 1 and x, p and ω^* are given by Proposition 2.

G. *Proof of Proposition 3*

Note that $\frac{\lambda(\omega^*)}{\mathbb{E}_\phi[\lambda(\omega)|\omega\leq\omega^*]}$ is increasing. Under the assumptions, the advantage rate is 1 when $\omega^* = 0$. At $\omega^* = 1$, the advantage rate is greater than 1. In contrast, $1 + r_x\left(q\mathbb{E}_\phi\left[\lambda(\omega)\,|\omega\leq\omega^*\right]\right)$ is decreasing in ω^*, and starts at a number greater than 1. Thus, if the two curves cross, they must cross at a single point. Otherwise, if they don't cross, $\omega^* = 1$ is an admissible solution.

H. *Proof of Proposition 5*

The proof of Proposition 5 is similar to one that appears in Bigio (2009) and relies on linear programming. Once $\iota(\omega)$ and x are determined, the problem of the i-entrepreneur becomes:

$$\hat{k}(x) = \max_{i^d,i^s} i - i^s + k^b$$

subject to,

$$\begin{aligned} i &= i^d + qi^s \\ \theta^I i &\geq i^s \\ qk^b + i^d &\leq xk. \end{aligned}$$

To solve this linear program we substitute for i to obtain an objective equal to:

$$\begin{aligned} \hat{k}(x) = \max_{k^b,i^d,i^s}\ & i^d + (q-1)\, i^s + k^b \\ & \theta^I i^d \geq \left(1 - q\theta^I\right) i^s \end{aligned}$$

$$qk^b + i^d \leq xk.$$

Here, there are several cases: (i) When $q = 1$ the objective becomes $i^d + k^b$, and the working capital constraint becomes $k^b + i^d \leq xk$. Since i^s reduces the objective, $i^s = 0$. Hence, the value of the problem is $\hat{k}(x) = xk$, and policies are indeterminate. (ii) When $q > 1/\theta^I$, the value of the problem is indeterminate since $i^s \to \infty$ is feasible. This clearly is a solution that cannot be part of an equilibrium. (iii) If $q \in [0, 1)$, $i^s = 0, i^d = 0$ and $k^b = xk/q$. The value of the problem is $\hat{k}(x) = xk/q$. Finally, when $q \in (1, 1/\theta^I)$, we obtain that $i^d = xk$, $k^b = 0$, and $\theta^I i^d = \left(1 - q\theta^I\right) i^s$. Substituting for i^s, the objective of the problem becomes: $i^d + \frac{(q-1)\theta^I}{\left(1-q\theta^I\right)} i^d = \frac{\left(1-\theta^I\right)}{\left(1-q\theta^I\right)} i^d$. Hence, $\hat{k}(x) = \frac{\left(1-\theta^I\right)}{\left(1-q\theta^I\right)} xk$. Using the definition in the text we obtain $\hat{k}(x) = \left(q^R\right)^{-1} xk$. Thus, if $q \in [1, 1/\theta^I)$, $\hat{k}(x) = \left(q^R\right)^{-1} xk$.

I. *Proof of Proposition 6*

The proof of Proposition 6 is similar to the proof of Proposition 2. Thus, I skip minor details. There is only one distinction. Due to the linearity in the production of capital and the constraints, in this case, the marginal value of an additional unit of liquidity is constant and equal to $\frac{q(x)}{q^R(x)}$, or Tobin's q. From Proposition 5 we know that for values of $q \in [1, 1/\theta)$ the value of the optimal financing problem is $\hat{k}(x) = \left(q^R\right)^{-1} xk$. Thus, the value of Problem 3 becomes:

$$W^i(k; p, q) = \max_{\iota(\omega)} \left(q^R\right)^{-1} xk + \int_0^1 (1 - \iota(\omega)) \lambda(\omega) k f_\phi(\omega) d\omega$$

subject to:

$$x = p \int_0^1 \iota(\omega) f_\phi(\omega) d\omega.$$

Following the same steps as in the proof of steps of Proposition 2, we can argue that the equilibrium is determined by a threshold quality, ω^i. Substituting x:

$$W^i(k; p, q) = \max_{\omega^i} \left(q^R\right)^{-1} p \left(\int_0^{\omega^i} f_\phi(\omega) d\omega \right) k + \left(\int_{\omega^i}^1 \lambda(\omega) f_\phi(\omega) d\omega \right) k. \tag{65}$$

Taking first order conditions yields:

$$\left(q^R\right)^{-1} p f_\phi\left(\omega^i\right) k \geq \lambda\left(\omega^i\right) f_\phi\left(\omega^i\right) k$$

and by substituting the zero-profit condition for intermediaries yields:

$$\left(q^R\right)^{-1} q \mathbb{E}_\phi\left[\lambda(\omega) | \omega \leq \omega^i\right] \geq \lambda\left(\omega^i\right)$$

which is the desired condition. The three cases in the statement of the proposition also follow from the proof of Proposition 2.

J. Proof of Proposition 7

From equation (65), the objective of the entrepreneur can be written as:

$$\begin{aligned}
& \left[\left(q^R\right)^{-1} pF\left(\omega^i\right) + \int_{\omega^i}^1 \lambda(\omega) k f_\phi(\omega) d\omega\right] k \\
= & \left[\left(q^R\right)^{-1} q\mathbb{E}_\phi\left[\lambda(\omega) | \omega \leq \omega^i\right] F\left(\omega^i\right) + \int_{\omega^i}^1 \lambda(\omega) k f_\phi(\omega) d\omega\right] k \\
= & \frac{1}{q^R}\left[q\int_0^{\omega^i} \lambda(\omega) k f_\phi(\omega) d\omega + q^R \int_{\omega^i}^1 \lambda(\omega) k f_\phi(\omega) d\omega\right] k \\
\equiv & \tilde{W}^i(q) k.
\end{aligned}$$

where the second line follows from the zero-profit condition for intermediaries.

K. Proof of Proposition of 8

Given a set of prices (p^S, p^F, q) a p-entrepreneur maximizes,

$$\begin{aligned}
W^p(k) = & \max_{I(\omega), \iota(\omega)} r(x) k + xk + ... \\
& k\int_0^1 (1 - I(\omega)) \iota(\omega)\left(q\lambda(\omega) - p^F\right) + (1 - \iota(\omega)) q\lambda(\omega) f(\omega) d\omega
\end{aligned}$$

subject to:

$$x = p^S \int_0^1 \iota(\omega) f(\omega) d\omega.$$

Let $\Omega^D \equiv \{\omega : I(\omega) = 1, \iota(\omega) = 1\}$ be the set of qualities that feature a default in a CD market equilibrium. Let $\Omega^{ND} \equiv \{\omega : I(\omega) = 0, \iota(\omega) = 1\}$. Finally, let $\Omega \equiv \Omega^D \cup \Omega^{ND}$. The first step is to show that if a given quality is defaulted, all lower qualities will feature participation and default. This means that $I(\cdot)$ is decreasing almost everywhere. The second is to show that without loss of generality we can treat $\iota(\cdot)$ as decreasing almost everywhere. By an almost-everywhere decreasing function I mean that there exist two intervals $[0, \omega^o]$ and $[\omega^o, 1]$ such that the function is 1 almost everywhere in $[0, \omega^o]$ and $I = 0$ in $(\omega^o, 1]$.

The value of the objective of the entrepreneur can be expressed in terms for these sets:

$$V = x + r(x, X) + \int_{\Omega^{ND}} \left(q(X)\lambda(\omega) - p^F\right) f(\omega) d\omega + \int_{[0,1]\setminus\Omega} q\lambda(\omega) f(\omega) d\omega$$

with

$$x = \int_{\Omega^{ND}} p^S d\omega + \int_{\Omega^D} p^S d\omega.$$

Suppose $I(\cdot)$ is not decreasing almost everywhere. Then, we can find two intervals: $(\omega_{N_1}, \omega_{N_2})$ and $(\omega_{D_1}, \omega_{D_2})$ such that $I = 0$ almost everywhere in $(\omega_{N_1}, \omega_{N_2})$ and $I = 1$ almost everywhere in $(\omega_{D_1}, \omega_{D_2})$. Moreover, since $f(\omega)$ is continuous, we can find intervals of same measure. We want to show that if $I(\cdot)$ is non-monotone, the p-entrepreneur is not optimizing. The strategy consists of setting $I = 1$ in $(\omega_{D_1}, \omega_{D_2})$ and vice versa in $(\omega_{N_1}, \omega_{N_2})$ to show that this improves his value. Since both sets have the same measure, x remains invariant and only the first integral in the objective changes with the policy perturbation. The value of the integral terms in the objective is then:

$$\begin{aligned}
& \int_{\Omega^{ND}\setminus(\omega_{D_1},\omega_{D_2})} \left(q(X)\lambda(\omega) - p^F(\omega)\right) f(\omega)\, d\omega + \int_{(\omega_{N_1},\omega_{N_2})} \left(q(X)\lambda(\omega) - p^F\right) f(\omega)\, d\omega \\
= & \int_{\Omega^{ND}\setminus(\omega_{D_1},\omega_{D_2})} \left(q(X)\lambda(\omega) - p^F(\omega)\right) f(\omega)\, d\omega + \int_{(\omega_{N_1},\omega_{N_2})} q(X)\lambda(\omega) f(\omega)\, d\omega ... \\
& + p^F \left[F(\omega_{N_2}) - F(\omega_{N_1})\right] \\
> & \int_{\Omega^{ND}\setminus(\omega_{D_1},\omega_{D_2})} \left(q(X)\lambda(\omega) - p^F(\omega)\right) f(\omega)\, d\omega + \int_{(\omega_{D_1},\omega_{D_2})} q(X)\lambda(\omega) f(\omega)\, d\omega + ... \\
& p^F \left[F(\omega_{N_2}) - F(\omega_{N_1})\right] \\
= & \int_{\Omega^{ND}\setminus(\omega_{D_1},\omega_{D_2})} \left(q(X)\lambda(\omega) - p^F(\omega)\right) f(\omega)\, d\omega + \int_{(\omega_{D_1},\omega_{D_2})} q(X)\lambda(\omega) f(\omega)\, d\omega + ... \\
& p^F \left[F(\omega_{D_2}) - F(\omega_{D_1})\right].
\end{aligned}$$

The first line is the value of the alternative strategy for the entrepreneur. The second line is an algebraic manipulation of the integral. The third follows from the monotonicity of λ, which holds by assumption. The third follows from the equivalence in the lengths of both intervals. The inequality shows that a non-monotone default strategy violates optimality.

We now turn to the non-monotonocity of $\iota(\omega)$. Observe that if $\iota(\omega) = 1$ and $I(\omega) = 0$, then the entrepreneur and the intermediary are indifferent between which qualities are brought to the contract. Collateral will be repurchased. Thus, without loss in generality, we can restrict attention to a decreasing $\iota(\omega)$. Thus, there are two threshold qualities: ω^p and $\bar{\omega}^p$. The first, defines a cutoff under which all qualities are defaulted. The second is a participation cutoff. An equilibrium where $\omega^p = \bar{\omega}^p$ is identical to the sales-only contract of Section II. Hence, $\omega^p \leq \bar{\omega}^p$. Thus, the objective for the entrepreneur becomes:

$$V = x + r(x) + \int_{\omega^p}^{\bar{\omega}^p} \left(q\lambda(\omega) - p^F\right) d\omega + \int_{\bar{\omega}^p}^{1} q\lambda(\omega)\, d\omega$$

subject to

$$x = \int_0^{\bar{\omega}^p} p^S d\omega.$$

The first-order conditions for ω^p is

$$q(X)\lambda(\omega^p) - p^F \geq 0, \tag{66}$$

but since λ is continuous and ω^p interior, the equation holds with equality. The first-order condition for $\bar{\omega}^p$ is:

$$\begin{aligned} (1 + r_x(x))\, p^S &\geq \left(p^F - q\lambda(\bar{\omega}^p)\right) + q\lambda(\bar{\omega}^p) \rightarrow \\ r_x(x)\, p^S &\geq \left(p^F - p^S\right). \end{aligned} \tag{67}$$

Finally, the zero-profit condition written in terms of ω^p and $\bar{\omega}^p$ yields:

$$p^F = \int_0^{\omega^p} q\lambda(\omega, \phi)\, d\omega + p^S \int_{\omega^p}^{\omega^*} d\omega. \tag{68}$$

Equations (66), (67) and (68) correspond to the equations that characterize equilibria.

L. *Obtaining Equivalent Problems 7 and 8*

By substituting the capital accumulation equation into the p-entrepreneur's budget constraint to obtain the following equivalent problem:

$$V^p(k, X) = \max_{c \geq 0, k' \geq 0, \iota(\omega), l, \sigma \in [0,1]} U(c) + \beta \mathbb{E}\left[V^j(k', X')|X\right], \; j \in \{i, p\}$$

subject to

$$c + q(X)\, k' = AF(k, l) - \sigma w(X)\, l + xk - (1 - \sigma)\, w(X)\, l + q(X) \int_0^1 (1 - \iota(\omega))\, \lambda(\omega)\, k f_\phi(\omega)\, d\omega$$

$$AF(k, l) - \sigma w l \geq \left(1 - \theta^L\right) A k^\alpha l^{1-\alpha}$$

$$(1 - \sigma)\, w l \leq xk$$

$$x = p^p(X) \int_0^1 \iota(\omega)\, d\omega.$$

His objective function is a function of c and k' and does not appear in the constraints below the budget constraint. In contrast, the choice of $\iota(\omega), l, \sigma$ only affects the right-hand side of the consolidated budget constraint and is constrained

through the additional constraints. Thus, the entrepreneur maximizes his value function by choosing $\iota(\omega), l, \sigma$ to maximize the right-hand side of his budget constraint. This problem is identical to Problem 1. Therefore, we can re-write the p-entrepreneur's problem as:

$$V^p(k,X) = \max_{c\geq 0, k'\geq 0, \iota(\omega), l, \sigma \in [0,1]} U(c) + \beta \mathbb{E}\left[V^j(k',X')|X\right],\ j \in \{i,p\}$$

subject to

$$c + q(X)\, k' = \tilde{W}^p(X) k$$

where $\tilde{W}^p(X)$ is the marginal value of capital in Proposition 4 for prices $p(X), q(X)$ are $w(X)$. This is a consumption-savings problem with linear returns. Similar steps can be followed to obtain the value for i-entrepreneurs in Proposition 8.

M. *Proof of Proposition 10*

Both statements of Proposition 10 follow from previous Propositions. I first prove the statements about labor inefficiency for any arbitrary state X. From Proposition 1, we know that if $\theta^L \geq (1-\alpha)$, then the labor-to-capital ratio of the individual entrepreneur is efficient for any choice of x. This proves the only if part. Instead, if $\theta^L < (1-\alpha)$, we know also from Proposition 1 that some positive amount of liquidity is needed to have the efficient labor-to-capital ratio. It is sufficient to show that amount is not obtained in equilibrium. From Proposition 2 we know that ω^p must satisfy

$$(1 + r_x(x))\, \mathbb{E}_\phi\left[\lambda(\omega)\,|\omega \leq \omega^p\right] \geq \lambda(\omega^p).$$

However, from Proposition 1 we also know that efficient employment implies that $r_x(x) = 0$. Thus, the above condition becomes $\mathbb{E}_\phi\left[\lambda(\omega)\,|\omega \leq \omega^p\right] \geq \lambda(\omega^p)$ which by Assumption 2 implies that this is true only for $\omega^p = 0$. This in turn implies that $x = q(X)\, \mathbb{E}_\phi\left[\lambda(\omega)\,|\omega \leq 0\right] F(0) = 0$. By Proposition 1 employment cannot be efficient as it requires some positive amount of liquidity.

I now prove the result for investment. Assume that $q(X) = 1$ and, thus, $q^R(X) = 1$. Therefore, by Proposition 6 we have that,

$$\mathbb{E}_\phi\left[\lambda(\omega)\,|\omega \leq \omega^i\right] = \lambda\left(\omega^i\right)$$

which implies that $\omega^i = 0$. This in turn implies $x^i = 0$ and, consequently, $i^d = 0$ from Proposition 3. Since $i^d = 0 \to i = 0$, we have that aggregate investment cannot be positive.

N. *Proof of Proposition 11*

Substitute the optimal policies described in Proposition 9 into the expression for $D(X)$ and $S(X)$ to obtain $I^s(X) = D(X) - S(X)$. Then use (37), (38) and (39) to clear out expressions for $I^s(X)$ and $I(X)$. In the proof the state X is fixed

so I drop the arguments from the functions. Performing these substitutions, the aggregate version of the incentive compatibility condition becomes:

$$\frac{(1-\pi)\left(\varsigma^{p}\left(r+q\psi^{p}\right)/q-\psi^{p}\right)K-(1-\pi)\,\varphi^{p}K-\pi\varphi^{i}K}{\theta}\leq\frac{\pi\left[\varsigma^{i}\left(W^{i}\right)K-\psi^{i}K\right]}{(1-\theta)}.$$

I have introduced the following variables:

$$\begin{array}{ll}\varphi^{p}=\int_{\omega\leq\omega^{p}}\lambda\left(\omega\right)f_{\phi}\left(\omega\right)d\omega & \varphi^{i}=\int_{\omega\leq\omega^{i}}\lambda\left(\omega\right)f_{\phi}\left(\omega\right)d\omega\\ \psi^{p}=\int_{\omega>\omega^{p}}\lambda\left(\omega\right)f_{\phi}\left(\omega\right)d\omega & \psi^{i}=\int_{\omega>\omega^{i}}\lambda\left(\omega\right)f_{\phi}\left(\omega\right)d\omega\end{array}$$

that correspond to the expectations over the sold and unsold qualities of both groups. K clears out from both sides. I then use the definition of q^{i} and rearrange the expression to obtain:

$$\begin{aligned}\frac{(1-\pi)\varsigma^{p}r-\left(\left(1-\pi\right)\left(1-\varsigma^{p}\right)\psi^{p}+\left(1-\pi\right)\varphi^{p}+\pi\varphi^{i}\right)q}{\theta q} &\leq \frac{\pi\left[\varsigma^{i}q\varphi^{i}-\left(1-\varsigma^{i}\right)\psi^{i}q^{R}\right]}{(1-\theta)\,q^{R}}\\ &\leq \frac{q\pi\varsigma^{i}\varphi^{i}}{(1-\theta q)}-\frac{\pi\left(1-\varsigma^{i}\right)\psi^{i}}{(1-\theta)}.\end{aligned}$$

I get rid of q from the denominators, rearrange terms and obtain,

$$\begin{aligned}&(1-\pi)\varsigma^{p}r\left(1-\theta q\right)-\left(\left(1-\pi\right)\left(\left(1-\varsigma^{p}\right)\psi^{p}+\varphi^{p}\right)+\pi\varphi^{i}\right)q\left(1-\theta q\right)\\ \leq\;&\theta q^{2}\pi\varsigma^{i}\varphi^{i}-\theta q\left(1-\theta q\right)\pi\frac{\left(1-\varsigma^{i}\right)\psi^{i}}{(1-\theta)}.\end{aligned}$$

By arranging terms, the inequality includes linear and quadratic terms for q. This expression takes the form:

$$\left(q^{*}\right)^{2}A+q^{*}B+C\geq0 \tag{69}$$

where the coefficients are:

$$A=-\theta\left(\left(1-\pi\right)\left(\left(1-\varsigma^{p}\right)\psi^{p}+\varphi^{p}\right)+\pi\left(1-\varsigma^{i}\right)\varphi^{i}-\pi\theta\frac{\left(1-\varsigma^{i}\right)}{(1-\theta)}\psi^{i}\right)$$

$$B=\theta(1-\pi)\varsigma^{p}r+\left(\left(1-\pi\right)\left(\left(1-\varsigma^{p}\right)\psi^{p}+\varphi^{p}\right)+\pi\varphi^{i}-\pi\theta\frac{\left(1-\varsigma^{i}\right)}{(1-\theta)}\psi^{i}\right).$$

$$C=-(1-\pi)\varsigma^{p}r$$

C is negative. Observe that

$$\begin{aligned}
& (1-\pi)\left(\left(1-\varsigma^p\right)\psi^p+\varphi^p\right)+\pi\varphi^i-\pi\frac{\left(1-\varsigma^i\right)}{(1-\theta)}\psi^i\theta \\
\geq\ & (1-\pi)\left(\left(1-\varsigma^p\right)\psi^p+\varphi^p\right)+\pi\left(1-\varsigma^i\right)\varphi^i-\pi\frac{\left(1-\varsigma^i\right)}{(1-\theta)}\psi^i\theta \\
\geq\ & (1-\pi)\left(\left(1-\varsigma^p\right)\psi^p+\varphi^p\right)+\pi\left(1-\varsigma^i\right)\varphi^i-(1-\pi)\left(1-\varsigma^i\right)\psi^i \\
\geq\ & (1-\pi)\bar{\lambda}-(1-\pi)\varsigma^p\psi^p+\pi\left(1-\varsigma^i\right)\bar{\lambda}-\pi\left(1-\varsigma^i\right)\psi^i-(1-\pi)\left(1-\varsigma^i\right)\psi^i \\
\geq\ & \bar{\lambda}-(1-\pi)\varsigma^p\psi^p-\pi\psi^i \\
\geq\ & 0
\end{aligned}$$

where the second line follows from the assumption that $(1-\theta)\geq\pi$. The third line uses the identity $\bar{\lambda}=\psi^p+\varphi^p=\psi^i+\varphi^i$. The fourth line uses the fact that $\left(1-\varsigma^i\right)<1$ and the last line uses the fact that ψ^p and ψ^i are less than $\bar{\lambda}$. This shows that A is negative and B is positive. Evaluated at 0, (69) is negative. It reaches a maximum at $-\frac{B}{2A}>0$. Thus, both roots of (69) are positive. Let the roots be (q_1,q_2) where q_2 is the largest. There are three possible cases:

Case 1: If $1\in(q_1,q_2)$, then $q=1$ satisfies the constraint.

Case 2: If $1<q_1$, then $q=q_1$, since it is the lowest price such that the constraints bind with equality.

Case 3: If $q_2<1$, then there exists no incentive compatible price. Thus, $I=0$ and i-entrepreneurs consume part of their capital stock.

O. *Proof of Proposition 12*

An identical proposition is shown in Bigio (2009). The proof is standard for consumption-savings problems with stochastic linear returns and homothetic preferences. The proof also implies that the economy admits aggregation.

X. Data Appendix (not for publication)

A. Macroeconomic Variables

Except for TFP and the capital stock, all the macroeconomic variables are obtained from the Federal Reserve Bank of St. Louis Economic Research Database, FRED® available at http://research.stlouisfed.org/fred2/. These series are used in the construction of figures 3 and 6. The sources of the series for output, investment and consumption are the National Income and Product Accounts (NIPAs) of the United States constructed by the Bureau of Economic Analysis (BEA). The data on hours is from the Bureau of Labor Statistics (BLS).

For TFP, I use the non-utilization series computed by Fernald (2012) available from the author's website http://www.frbsf.org/economic-research/economists/john-fernald/. The macroeconomic data is downloaded directly into MATLAB® using the Datafeed Toolbox®. The MATLAB code *FRED_TFP_accounting_iii.m* downloads the time series for these variables and reads the TFP data from Fernald's website after saved to a computer —as a .csv file.

All the data is quarterly, converted into real terms and adjusted for seasonality by the original source. The data begins at 1983:IV and ends at 2013:II. Fernald's TFP series is published in growth rates. I normalize the first value by 100. The following table summarizes the list of variables:

Variable in Model	Data Analogue Used	Source Acronym	Source
Output (Y_t)	Real Gross Domestic Product, 3 Decimal	$GDPC1$	BEA
Investment (I_t)	Real Private Nonresidential Fixed Investment	$PNFIC1$	BEA
Consumption (C_t)	Real Personal Consumption Expenditures	$PCECC96$	BEA
Labor (l_t)	NFBS: Hours of All Persons	$HOANBS$	BLS
TFP (A_t)	TFP	dtfp	Fernald (2012)
Wages (w_t)	NFBS: Real Compensation Per Hour	$COMPRNFB$	BLS

Ratios. I use the series of labor and output described above to compute output-per-hour. Fernald also reports series for the growth rates of output and capital —acronyms dY_prod and dk. I also normalize initial values to 100. I use this data to compute an investment-to-capital ratio consistent with Fernald's TFP measure. For this, I use the invshare share published by Fernald —the series invshare– and multiply it by Fernald's output series and the capital stock series. To compute investment-to-capital, I multiply the investment share series by the ratio of the normalized capital stock and output. I then compute the deviations from the mean of this series, and multiply it by $(1-0.9\hat{}(1/4))$ to make the series consistent with an average 10% depreciation.

Detrending. As noted in the main text, I use a combination of the HP filter and a linear trend to extract cycles. First, I compute the linear trend of every series for 2007:IV-2013:II. I then construct an auxiliary time series where the original data is replaced by the linear trend for 2007:IV-2013:II. Finally, I run the HP filter on the auxiliary series with a parameter of 1600 and treat the HP trend of the auxiliary series as the trend of the original data. I detrend the data subtracting the trend of the auxiliary data from the original time series. To

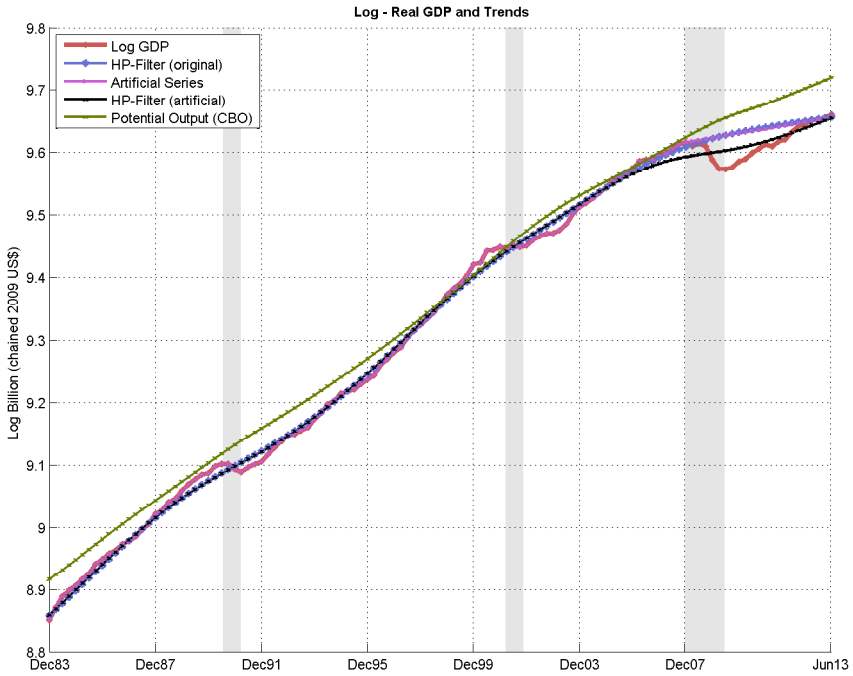


FIGURE 12. US OUTPUT — LEVEL AND TREND COMPARISONS.

clarify the procedure, Figure 12 plots the original series for the original log of Real Output together with four other series. These series correspond to the artificial series, the trends of the original and artificial series and —for comparison— the log of Real Potential Gross Domestic Product from the Congressional Budget Office —also available from FRED. One can observe that the deviation of output from the HP filter predicts a boom during the first three quarters of the Great Recession. Moreover, the magnitude of the deviation from trend during the Great Recession is small compared to the distance from potential output. The trend of the artificial data lies in the middle and is consistent with a return to trend by the end of the sample. With this procedure, the cycle component of —for example— real output coincides with the NBER recession dates and shows a deep recession.

B. Credit Market Variables

Credit Market Data. Credit market data is obtained from several sources. I build the time series for liquidity using data from the Flow of Funds. Liquidity is the sum of the series for Net Worth and Total Credit Market Instruments for both Noncorporate and Corporate Non-Financial Business.

Data	Source Acronym	Source
Nonfinancial Noncorporate Business; Net worth	$TNWBSNNB$	FoF
Nonfinancial Corporate Business; Net Worth at Historical Cost	$TNWMVBSNNCB$	FoF
Nonfinancial Noncorporate Business; Credit Market Instruments	$TCMILBSNNB$	FoF
Nonfinancial Corporate Business; Credit Market Instruments	$TCMILBSNNCB$	FoF

This data is also available from FRED. The code FRED_NFNCB.m downloads the data and constructs the series for aggregate liquidity. I use the same method described above to detrend this data.

Syndicated Loans. The data on syndicated loans is obtained from the Thomson Reuters LPC DealScan© dataset. The data is downloaded from the Wharton Research Database Site, WRDS©. The dataset covers almost the entire universe of syndicated bank loans world-wide. I use loans only for the US. I use quarterly data from 2000:I to 2013:II. The data format is a cross section of loans which include several characteristics. The STATA® do-file *DealScanBuild.do* creates time series for aggregate total amounts of loans and the number of loans. To construct the aggregate total amounts of loans, I sum across all loans the variable *dealamount* which is the descriptor for loan size. I count the number of loans across time to obtain the average loan size. DealScan does include data on interest rate spreads —*spreadoverdefaultbase*— but this data is not available for all loans.

DealScan includes information on the purpose of each loan which is encoded in the variable *purpose*. The STATA code *DealScanBuild.do* saves these time series into a .csv file labeled *SyndicatedLoans.csv*. The MATLAB code *DealScanBuild.m* loads the data from the .csv file and generates quarterly sums and average sizes for the categories used in the paper: those with an investment (INV) purpose and those with a working capital (WC) purpose. Time series for loans where the value of *purpose* is *Working Capital* end in 40 in the .csv file. For the investment-purpose time series, I use the series whose purpose variable takes values *Acquisitions line*, *Levered Buyout (LBO)*, *Project finance*, or *Takeover* —the time series ending in 1, 18, 25, 36 in the .csv file. An earlier version of the paper used these series separately. The latest version uses their weighted average.

C&I. The series for Commercial and Industrial Loans (C&I) is downloaded from FRED and corresponds to the series in Loans Assets and Liabilities of Commercial Banks in the United States — Table H.8 of the statistical release of the Board of Governors of the Federal Reserve System. The FRED acronym for this variable is BUSLOANS. The same source provides the series for Charge-Off Rates on Business Loans at all Commercial Banks. The FRED acronym is CORBLACBS.

Bond Spreads. The A and BBB spread indices correspond to the series of effective yield of the BofA Merrill Lynch US Corporate A and BBB index. These series are part of the BofA Merrill Lynch US Corporate Master Index for US dollar-denominated investment-grade-rated corporate debt publicly issued in the US domestic market. The FRED acronyms are BAMLC0A3CAEY and BAMLC0A4CBBBEY for the A and BBB ratings.

Survey of Terms of Business Lending (STL). The Data from the Survey of

Terms of Business Lending, also available from FRED, collects information on loans which includes the size of loans made to businesses the first full business week of the mid-month of each quarter (February, May, August, and November). The information from the reports includes average maturity in days, average loan size, and total loan amount separately for different risk-level assessments. I report the average loan size weighted by the total volume of each series for each risk assessment level. The variable descriptor acronym is EVA (volume) and EAA (average size for within-class loan). The acronyms for risk are N (minimal), L (low), M (medium) and O (other). The series acronyms join the variable descriptor with the risk descriptor. The data series for C&I, Bonds Spreads and STL are downloaded together from FRED by the code FRC_FRED_data_upload_v5.m.

C. Data Used in Earlier Versions

Firm Cross-Section Data: An earlier version of the paper uses the cross-sectional standard deviation of sales for all firms as an indirect measure of dispersion. This data is found in COMPUSTAT© – North America – Fundamentals Quarterly under the acronym *salesq*. The data is downloaded from WRDS. I use quarterly data from 2000:I to 2012:II. The code *createCCCdata2.do* and *data_analysis_TS2.do* aggregates across firms to generate time series for different firm sizes for the quarterly cross-sectional deviation. I use the entire sample for the computation of the dispersion of sales.