Discussion of "Beyond Hulten's Theorem"

by Baqaee and Farhi

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Why is this paper important?

Neglect of Production Theory

Theorem

I-O matrix Ω, unit CES everywere. Then,

$$GDP = \bar{z}(\mathbf{A})\,\ell^{\bar{\eta}},$$

and

$$\bar{z}(\mathbf{A}) \equiv \exp\{\lambda \log \mathbf{A}\},$$

$$\lambda \equiv \left(\mathbf{b}'[\mathbf{1}-\Omega]^{-1}\right).$$

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Any IO with CRS

- *first-order effects of A_i coincide with λ_i*
- *Furtheremore, λ_i is vector of relative sales*

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Literature and Motivate the Paper

- Daron, Vasco, Asu, and Ali (ECMA, 2012) and Daron and Ali's (AER, 2016)
    - What properties on Ω produce $\lambda_i^{\prime s}$ that break classic limit theorems?
- Motivation in Bigio and La'O
    - Amplification of shocks to spreads: $\bar{z}(\mathbf{A})\,\zeta(Spreads_t)\,\ell^{\bar{\eta}}$
        - No Hulten result for $\zeta(Spreads_t)$
    - However: "Our theory is, of course, incomplete... Departingfrom **perfectly mobile labor** or **relaxing unit CES** will generate more amplication."
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A General Environment Here

- $\mathscr{C}$ commodity aggregator
- Individual production function:

$$y_i = A_i F_i(l, x) \text{ (in vector form)}$$

- Define the "Macro Elasticity":

$$\frac{1}{\rho_{ij}} \equiv \frac{dlog(C_i/C_j)}{dlog(A_i)} = 1 - \frac{dlog(\lambda_i/\lambda_j)}{dlog(A_i)}.$$

Key Formula in the Paper

Second Order Expansion:

$$C \simeq \underbrace{diag(\lambda)\log(A)} + M(\log(A))^2$$

With diagonal terms:

$$M_i = \frac{\lambda_i}{\xi}\sum_{j\neq i}\lambda_j(1-\frac{1}{\rho_{ij}}) + \lambda_i\frac{dlog\xi}{dlogA_i}.$$

Implication:

- Non-linearities increase as $\rho \to 0$
- Homogeneity $\frac{dlog\xi}{dlogA_i} = 0$
- Also, linear term is:

$$\frac{\lambda_i}{\xi}\sum_{j\neq i}\lambda_j\frac{dlog(\lambda_i/\lambda_j)}{dlog(A_i)}.$$

- Skip off-diagonal terms

Producing an Operational Result

- 2 Challanges:
  - A from non-parametric micro to macro
  - B from parametric micro to macro
- Theorem (above) provides a path for A (in paper)
- B provides intuition, but we need examples

Specialization to Standard Model

- Challange structure in the paper:
- CES Household with σ and b_k
- CES labor-materials aggregator: θ_i and a_k
  - Each sector has associated Labor and Intermediates Supplier
  - Labor is Cobb-Douglass in fixed factors and mobile labors β_k
  - CES ε_k and ω_k
- Question:
  - Could you pick a notation that is harder to mememorize?

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Structure of the Excercises

- 4 - Highlight Role of Mobility
  - No intermediate inputs
  - Uniform labor reallocation produce

$$\rho_{ij} = \frac{\sigma(1-\beta)+\beta}{\sigma(1-\beta)+\beta+(1-\sigma)}; \ \lambda_i = b_i$$

- 5 - Uniform CES $\sigma = \theta_i = \varepsilon_k$

$$\rho = \sigma \text{ when mobility } \beta = 1$$

$$\rho = 1/(2\text{-}\sigma) \text{ when mobility } \beta{=}0$$

$$\lambda_i = \text{labor share}$$

$$M_i = \lambda_i(1-\lambda_i)(1-\frac{1}{\rho})$$

- 6 - Multiplier is a classic excercise

The General - Parametric Result

- Assume perfect mobility (CRS)
- Take the I-O matrix from the Cobb-Douglass case:

$$\Omega_{[ij]} = \frac{p_j x_{ij}}{p_i y_i}$$

  and the Leontief Inverse:

$$\Psi = (1 - \Omega)^{-1}$$

- Think of Ω as a Probability Matrix

$$COV_i(j,k) = Cov(\Psi_{mj}, \Psi_{mk})\text{under } \Omega_{[i:m]}$$

- Then:

$$M_i = (\sigma - 1)COV_b(i,i) + \sum(\theta_j - 1)COV_j(i,i)$$

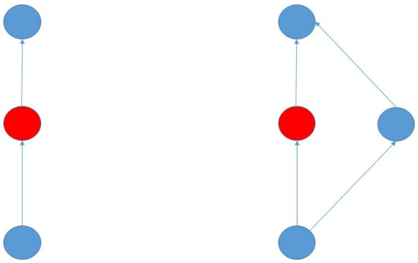
  - with further generalizetion to DRS-Immobility case.

Comments

- Great Foundational Work
- ... examples don't illustrate role of "network architecture"
  - the machinery is there, but not the applications
- Questions for the group

C1: Role of "Architecture"?

- Consider two networks and effects of a bottleneck



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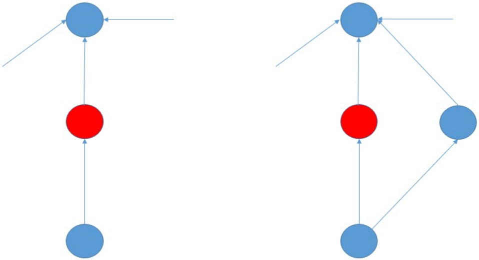

C1: Role of "Architecture"?

- There's info in one of the second order of components:

$$\sum(\theta_j - 1)COV_j(i,i)$$

- In my example, the effect is going to pop-out in $COV_j(i,i)$
  - what archtectures produce high covarinaces?

C2: Strategic Sectors

- The energy example is great!
- Notional View: Leontief at final layer in every final good
  - "Strategic" industries: food, energy, water, superconductors (indium), super metals (tungstenum)
- What configurations lead to similar notions of strategic?
  - Employ sufficient statistics:

$$\sum(\theta_j - 1)COV_j(i,i)$$

  - What configurations of Ω produce same statistics (symmetric)

C3: When is the Second Order Enough?

- We can solve general network via Negishi's theorem
- When is the second order good? Are higher expansions monotone in second order effects?
- Take Leontief limit (S5): then

$$M_i = \lambda_i(1-\lambda_i)(1/2)$$

should effect be larger?

C4: On-Impact vs. Steady State Effect

- These are "new" steady-state effects
- Are there differences when you consider the impact effect?
    - does the answer change with CES?

Summary

- Paper: import effort to flesh out production theory
- Understand the role of factor mobility, elasticities of substitution
- Still, I would like to see more work on network configurations and interaction CES