Financial Risk Capacity*

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Abstract

Financial crises are particularly severe and lengthy when banks fail to recapitalize after bearing large losses. We present a model that explains the slow recovery of bank capital and economic activity. Banks provide intermediation in markets with information asymmetries. Large equity losses force banks to tighten intermediation, which exacerbates adverse selection. Adverse selection lowers bank profit margins which slows both the internal growth of equity and equity injections. This mechanism generates financial crises characterized by persistent low growth. The lack of equity injections during crises is a coordination failure that is solved when the decision to recapitalize banks is centralized.

Keywords: Financial Crisis, Adverse Selection, Capacity Constraints.

JEL Classifications: E32, E44, G01, G21

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1 Introduction

Financial crises that originate from extreme bank losses are severe in depth and duration.[1] These episodes suggest that the recapitalization of banks is critical for the recovery of overall economic activity. After the recent financial crisis, the slow recovery of bank equity has been a major concern for policy makers, academics, and practitioners.[2] When the then Chairman of the Federal Reserve, Ben Bernanke, was asked when the crisis would be over, he answered, "When banks start raising capital on their own."[3] But why do banks struggle to recapitalize after a financial crisis?

To answer this question, macroeconomic theory introduces frictions that prevent banks from raising equity or barriers to entry that deter the creation of new banks. This explains why banks do not recapitalize, but does not rationalize why banking crises last so long. To explain slow recoveries, any theory must also rely on low profit margins from intermediation after banks suffer equity losses. Otherwise, high profit margins translate into rapid revenue retentions that accelerate the recovery of inside equity. Recent macroeconomic theories of financial intermediation cannot explain declines in bank profit margins, because frictions only limit the ability to raise debt and equity.[4] A simple supply-demand analysis suggests that financial intermediation profits should rise, not fall, when equity capital is limited. Thus, those models predict an accelerated recovery of bank equity and, in turn, of economic activity. A theory of low profit margins during crises is required.

In this paper, we present a model that explains the slow recovery of bank capital and economic activity. We show that asymmetric information can explain persistent low economic growth after financial crises, even though bankers have the funds to recapitalize their banks.

A natural role for banks is to deal with asymmetric information between borrowers

[1]See Cerra and Saxena (2008); Reinhart and Rogoff (2009).

[2]The slow recovery of intermediary capital is the subject of Darrell Duffie's 2010 Presidential Address to the American Finance Association (Duffie, 2010) and a centerpiece of Bagehot (1873).

[3]"The Chairman," *60 Minutes*, CBS News, March 15, 2009.

[4]In most macroeconomic models of intermediation developed after the crisis, banks cannot raise equity because bankers are fully invested specialists who face agency frictions. See below for a review of the literature.

and lenders. Banks can diversify transaction risks caused by asymmetric information, because they can exploit their scale to pool assets.[5] Yet, banks are not immune to large losses. When banks lose their *financial risk capacity*, they must scale down their operations. This decline in intermediation volumes exacerbates adverse selection. In turn, heightened adverse selection lowers profit margins for banks and incentives to recapitalize. Eventually, the financial system recovers—but this only comes through retained earnings, an essentially lethargic process when volumes and profit margins are low. This mechanism delivers financial crises characterized by persistent low economic growth.

An outcome of our theory is that the depth and length of financial crises can be mitigated if bankers coordinate their equity injections. When an individual banker observes low net worth in the banking sector, he takes as given that adverse selection is strong, and therefore profit margins are low. Thus, the crisis persists even when capital is readily available to recapitalize banks. If bankers coordinated to recapitalize banks, they would factor in that their equity injections could moderate adverse selection and improve profits for the industry, alleviating the severity of the crisis.

We do not provide an ultimate theory of financial crises, but rather present a mechanism that delivers long-lasting recessions after a banking crisis with a minimal set of ingredients: (i) reallocation of capital across sectors fuels real economic activity; (ii) banks face a limited liability constraint to tie financial activity to bank equity; (iii) intermediation is risky such that bank equity can suffer losses; and (iv) financial intermediation is subject to asymmetric information, which lowers profitability when bank financial risk capacity is low. Thus, our theory attributes the lack of entry to the decline in profitability. We argue that low profit margins would exacerbate other explanations of slow-moving equity. For example, low profits reduce the value for outside investors, worsen debt-overhang problems (Myers, 1977), and add stress to agency frictions that limit the ability to raise equity. These include moral hazard (Holmstrom and Tirole, 1997), limited enforcement (Hart and Moore, 1994), or asymmetric information (Myers and Majluf, 1984).

[5] This view is rooted in classic banking theory: for example, Freixas and Rochet (2008), Leland and Pyle (1977), Diamond (1984), or Boyd and Prescott (1986).

Our paper is related to two branches of financial macroeconomics. The first links the net worth of the financial sector to the amount of financial intermediation through agency frictions. This literature builds on earlier work by Bernanke and Gertler (1989) that focused on financial factors affecting firms.[6] Since the onset of the Great Recession, several papers have incorporated similar intermediaries into state-of-the-art business cycle models. Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) study business cycle effects after intermediaries suffer equity losses. Our paper is closer to the continuous-time models of He and Krishnamurthy (2011) and Brunnermeier and Sannikov (2014), because those papers also stress the non-linear nature of intermediation dynamics. In He and Krishnamurthy (2011), equity shocks are amplified through a substitution of equity financing for debt. In Brunnermeier and Sannikov (2014), amplification operates through fire sales. Our paper differs from the literature in some important respects. First, intermediaries do not operate production; they reallocate capital. Second, they issue liabilities that become means of payment. Third, frictions do not limit the ability to raise equity; here bankers have equity, but choose not recapitalize banks. These elements bring the model closer to institutional details of banking. Yet, nonlinear effects still emerge from the interplay between low bank capital, asymmetric information, and low profitability. Finally, the source of the inefficiency is different. Typically, models introduce pecuniary externality that operate through collateral constraints. Rather, here the externality stems from the individual impact on the average quality of capital.

The second branch investigates the effects of asymmetric information in financial market intermediation. Stiglitz and Weiss (1981) investigate asymmetric information on the side of borrowers. Our paper is closely related to Eisfeldt (2004), who studies an asset market with asymmetric information. There, adverse selection induces a cost to insure against investment risks. Bigio (2015) and Kurlat (2013)

[6]Fire-sale phenomena were first described by Shleifer and Vishny (1992). A feedback between losses in intermediary capital and reductions in asset values is also a theme in Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2008). Maggiori (2017) extends this framework to a two-country setup to study current account dynamics. Diamond and Rajan (2011) study strategic behavior by banks to exploit fire sales by their competitors. Vayanos and Wang (2011) introduce asymmetric information to a related setup.

study models in which assets are also sold under asymmetric information, but to fund production. The novelty here is the interaction between intermediary capital and asymmetric information. This interaction is important, because those models lack a strong internal propagation: The persistence of adverse selection corresponds exactly to the persistence of exogenous shocks. Here, low bank equity leads to a persistent aggravation of adverse selection.[7] This feature is connected to the business cycle decompositions in Christiano, Motto, and Rostagno (2014) and Ajello (2016), who find a prevalence of exogenous shocks that exacerbate asymmetric information. Although those models lack intermediaries, their filtering exercises find that dates associated with stronger adverse selection coincide with dates on which financial institutions were in distress. Ordonez (2013) shows that asymmetric information with financial frictions can amplify the asymmetric movement of economic variables in the business cycles. Stiglitz, Stiglitz, and Greenwald (2003) argue that credit quality deteriorates when banks provide little intermediation, and regards this as being essential to understanding cycles, monetary policy, and the evolution of bank equity and profits after crises. Gennaioli, Shleifer, and Vishny (2013) study an environment in which intermediaries increase leverage when they can mutually insure against idiosyncratic credit risk. However, their higher leverage increases aggregate credit risk. In Martinez-Miera and Suarez (2012) and Begenau (2019), banks can choose the risk of their assets directly. As an outcome, those models deliver procyclical credit risk, but they cannot explain declines in margins in crises. In our paper, credit risk and returns are endogenous. Finally, the mechanism here relates to the mechanisms in Gorton and Ordonez (2014) and Dang, Gorton, Holmström, and Ordonez (2017). In those models, the equity of constrained agents determines their incentives to acquire information. Thus, equity losses may trigger adverse selection because the economy swings from states in which information is symmetric and assets are liquid to states in which information is asymmetric and assets illiquid. Here, what triggers adverse selection is that low bank equity induces low volumes of intermediation.

The next section provides an intuitive description of the mechanics of the model.

[7]Other models that study lemons markets, such as Hendel and Lizzeri (1999), Guerrieri and Shimer (2014), Plantin (2009), or Daley and Green (2012), obtain persistence through learning.

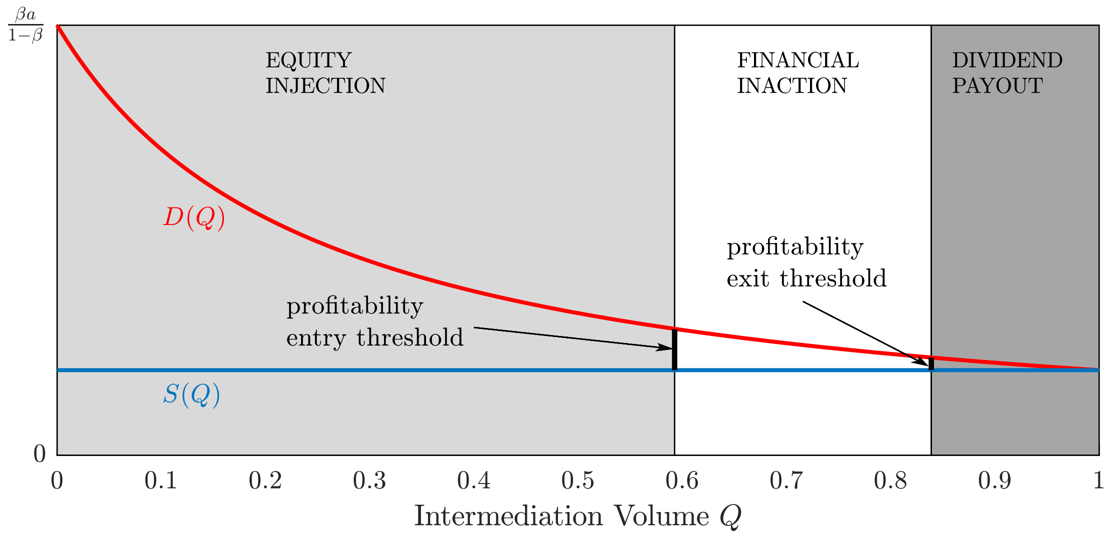


Figure 1: Financial Intermediation without Asymmetric Information

The model is laid out in Section 3 and characterized in Section 4. Section 5 presents the model's dynamics. Section 6 discusses the failure to coordinate banks' recapitalization after a financial crisis, and Section 7 concludes.

2 Nutshell

Fisher (1933) compares financial crises to the capsizing of a boat that "under ordinary conditions, is always near a stable equilibrium but which, after being tipped beyond a certain angle, has no longer this tendency to return to equilibrium." In the aftermath of the Great Depression, Fisher was providing us with a rudimentary description of the nonlinear nature of financial crises. The main insight of this paper is that asymmetric information can induce these "rocking boat" dynamics. We illustrate the underlying mechanism in Figures 1, 2, 3, and 4.

In Figures 1 and 2, the two curves represent aggregate demand and supply schedules for intermediation of capital, $D(Q)$ and $S(Q)$, respectively. In any intermediated market, intermediaries buy assets from suppliers and resell assets to final buyers. For a given aggregate quantity of trade Q, the intermediaries' marginal

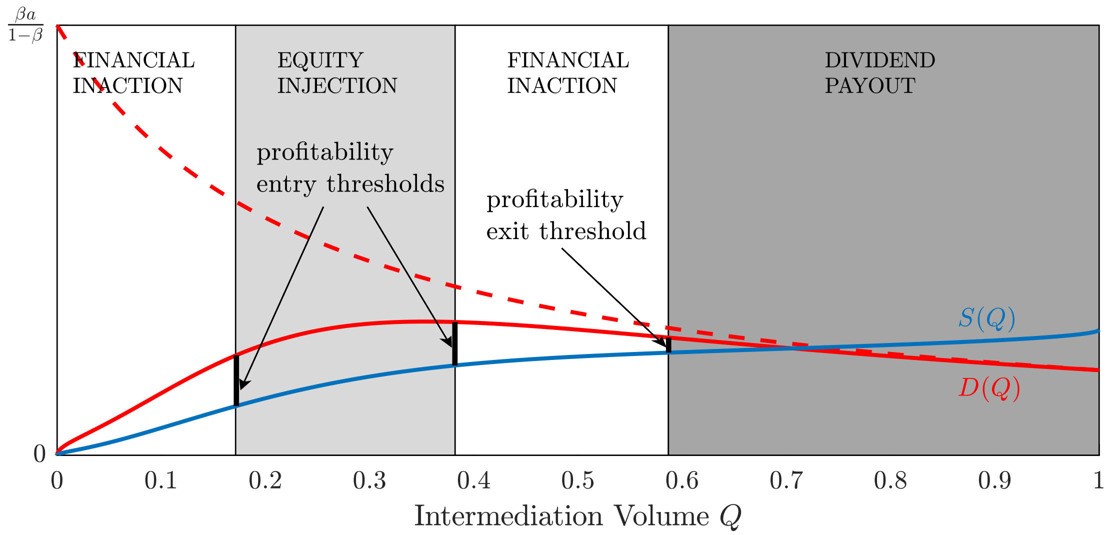


Figure 2: Financial Intermediation with Asymmetric Information

profit $\Pi(Q)$ is the distance between supply schedule price to the demand schedule price: $\Pi(Q) = S(Q) - D(Q)$. If some friction imposes a limit on the volume of intermediation, there is a positive arbitrage. In models with financial frictions, the net worth of intermediaries caps Q. Thus, the quantity of intermediation is increasing in the financial sector's net worth and Q can be represented as a function of bankers' net worth.

The shapes of the demand and supply schedules govern the behavior of marginal profits. In Figure 1, without information asymmetries, marginal profits are decreasing in Q, and thus also in net worth. Conversely, the evolution of net worth is influenced by marginal profits in two ways: directly, by affecting retained earnings, and indirectly, by attracting outside equity injections.

To understand this relation, suppose that there is a level of marginal profits below which dividends are paid out. Similarly, suppose there is another profitability threshold above which equity injections are attracted. Whenever net worth is above the level that induces exit-threshold profits, dividends are paid out. The opposite occurs whenever net worth is below the level that induces entry-threshold profits: Equity injections replenish net worth. Because the entry and exit profit levels are not the

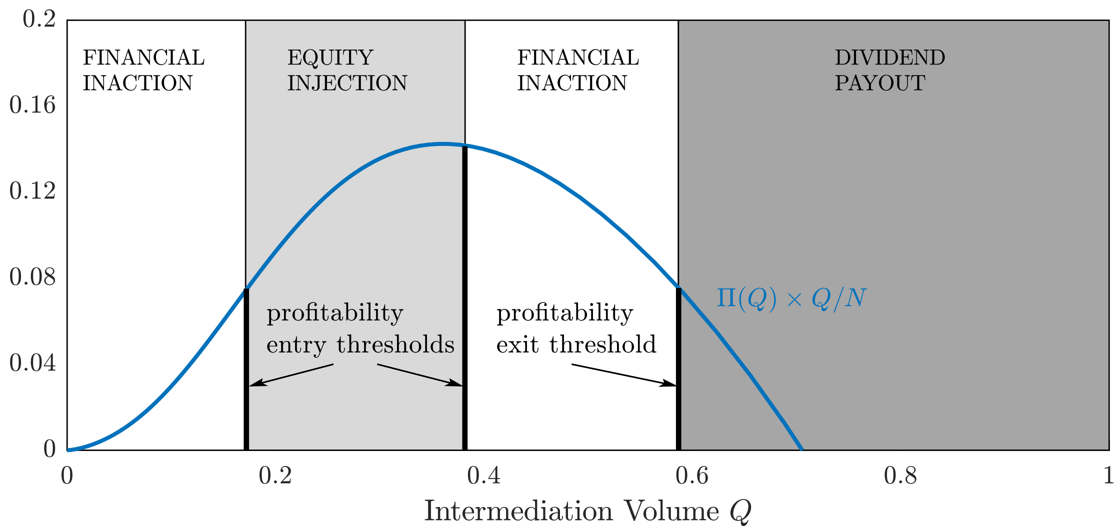


Figure 3: Profits per Unit of Net Worth $\Pi(Q) \times Q/N$

same, there is also an intermediate inaction region in which intermediaries neither pay dividends nor raise equity. Within that region, equity has a tendency to increase, but only through retained earnings. This illustrates an economic force that triggers financial stability. If anything reduces net worth below (above) the equity entry (exit) point, intermediaries raise (decrease) equity. In that world, intermediation, equity, and profits live within a bounded region.

Asymmetric information alters this stabilizing force. This situation is depicted in Figure 2, and emerges from an environment in which intermediaries buy individual assets under asymmetric information and resell them as a pool of homogeneous quality. When intermediaries purchase capital under asymmetric information, both the quantity and the quality of assets increase with the purchase price. This is why the supply schedule is also increasing. However, what has changed is that the demand faced by the intermediaries has a backward-bending portion. Standard consumer theory dictates that on the margin, the value of a unit of any normal good—savings instruments included—is lower than the marginal value of the previous unit, provided that all units are homogeneous. When qualities improve with quantities, the marginal valuation may actually rise with quantities—if qualities improve sufficiently

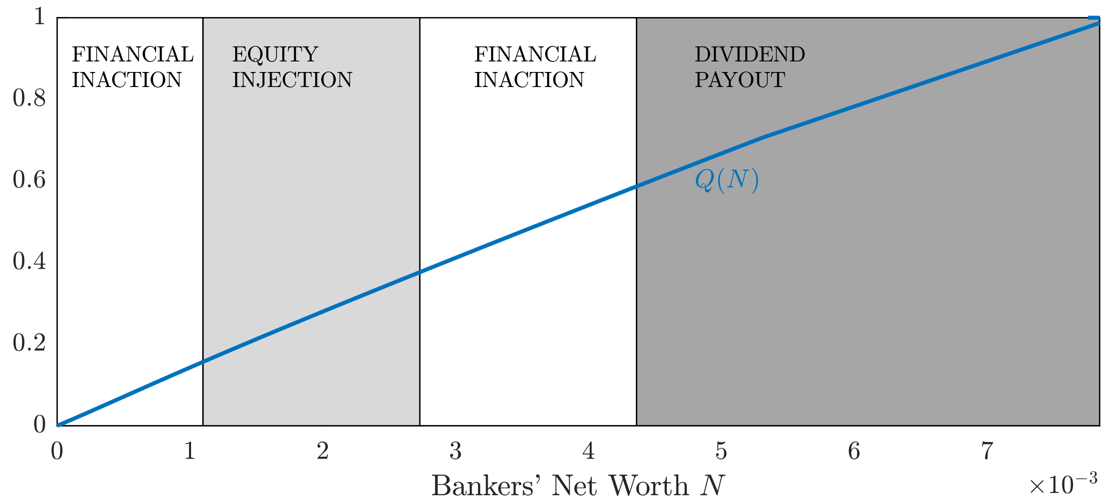


Figure 4: Volume of Intermediation $Q(N)$

fast. The result is an "effective" demand curve that can be backward-bending. A direct consequence of this backward-bending demand is that marginal profits are no longer necessarily decreasing, as in Figure 3. Instead, marginal profits are potentially hump-shaped. In the case of Figure 2, the hump shape in marginal profits generates two inaction regions instead of the single region found in Figure 1.

The inaction regions impair the stability of financial intermediation. Assume that net worth is in the rightmost inaction region of Figure 3. In that region, the dynamics of equity and intermediation depend on the size of intermediation losses, as in Fisher's rocking-boat analogy. A shock that produces equity losses, but only sends the economy to the neighboring injection region to the left, will be counterbalanced by quick equity injections. As a result, small shocks are stabilized, as in Figure 1. However, if losses are large enough to send the economy to the leftmost inaction region, the economy loses the tendency to return to equilibrium. Because profits are low, intermediaries lack individual incentives to inject equity. All in all, large shocks can capsize this economy. Eventually, this economy can recover, but slowly as intermediaries retain earnings.

The next section presents the dynamic environment for which Figures 1, 2, 3, and

4 are the solution. The rest of the paper characterizes the dynamics of the model.

3 Model

Time is discrete and the horizon is infinity. There are two goods: consumption goods (the numeraire) and capital goods. There is an aggregate shock, $\phi_t \in \Phi \equiv \{\phi_1, \phi_2, \dots, \phi_N\}$, that affects capital depreciation and follows a Markov process with standard assumptions. The model is populated by a continuum of producers and bankers. The banking sector intermediates capital from capital-goods producers to consumption-goods producers. The source of risk for intermediaries follows from the assumption that ϕ_t is realized after funding decisions are made.

3.1 Environment

Demographics There are two populations of agents: producers and bankers. Each population has unit mass. At the beginning of each period, producers are randomly segmented into two groups: capital-goods producers (k-producers) and consumption-goods producers (c-producers). Producers become k-producers with probability Δ, independent of time and ϕ_t.[8] Each agent carry a capital stock k_t, and because agents operate linear technologies, their problem is homogeneous within their own group.

Timeline There are three consecutive trades in this economy. First, k-producers sell capital under asymmetric information to bankers against IOUs (see Figure 5). After realization of the depreciation shock, c-producers sell consumption goods to k-producers against the IOUs of bankers (see Figure 6). Finally, c-producers redeem the IOUs of bankers against capital (see Figure 7).

[8]The real sector is directly borrowed from Kiyotaki and Moore (2019). These random assignments reduce the state space of the model.

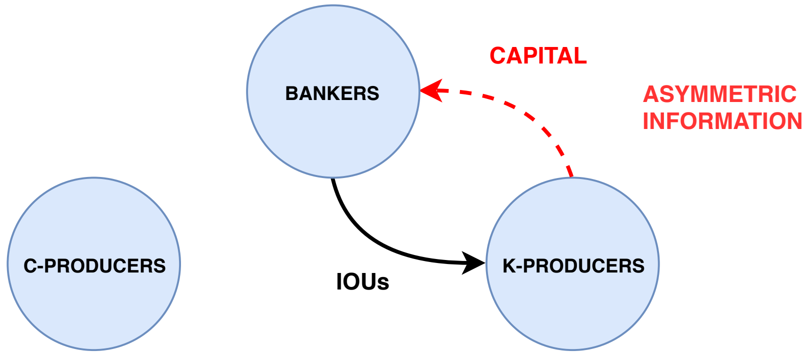


Figure 5: Flow of Funds between Bankers and K-producers. K-producers sell pools of capital under asymmetric information to bankers in exchange of riskless IOUs.

Producers Producers have logarithmic preferences over their stream of consumption c_t with time discount rate β:

$$\mathbb{E}_t\left[\sum_{\tau=t}^{\infty}\beta^t\log(c_\tau)\right].$$

C-producers operate a linear technology that produces ak_t units of consumption. Their output may be consumed or converted into capital by k-producers. In turn, k-producers have access to a linear investment technology that transforms κ units of consumption good into one unit of new capital. Each type of producer can only operate their corresponding technology.

The segmentation of production induces the need for trade: On the one hand, k-producers need consumption goods to operate their investment technologies. On the other hand, c-producers produce those resources but lack access to the investment technology. Consumption goods must flow from c-producers to k-producers, and capital must flow in the opposite direction.

Capital At the beginning of each period, capital k_t is divided into a uniform distribution of capital units, each identified by some quality $\varphi \in [0,1]$ that can be sold individually. The quality φ and the realization of ϕ_t determine the depreciation rate of each capital unit through the function $\lambda : [0,1] \times \Phi \to \mathbb{R}_+$. In particular, $\lambda(\varphi, \phi_t)$

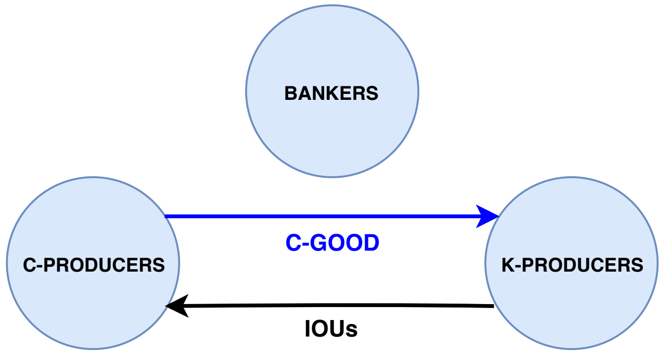


Figure 6: Flow of Funds between K-producers and C-producers. K-producers buy consumption goods from c-producers in exchange for riskless bankers' IOUs.

denotes the capital that will remain out of a φ-unit given ϕ_t. Once a φ-unit of capital is scaled by $\lambda(\varphi, \phi_t)$, it becomes homogeneous capital and can be merged with other units to form the next period stock of capital.[9] Thus, after realization of the depreciation shock ϕ_t on the stock of capital k_t, the remaining capital available for production next period is given by

$$k_t \int_0^1 \lambda(\varphi, \phi_t) d\varphi.$$

However, a producer can sell individual φ-units of capital. This decision is summarized by the indicator $\mathbb{1}(\varphi) : [0, 1] \to \{0, 1\}$ where $\mathbb{1}(\varphi)$ takes a value of 1 if φ is sold. Thus, the producer sells

$$k_t \int_0^1 \mathbb{1}(\varphi) d\varphi \tag{1}$$

and keeps

$$k_t \int_0^1 (1 - \mathbb{1}(\varphi))\lambda(\varphi, \phi_t) d\varphi.$$

To discipline our analysis, we make four assumptions on the depreciation function. Assumption 1 normalizes the depreciation function and facilitates the characteriza-

[9]This assumption greatly simplifies the state space, as we do not need to keep track of the distribution of qualities over time.

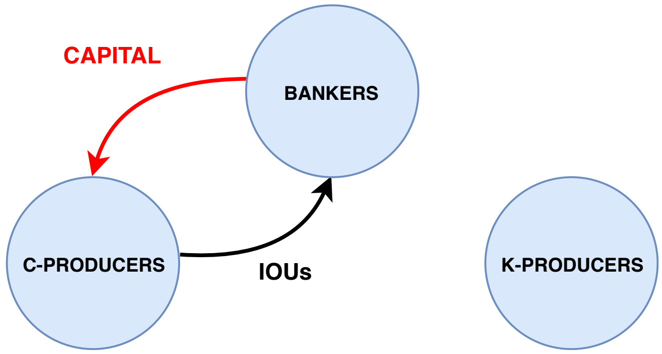


Figure 7: Flow of Funds between C-producers and Bankers. Bankers settle their IOUs by selling homogeneous units of capital to c-producers. Bankers' profits or losses arise from the difference between the proceeds of the sale of depreciated capital to c-producers and the debt contracted with k-producers.

tion in Section 4. Assumption 2 is without loss of generality and allows us to order φ-units of capital from the lowest to the highest quality. Assumption 3 preserves the ordinality of depreciation shocks ϕ over the whole range of φ. Assumption 4 allows the analysis to focus on intermediation risk and abstract from aggregate depreciation risk.

Assumption 1. *The depreciation function is such that* $\lambda(0, \phi) = 0$.

Assumption 2. *The depreciation function* $\lambda(\varphi, \phi)$ *is monotone and increasing in* φ.

Assumption 3. *If the following holds for* $\overline{\varphi} \in [0, 1]$*:*

$$\int_0^{\overline{\varphi}} \lambda(\varphi, \phi) d\varphi \leq \int_0^{\overline{\varphi}} \lambda(\varphi, \phi') d\varphi,$$

then it is also true for any other $\overline{\varphi}' \in [0, 1]$.

Assumption 4. *There is no aggregate depreciation risk:* $\int_0^1 \lambda(\varphi, \phi) d\varphi = \overline{\lambda} \; \forall \; \phi$.

Private Information The quality φ of each capital unit is known only to its owner. Buyers can only observe the quantity of a pool of sold units, as given in equation (1), but cannot discern the composition of φ-units within that pool. After ϕ_t is realized, the remaining capital available for production next period from that pool is given by

$$k_t \int_0^1 \mathbb{1}(\varphi)\lambda(\varphi, \phi_t) d\varphi.$$

As stated in Proposition 1, this information asymmetry between the seller and the buyer of capital incentivizes the seller to always sell the worst φ-units of capital first.[10] In equilibrium, agents can infer the threshold quality $\overline{\varphi}_t$ by observing the aggregate quantity of capital intermediated.

Proposition 1. *The decision to sell as a function of quality $\mathbb{1}(\varphi)$ is decreasing. Thus, it is equivalent to choose a threshold quality $\overline{\varphi}_t$ under which each unit of capital is sold. See Appendix A for the proof.*

Bankers Risk-neutral bankers provide intermediation by buying capital from k-producers and reselling these units to c-producers after the realization of the depreciation shock ϕ_t. Banks issue money to finance these investments and pay k-producers with riskless IOUs. These IOUs can be thought of as inside money and entitle their holders to a riskless unit of consumption. K-producers use these IOUs to buy consumption goods from c-producers and invest these resources to produce capital.

Bankers have access to an exogenous endowment of consumption goods. Every period, bankers can transfer wealth, as an equity injection, into legal institutions called banks. Once in a bank, bankers' net worth n_t is liable to intermediation losses, but personal endowments are protected by limited liability. He can also transfer a fraction of his net worth $d_t \in [0, 1]$ as dividends to his personal account. Access to an exogenous endowment of consumption goods is key to show that financial crises can occur even if banks have enough funds to recapitalize.

We make two assumptions regarding equity injections and dividend payments. First, when a banker is paid dividends, he must pay a tax τ. This tax can be

[10] This unique cutoff resembles the solution to the lemons problem of Akerlof (1978), with the distinction that there is adverse selection about risky assets as opposed to riskless assets.

interpreted as an exogenous wedge that emerges from agency frictions or government policies that are not modeled. The role of this tax is to induce a wedge between the marginal cost of equity and the marginal value of dividends. This wedge is essential to obtain inaction regions in which bankers neither pay dividends nor inject equity. If the tax rate is set to zero, inaction regions become inaction points.

Second, when a banker makes equity injections $e_t \times n_t$ where $e_t \in \mathbb{R}_+$, it comes with a cost $\Gamma(e_t) \times n_t$ such that

$$\Gamma(0) = 0, \qquad \frac{\delta \Gamma(e_t)}{\delta e_t} > 0, \qquad \frac{\delta^2 \Gamma(e_t)}{(\delta e_t)^2} > 0.$$

This cost can be interpreted as the liquidity premium of selling a large quantity of illiquid assets to recapitalize banks in a short period of time. Importantly, this assumption prohibits bankers from being indifferent between multiple optimal equity injection policies and prevents the existence of sunspot equilibria, which are not the focus of this paper.[11]

The diversification of idiosyncratic risk by bankers follows from the implicit assumption that only banks can buy large pools of capital. This gives banks an advantage over c-producers, who would otherwise bear the risk of getting a low φ-unit of capital when trading with a k-producer directly. This role emerges as an equilibrium outcome in Boyd and Prescott (1986), in which banks are coalitions of agents that join together to mitigate idiosyncratic risk.

When bankers buy a pool of capital from k-producers, they cannot distinguish the quality of φ-units of capital. Moreover, they hold on to the pool of capital until ϕ_t is realized. After the φ-units of capital in the pool depreciate, the pool is resold as homogeneous capital to c-producers and bankers settle all of their IOUs.

The depreciation shock ϕ_t renders bankers' assets risky, while their liabilities are riskless. Thus, in the event of a bad depreciation shock, bankers face equity losses if their IOUs exceed the value of their purchased capital pool. In principle, they could finance losses with their personal endowment, but limited liability protects

[11]Without convex costs in equity injections, multiple equilibrium equity injection policies can arise in equilibrium, as bankers' decisions to inject equity are strategic complements.

their personal wealth. Thus, the limited liability constraint caps the amount of intermediation a bank can provide: The greater the volume of capital bought, the greater the risk, and the greater the need for an equity cushion. While bankers can inject equity to scale up their operations, they need incentives to do so.

State Variables There are two aggregate quantities of interest: the aggregate capital stock,

$$K_t = \int_0^1 k_t(z)dz,$$

and the equity of the entire financial system,

$$N_t = \int_0^1 n_t(j)dj,$$

where individual producers and bankers are identified by $z \in [0,1]$ and $j \in [0,1]$, respectively.

In Section 4, we show that producers' policy functions are linear in their capital stock and bankers' policy functions are linear in their net worth. Thus, it is only necessary to keep track of the relative net worth of bankers relative to the size of the economy—the economy's financial risk capacity—which is defined as

$$\eta_t \equiv N_t/K_t.$$

The aggregate state of the economy is summarized by $\{\eta_t, \phi_t\}$.

3.2 Agents' Problems

Producers Using a recursive notation and dropping the time subscript, the value function of producers before being assigned to produce consumption goods or capital is given by

$$U(k,\eta) = (1-\Delta)U^c(k,\eta) + \Delta U^k(k,\eta),$$

where $U^c(k,\eta)$ and $U^k(k,\eta)$ are the value functions of c- and k-producers, respectively.

C-Producers We can write the problem of a c-producer as

$$U^c(k,\eta) = \mathbb{E}_\phi \left[\max_{c^c \geq 0, i^c \geq 0} \left\{ \log(c^c) + \beta U(k', \eta') \right\} \right],$$

subject to their budget constraint:

$$c^c + p^d i^c = ak,$$

and the law of motion for capital:

$$k' = k \int_0^1 \lambda(\varphi, \phi) d\varphi + i^c.$$

C-producers use their stock of capital to produce consumption goods. They can either consume c^c or invest i^c in new units of capital at price p^d. Consumption and investment decisions are made after the realization of the depreciation shock ϕ.

K-Producers The problem of a k-producer is given by

$$U^k(k,\eta) = \max_{\overline{\varphi}} \left\{ \mathbb{E}_\phi \left[\max_{c^k \geq 0, i^k} \left\{ \log(c^k) + \beta U(k', \eta') \right\} \right] \right\}$$

subject to their budget constraint:

$$c^k + \kappa i^k = p^s \overline{\varphi} k,$$

and the law of motion for capital:

$$k' = k \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi) d\varphi + i^k.$$

Before realization of the depreciation shock ϕ, k-producers choose the threshold quality $\overline{\varphi}$ under which they sell each φ-units of capital. Since bankers cannot differentiate φ-units of capital, k-producers sell each unit of capital at the same price p^s. If bankers were able to differentiate between φ-units of capital, there would be no asymmetries of information.

After realization of the depreciation shock ϕ, they can either consume c^k or produce new units of capital i^k at cost κ. Proposition 2 provides the parameter restriction on κ such that it is never optimal for k-producers to purchase intermediated capital from bankers. Note that we allow k-producers to transform capital into consumption goods—i^k can be negative—when selling capital is not sufficiently profitable to sustain their desired consumption level.[12]

Proposition 2. *If the following condition holds:*

$$\kappa < \frac{\beta a(1-\Delta)}{\Delta\overline{\lambda} + (1-\beta)\overline{\lambda}(1-\Delta)}, \tag{C1}$$

then in equilibrium, $p^d > \kappa$ is always true and k-producers never purchase intermediated capital from bankers. See Appendix E for the proof.

Bankers The problem of a banker is given by

$$U^b(n,\eta) = \max_{q\geq 0, e\geq 0, 1\geq d\geq 0} \left\{ \left(d-e\right)n + \mathbb{E}_\phi\left[\beta U^b(n',\eta')\right]\right\}$$

subject to the law of motion for wealth:

$$n' = n\Big(1 + e - \Gamma(e) - (1+\tau)d\Big) + q\pi(\overline{\varphi},\phi),$$

and the limited liability constraint:

$$n' \geq \varepsilon q, \qquad \forall \phi, \tag{2}$$

[12]Restricting i^k to be positive would not change our results but would introduce a new regime that unnecessarily complicates characterization of the solution.

where ε is a small positive number.

The intermediation profit $\pi(\overline{\varphi}, \phi)$ per unit of capital is the difference between the resale value of capital left after the depreciation shock and the cost of purchasing the pool of φ-units of capital:

$$\pi(\overline{\varphi}, \phi) \equiv p^d \Lambda(\overline{\varphi}, \phi) - p^s,$$

where $\Lambda(\overline{\varphi}, \phi) \equiv \frac{\int_0^{\overline{\varphi}} \lambda(\varphi, \phi) d\varphi}{\overline{\varphi}}$ is the average quality of φ-units of capital sold by k-producers.

Bankers take all of their decisions before realization of the depreciation shock ϕ. Bankers can either transfer wealth as an equity injection e into the bank, pay dividends d, or remain inactive. When paying dividends, bankers have to pay a tax τ while injecting capital costs $\Gamma(e)$.

Bankers then decide how much capital to intermediate q, subject to the limited liability constraint from equation (2). If ε were equal to 0, bankers would need just enough wealth to be able to pay any potential intermediation losses. With $\varepsilon > 0$, we ensure that bankers' wealth is never completely wiped out. This limited liability constraint is akin to a value-at-risk constraint that scales with the size of the balance sheet q and is such that the volume of intermediation per unit of net worth, q/n, is bounded above by $1/\varepsilon$.

Recursive Competitive Equilibrium Market-clearing conditions require that the demand for capital by bankers equals the amount of capital sold by k-producers at price p^s before the depreciation shock ϕ. After the depreciation shock ϕ, the supply of capital by bankers must equal the demand for capital by c-producers at price p^d. The definition of a recursive competitive equilibrium is a set of allocations that solve the agents' problems and prices p^s and p^d such that markets clear in both stages. The definition of the equilibrium is presented as Definition 1.

Definition 1 (Recursive Competitive Equilibrium). *A recursive competitive equilibrium is (i) a set of price functions $\{p^s(\eta), p^d(\eta, \phi)\}$, (ii) a set of policy functions for c-producers $\{c^c(k, \eta, \phi), i^c(k, \eta, \phi)\}$, (iii) a set of policy functions for k-*

producers $\{\overline{\varphi}(k,\eta), c^k(k,\eta,\phi), i^k(k,\eta,\phi)\}$*, (iv) a set of policy functions for bankers* $\{e(n,\eta), d(n,\eta), q(n,\eta)\}$*, (v) a set of value functions* $\{U^c(k,\eta), U^k(k,\eta), U^b(n,\eta)\}$*, and (vi) a law of motion for the aggregate state* $\eta'(\eta,\phi)$ *such that:*

1. *The agents' policy functions (ii), (iii), and (iv) are solutions to their respective problems given prices (i) and the law of motion for* η *(vi).*

2. *Markets for intermediation of capital, depreciated capital, and consumption goods clear.*

3. *The law of motion for the state variable* $\eta'(\eta,\phi)$ *is consistent with equilibrium functions and demographics.*

3.3 Discussion of the Environment

Role of Banks We interpret banks as commercial banks. In the model, banks invest in risky pools of capital sold by k-producers and finance the position with risk-free IOUs. The IOUs are used by c- and k-producers to trade goods and capital. In doing so, banks perform three roles stressed by the banking literature.[13] First, banks diversify idiosyncratic transaction risk by pooling assets. Second, they create a means of payment. Third, they provide insurance against aggregate risk, because they purchase capital prior to the realization of ϕ. We do not model the emergence of banks nor do we provide a theoretical foundation for this institutional arrangement. Nevertheless, the paper is motivated by the banking literature and the institutional features we see in practice.

The diversification of idiosyncratic risk follows from an implicit assumption on bank size. The idea is that banks are large agents and therefore can buy capital pools and, in turn, dilute idiosyncratic trade risk. By contrast, we envision that producers are small agents and cannot exploit the law of large numbers. If a c-producer were to contract privately with a k-producer, it would bear the risk of ending with low-quality capital. A similar role emerges in equilibrium in Boyd and

[13]See Freixas and Rochet (2008), Section 1.2.

Prescott (1986), in which banks are coalitions of small agents that join together to exploit economies of scale. We use that insight to motivate bank intermediation.

Second, banks provide liquidity services because they create risk-free liabilities. This aspect is fundamental. If the bank were to purchase capital by issuing debt contingent on the quality of the k-producer's capital, the bank would be transferring the asymmetric information back to the c-producer. As a result, trade opportunities would be lost. This is precisely the problem that banks are there to solve. Because banks offer risk-free IOUs, they mitigate the asymmetric information problem. This financial arrangement is consistent with the view in Gorton (2010) that the essential function of banking is to create a special kind of debt that is immune to adverse selection. Gorton and Ordonez (2014), Farhi and Tirole (2015), and Bigio and Weill (2016) are examples of related models in which the issuance of risk-free liabilities by banks emerge as an optimal security design problem tailored to improving trade under private information.

A third role for banks is to absorb the risk implied by ϕ. Liquidity provision requires that deposits be independent of φ, but it does not require these to be independent of ϕ. While we do not model this aspect explicitly, this role emerges if banks cannot contract on the aggregate state.

These insights motivate the building blocks of our model but we do not establish the optimal use of debt as means of payments nor the emergence of banks in equilibrium. We rather focus on the mechanics of slow recoveries following financial crises. Recent models that provide foundations for the emergence of banks and their role as liquidity providers include Gu, Mattesini, Monnet, and Wright (2013) and Donaldson, Piacentino, and Thakor (2018).

Asymmetry of Information and the Nature of Financial Activities While in the model banks intermediate capital, in the real world they issue loans. Loans are typically collateralized with residential or commercial mortgages, equipment, intellectual property, or accounts receivable. However, buying and selling assets is not all that different from lending with collateral. Furthermore, problems of asymmetric information in a market in which assets are sold produce similar effects on liquidity as in markets where the quality of collateral is private information. To see this connection, consider a loan collateralized with an asset whose quality is private information. The loan agreement sets a loan size, which we can think of as the analogue of the price by financial intermediaries, p^s. A loan agreement also determines a principal payment to be repaid by the borrower. There is default if the collateral quality is lower than the principal. Upon default, the bank seizes the collateral and resells it at its full information price, as in this paper. If there is no default, it is effectively as if the bank resells the collateral to the borrower—with the distinction that it sells it at a price determined by the face value of the debt.

A connection between intermediation under private information and collateralization under private information is studied formally by Bigio and Shi (2019). That paper presents a model in which deep-pocketed banks compete by offering collateralized loans and asset purchase contracts. The motive for trade is similar to the motive in this paper, although there is no aggregate risk or dynamics of bank equity. The important connection with this paper is that although collateralized loans provide more liquidity than outright sales, the environments are observationally equivalent. In particular, asset sales contracts, given a low dispersion of quality, produce the same amount of liquidity as collateralized loans given a greater dispersion of quality.

Asymmetries of information have been documented as an important feature of loan markets. In particular, Stroebel (2016) documents the importance of adverse selection on collateral quality in the residential mortgage market. Similarly, Sufi (2007) emphasizes that information asymmetry between lenders and borrowers influences the structure of syndicated loans. We envision that if we were to introduce collateralized debt contracts in the paper, the main message would be unaltered. We therefore keep the analysis simple by working with outright sales only, but our intermediation of capital is meant to capture actual lending practices.

Interpretation of $\lambda(\varphi, \phi)$ In modern economies, firms operate in complex production and financial networks. They produce in multiple interrelated product lines and hold risky claims on others. This amalgamation of physical and financial assets is represented by the collection of φ-units of capital held by producers. The quality

φ is an ordered index that maps the different attributes of assets into a comparable number, the efficiency units $\lambda(\varphi, \phi)$. The aggregate shock ϕ generates intermediation risk and captures distributional changes in asset values at business cycle frequencies. Bloom (2009) provides evidence of such increases in return dispersion in recessions. Through λ, φ, and ϕ the model parsimoniously captures these complex forces. In Section 4, we characterize the relationship between the shape of $\lambda(\varphi, \phi)$ and the strength of asymmetries of information.

Private Rationing In the model, banks take market prices and capital quality as given. When the net worth of banks is low, the clearing mechanism lowers the price at the same time as quality falls. A natural question is whether banks with low net worth could improve the equilibrium by rationing sellers while offering a higher price to attract assets of better quality. A potential concern is that banks could offer a constant pooling price and ration $k-$producers when they have low risk-bearing capacity. This would reduce quantities but keep the average quality constant, and thus the profitability per unit of intermediation. However, this mechanism does not take competition into account. In a related environment that allows for quantity and price competition under competitive search, Guerrieri, Shimer, and Wright (2010) find that this form of competition actually leads to separating equilibria, which is typically worse than the pooling of assets that occurs here. Interestingly, the comparative statics for asset dispersion are similar to our case. We keep the analysis simple by focusing on pure asset sales and not studying asset rationing. Bigio and Shi (2019) also compare lemons equilibrium with Guerrieri, Shimer, and Wright (2010).

Lack of Screening and Flight to Quality In our model, intermediaries cannot observe any attribute of φ. This assumption reduces the model's dimensionality at the expense of realism. In actuality, assets have observable and unobservable characteristics. Asset prices should condition on observable variables and there is evidence of flight to quality during downturns (see Bernanke, Gertler, and Gilchrist, 1994). At first, flight-to-quality behavior may seem to conflict with the idea that adverse selection worsens during crises. However, adverse selection and flight-to-quality can occur simultaneously: Adverse selection can worsen within assets of common observed characteristics, while banks recompose portfolios between assets of different observed characteristics (see Malherbe, 2014).

Similarly, the model abstracts from any screening because banks cannot invest to improve the information about the quality of the assets they invest in. However, in practice, banks employ resources to improve screening. Nonetheless, the important question is whether introducing a screening technology would offset the decline in profitability when banks scale down their operations. With fixed screening costs, profitability would also decline as banks shrink their scale.

4 Characterization

This section characterizes the key elements of the mechanism illustrated in Figures 1, 2, 3, and 4. First, we describe the problems of producers and bankers and show that all policy functions for producers are linear in their capital stock, and those for bankers are linear in their net worth. Thus, η is the only endogenous state. From the policy functions of k-producers, we deduce the supply schedule $S(Q)$. Then, we derive the regions in which banks inject equity, pay dividends, or remain inactive. Next, we derive the expected intermediation revenues for bankers to pin down the demand schedule $D(Q)$. Finally, we show that intermediation profits $\Pi(Q)$ are nonmonotone in the presence of information asymmetries. We relegate all derivations and proofs to the appendices.

4.1 Policy Functions

As a result of the homogeneity of production technologies and preferences, producers' policy functions are linear in their wealth. The wealth of c- and k-producers are functions of their stock of capital, and take a different form as their valuation of capital changes with their investment options. Lemma 1 provides the policy functions of c- and k-producers as a function of prices and their stock of capital.

Lemma 1. *C- and k-producers' consumption policies are given by*

$$c^c = (1-\beta)w^c, \qquad \textit{where} \quad w^c = qk\int_0^1 \lambda(\varphi,\phi)d\varphi + ak,$$
$$c^k = (1-\beta)w^k, \qquad \textit{where} \quad w^k = \kappa k\int_{\overline{\varphi}}^1 \lambda(\varphi,\phi)d\varphi + p^s\overline{\varphi}k.$$

From the problem of k-producers, it follows directly that the threshold policy is such that

$$\overline{\varphi}(p^s) = \arg\max_{\widetilde{\varphi}} \mathbb{E}_\phi\left[\log\left(\kappa\int_{\widetilde{\varphi}}^1 \lambda(\varphi,\phi)d\varphi + p^s\widetilde{\varphi}\right)\right].$$

Moreover, $\overline{\varphi}(p^s)$ is unique, strictly increasing in p^s, $\overline{\varphi}(0)=0$, and $\overline{\varphi}(\kappa\mathbb{E}_\phi[\lambda(1,\phi)]) = 1$.

Thus, the threshold policy $\overline{\varphi}$ does not depend on the holding of capital and the corresponding distribution of capital across k-producers. Deriving the aggregate quantity of capital supplied by k-producers is then straightforward: $Q = \overline{\varphi}(p^s)\Delta K$. The supply schedule from Figure 2 is the inverse of that function: $S(Q) = \overline{\varphi}^{-1}\left(\frac{Q}{\Delta K}\right)$.

Lemma 2. *Because $\overline{\varphi}(p^s)$ is strictly increasing in p^s, $S(Q)$ is also strictly increasing in Q. Furthermore, $S(0)=0$ and $S(\Delta K) = \kappa\mathbb{E}_\phi[\lambda(1,\phi)]$.*

Equivalently, Proposition 3 demonstrates that $e(\eta)$ and $d(\eta)$ do not depend on n, and q is a linear function of wealth: $q(n,\eta) = \mathfrak{q}(\eta)n$. Thus, we do not need to keep track of the corresponding distribution of net worth across bankers, and the aggregate quantity of capital intermediated by bankers from Figure 4 is given by $Q(N) = \mathfrak{q}(N/K)N$.

Proposition 3. *The bankers' value function is linear in the net worth of bankers. That is, $U^b(n,\eta) = u^b(\eta)n$ and $u^b(\eta)$ satisfies*

$$u^b(\eta) = \max_{\mathfrak{q}\geq 0, e\geq 0, 1\geq d\geq 0}\left\{d - e + \beta\mathbb{E}_\phi\left[u^b(\eta')\Big(1 + e - \Gamma(e) - (1+\tau)d + \mathfrak{q}\pi(\overline{\varphi},\phi)\Big)\right]\right\},$$

subject to the limited liability constraint:

$$\mathfrak{q} \leq \frac{1 + e - \Gamma(e) - (1+\tau)d}{|\pi(\overline{\varphi}, \underline{\phi}) - \varepsilon|},$$

where $\underline{\phi}$ is defined as the worst realization of the depreciation shock such that $\pi(\overline{\varphi}, \underline{\phi}) \leq \pi(\overline{\varphi}, \phi)\ \forall\ \phi \in \Phi$. Furthermore, $\pi(\overline{\varphi}, \underline{\phi})$ is always negative in equilibrium.

To characterize the bankers' policy functions, we define the value of inside equity as

$$\theta(\eta) \equiv \beta \mathbb{E}_{\phi}\left[u^b(\eta')\right] + \max\left\{\beta \mathbb{E}_{\phi}\left[u^b(\eta') \frac{\pi(\overline{\varphi}, \phi)}{|\pi(\overline{\varphi}, \underline{\phi}) - \varepsilon|}\right], 0\right\}.$$

The formula for $\theta(\eta)$ is intuitive. The first term is the marginal value of equity in the next stage. The second term is the shadow value of relaxing the limited liability constraint by holding an additional unit of net worth. This shadow value is equal to 0 when expected profits are negative and bankers do not intermediate capital. Using this definition for the value of inside equity, we can rewrite the bankers' value function as

$$u^b(\eta) = \max_{e \geq 0, 1 \geq d \geq 0} \left\{d - e + \theta(\eta)\Big(1 + e - \Gamma(e) - (1+\tau)d\Big)\right\}. \tag{3}$$

From Proposition 4, we get that $\mathfrak{q}(\eta)$ follows a linear program: the limited liability binds when the marginal intermediation profits are positive, $\mathbb{E}_{\phi}\big[\pi(\overline{\varphi}, \phi)\big] > 0$. Otherwise $\mathfrak{q}(\eta)$ is zero when expected profits are negative.

Likewise, the decisions to inject equity and pay dividends are also characterized by a linear program. When $\theta(\eta) < 1/(1+\tau)$, the banker pays dividends as the marginal value of inside equity is below the after-tax benefit of dividends. Conversely, if $\theta(\eta) > 1$, the banker injects equity because the value of inside equity exceeds the cost of forgone consumption. When $\theta(\eta) \in (1/(1+\tau), 1]$, there is financial inaction. Finally, if $\theta(\eta)$ is equal to $1/(1+\tau)$, bankers are indifferent about the scale of dividends. In equilibrium, the aggregation of dividends has to be consistent with the law of motion for η. Without loss of generality, we study the case with symmetrical dividend policies.[14]

Proposition 4. *The bankers' intermediation policy is given by*

$$\mathfrak{q}(\eta) = \frac{1 + e - \Gamma(e) - (1+\tau)d}{|\pi(\overline{\varphi}, \underline{\phi}) - \varepsilon|} \qquad \textit{if} \;\; \mathbb{E}_\phi\big[\pi(\overline{\varphi}, \phi)\big] > 0.$$

The intermediation policy is indeterminate at the individual level if $\mathbb{E}_\phi\big[\pi(\overline{\varphi}, \phi)\big] = 0$ *and equal to 0 if* $\mathbb{E}_\phi\big[\pi(\overline{\varphi}, \phi)\big] < 0$*. Similarly, the dividend policy is given by*

$$d(\eta) = 1 \qquad \textit{if} \;\; \theta(\eta) < 1/(1+\tau).$$

The dividend policy is indeterminate at the individual level if $\theta(\eta) = 1/(1+\tau)$ *and equal to 0 if* $\theta(\eta) > 1/(1+\tau)$*. Finally, the equity injection policy satisfies:*

$$e(\eta) = \Gamma_e^{-1}\left(\frac{\theta(\eta) - 1}{\theta(\eta)}\right) \qquad \textit{if} \;\; \theta(\eta) \geq 1 \qquad \textit{where} \;\; \Gamma_e(e) = \frac{\partial \Gamma(e)}{\partial e}.$$

The equity injection policy is equal to 0 if $\theta(\eta) < 1$*.*

4.2 Expected Intermediation Profits

Using the market-clearing condition for consumption goods, we derive the price for intermediated capital p^d:

$$p^d(\overline{\varphi}, \phi) = \frac{\beta a(1-\Delta)}{\overline{\varphi}\Lambda(\overline{\varphi}, \phi)\Delta + (1-\beta)\overline{\lambda}(1-\Delta)}.$$

The price $p^d(\overline{\varphi}, \phi)$ is decreasing in both $\overline{\varphi}$ and ϕ. Both outcomes are natural, because capital is a normal good. In essence, p^d falls when the supply increases and captures a substitution effect with consumption.

Risk-neutral bankers are willing to intermediate capital as long as the expected intermediation revenue, $D(Q)$, is higher than the cost of capital purchased from k-

[14] Since bankers' policy functions do not depend on their quantity of net worth and are linear in their net worth, the cross-sectional distribution of net worth has no impact on the equilibrium.

producers, $S(Q)$. Given the depreciation shock ϕ, the intermediation revenue per unit of capital, $d(\overline{\varphi}, \phi)$, is the product of the price of intermediated capital sold to c-producers and the average quality of φ-units of capital sold by k-producers:

$$d(\overline{\varphi}, \phi) = \underbrace{p^d\Big(\overline{\varphi}, \phi\Big)}_{\substack{\text{substitution} \\ \text{effect}}} \ \underbrace{\Lambda\Big(\overline{\varphi}, \phi\Big)}_{\substack{\text{composition} \\ \text{effect}}} . \tag{4}$$

Following Assumption 2, the average quality increases with $\overline{\varphi}$. Thus, the composition effect—the novelty of this paper—responds opposite to the substitution effect.

We have already defined the aggregate quantity of capital intermediated as a function of the threshold quality $\overline{\varphi}$: $Q = \overline{\varphi}\Delta K$. Thus, we can construct the demand schedule of capital from Figure 2:

$$D(Q) = \mathbb{E}_\phi\Big[d\big(\overline{\varphi}(Q), \phi\big)\Big] \quad \text{where } \overline{\varphi}(Q) = \frac{Q}{\Delta K}.$$

In Appendix E, we show that the composition effect always dominates the substitution effect with respect to the depreciation shock ϕ. In contrast, the effect of $\overline{\varphi}$ (and therefore Q) can be nonmonotone. Proposition 5 provides the necessary and sufficient condition for the composition effect to dominate. This condition is analogous to requiring that the depreciation function $\lambda(\varphi, \phi)$ be sufficiently steep in φ.

Proposition 5. *Given ϕ, the intermediation revenues are increasing in $\overline{\varphi}$,*

$$\frac{\partial d(\overline{\varphi}, \phi)}{\partial \overline{\varphi}} > 0,$$

if and only if the following condition holds:

$$\frac{\lambda(\overline{\varphi}, \phi) - \Lambda(\overline{\varphi}, \phi)}{\overline{\varphi}} > \frac{[\Lambda(\overline{\varphi}, \phi)]^2 \, \Delta}{(1-\beta)\overline{\lambda}(1-\Delta)}. \tag{C2}$$

The steepness of $\lambda(\varphi, \phi)$ is what the literature refers to as "information sensitivity." The further the marginal quality $\lambda(\varphi, \phi)$ is from the average quality of the pool $\Lambda(\overline{\varphi}, \phi)$, the stronger are the information asymmetries. Thus, (C2) is akin to imposing that information asymmetries weaken sufficiently fast as more capital is intermediated. Without information asymmetries, the price of each φ-unit of capital would be scaled by its quality $\lambda(\varphi, \phi)$ in equation (4) instead of $\Lambda(\overline{\varphi}, \phi)$, and the substitution effect would always dominate.

Lemma 3. *Intermediation revenues are such that* $d(0, \phi) = 0 \ \forall \ \phi \in \Phi$ *and, thus,* $D(0) = 0$.

The expected intermediation profits from Figure 3 can be written

$$\Pi(Q) = \mathbb{E}_\phi\Big[\pi\big(\overline{\varphi}(Q), \phi\big)\Big] = D(Q) - S(Q).$$

Because both the demand and the supply meet at the origin, $\Pi(0) = 0$ as well. Proposition 6 establishes that these profits increase when the volume of intermediation is low. That is, when Q is low, an increase in intermediation volume alleviates information asymmetries more than it satiates the need of c-producers for investment in capital. (C3) ensures that intermediation of capital is profitable in expectation, even when the volume of intermediation is low and information asymmetries are the strongest.

Proposition 6. *If the following sufficient condition holds,*

$$\kappa < \frac{1}{2}\frac{\beta a}{(1-\beta)\overline{\lambda}}, \tag{C3}$$

then expected intermediation profits increase as $\overline{\varphi}$ *tends to 0. That is,*

$$\lim_{Q \to 0} \frac{\Pi(Q)}{\partial Q} > 0.$$

Proposition 7 provides the last component to prove the nonmonotonicity of expected intermediation profits. This condition ensures that the bankers never intermediate all of the stock of capital of k-producers. This could happen if capital production were so efficient that k-producers could sell their entire stock of capital and produce enough capital to replenish their stock. In that situation, there would be no information asymmetries or intermediation risk for bankers. Importantly, (C4) also implies that $\Pi(\Delta K) < 0$.

Proposition 7. *If the following condition holds,*

$$\kappa > \frac{1}{\mathbb{E}_\phi\left[\lambda(1,\phi)\right]} \frac{\beta a(1-\Delta)}{\Delta + (1-\beta)(1-\Delta)}, \tag{C4}$$

then in equilibrium, k-producers never sell their entire stock of capital ($\overline{\varphi} < 1$).

Therefore, we have characterized the shape of expected intermediation profits from Figure 3. The function $\Pi(Q)$ is equal to 0 for $Q = 0$, then increases and becomes positive, and reaches negative territory as it gets closer to $Q = \Delta K$.

4.3 Quantity of Intermediated Capital

The last object to characterize is the quantity of intermediated capital as a function of bankers' total net worth, $Q(N)$, from Figure 4. This mapping arises from the market-clearing condition for capital intermediation:

$$\overline{\varphi}(p^s)\Delta = \mathfrak{q}\left(\eta, p^s\right)\eta.$$

The left-hand side is the aggregate supply of capital by k-producers and the right-hand side is the aggregate quantity of capital that bankers can intermediate while satisfying the limited liability constraint. The bankers' policy function $\mathfrak{q}$ is a function of the price p^s because it is constrained by the worst realization of intermediation profits (see Proposition 4),

$$\pi(\overline{\varphi}, \underline{\phi}) = p^d(\overline{\varphi}, \underline{\phi})\,\Lambda(\overline{\varphi}, \underline{\phi}) - p^s.$$

In general, there may be multiple solutions to that equation, as $\pi(\overline{\varphi}, \underline{\phi})$ might be nonmonotone and the aggregate net worth of bankers η could support losses associated with two different volumes of intermediation. This price multiplicity is not the focus of this paper. We can exclude the occurrence of multiple equilibria by ensuring that the worst realization of intermediation profits is monotone and decreasing in the quantity of intermediated capital. We provide explicit sufficient conditions such that $\partial \mathfrak{q}(\eta, p^s) / \partial p^s < 0$ is satisfied and p^s is unique in Appendix G. Given that this condition is satisfied, $Q(N)$ is an increasing function in bankers' total net worth N.

5 Dynamics

In this section, we describe the nonlinear dynamics of the model, and demonstrate that it features rocking-boat dynamics. To ensure that bankers never decide to completely deplete the net worth in their banks, we restrict dividend payments when banks' leverage ratio is above 10, in line with Basel capital requirements.[15] Without this restriction, when the net worth of the banking sector is sufficiently low, the value of inside equity might be so low that bankers withdraw all their net worth from the banks and the economy collapses in an absorbing steady state with no financial sector.

Calibration We solve numerically for the solution of the model with the following parameter values:

$$\{\beta, \overline{\lambda}, \Delta, a, \kappa, \alpha, \tau, \varepsilon\} = \{0.97, 0.95, 0.10, 0.05, 0.15, 0.20, 0.08, 0.10\}.$$

Importantly, we use the (α_1, α_2)-Beta cumulative distribution function for $\lambda(\overline{\varphi}, \phi)$ and two possible shocks for $\phi = (\alpha_1, \alpha_2) \in \{(8,4), (8,1)\}$ with probabilities $(0.5, 0.5)$. Finally, the functional form for the cost of equity injection is given by:

$$\Gamma(e) = \frac{\alpha}{2} e^2.$$

We set β so that the annualized risk-free rate is 3% at the stochastic steady state.

[15] We measure the leverage of the bank as the value of capital at the midpoint between the selling and demand price over the net worth of the bank: $q/n \times (p^s + p^d)/2$.

The depreciation rate $\overline{\lambda}$ is consistent with an average annual depreciation of the total capital stock of 5%. The fraction Δ is set to 0.10, as in Bigio (2015). The productivity of capital a is set such that the return on producing consumption goods fluctuates between 2% and 5%. With a capital production cost κ of 0.15, the return on equity of the bank is at most 10% and (C1) is satisfied. The equity injection cost parameter α is set sufficiently high to eliminate multiple equilibrium equity injection policies. The cost of dividend payments τ is set to 8% to be consistent with the estimates in Hennessy and Whited (2005). The limited liability parameter ε is equal to 10% to approximate Basel capital requirements.[16]

To construct the model moments, we simulate the economy one million times after a negative shock to bankers' net worth and plot the average response in Figure 8. The size of the shock is defined as the initial deviation of the equity of bankers from the stochastic steady state of the economy.

Recovery Time Figure 8a shows the average time it takes for the economy to recover after a negative shock to bankers' net worth, for two shocks of different sizes. The recovery time is measured as the expected time it takes for net worth to reach its stochastic steady-state level. The highly nonlinear response is noteworthy: The response to large shocks is extremely persistent, although the shock itself has no memory. For shocks that are above 60% of bankers' net worth, the recovery time increases exponentially with the size of the shock. For smaller shocks below 50%, the recovery time is quick and lasts only a few years.

Rocking-boat Dynamics Figures 8b to 8f show the responses of intermediation volume Q; expected intermediation profits $\Pi(Q) \times Q/N$; average quality of intermediated capital $\mathbb{E}_{\phi}[\Lambda(\overline{\varphi}, \phi)]$; growth rate of the economy K'/K; and equity injections e to a net worth shock of -50% and a net worth shock of -70%. These figures demonstrate that in response to the smaller shock, the economy stays near a stable equilibrium, while after a large shock, the economy loses this tendency to return to equilibrium.

[16] With this parameter value, the leverage of the bank is at most 10 over the state space.

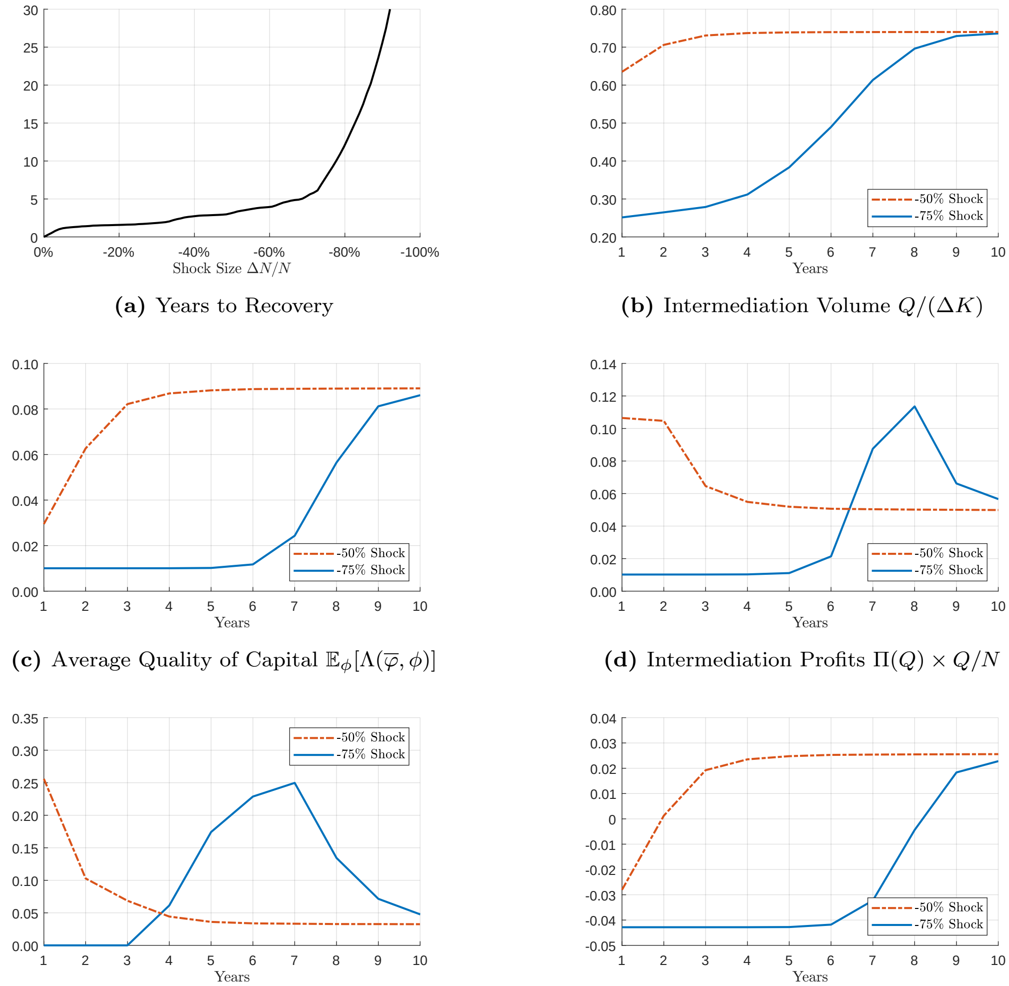
These distinct dynamics with respect to shocks of different sizes are driven by expected intermediation profits. Expected profits increase after the smaller shock, while following the larger shock, profits remain low for a long time. This difference follows from the interaction between the substitution and composition effects. The decline of financial risk capacity causes a simultaneous decline in intermediation volume and quality of intermediated capital (see Figures 8b and 8c). For the large shock, the composition effect is so dramatic that it overcomes the substitution effect and marginal profits decline (see Figure 8d).

In Figure 8e, we see large equity injections after the small shock, and these continue for a while until bank equity recovers. For the large shock, recapitalization of the banking sector does not occur until almost 8 years into the crisis. After the large shock, persistently low intermediation profits lead to the inability of bankers to coordinate equity injections.

Importantly, as shown in Figure 8f, the collapse of financial intermediation translates into an investment decline and slowdown of the growth rate of the economy. The long-lasting impact on growth after the large shock creates a large shift in the trend of output, a concurrent theme during the last crisis.

It is useful to contrast these results with those of He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014). In He and Krishnamurthy (2013), bankers require a higher premium to manage risky investments when their net worth is low. In Brunnermeier and Sannikov (2014), the surge in the risk premium is dampened by the participation of less productive households. Nevertheless, in both models, intermediation margins increase during financial crises and speed up recoveries. If there were free entry into the banking sector in either model, we would see banks being recapitalized even faster. In this paper, recoveries are slow despite free entry, because profitability is low.

Thus, our framework characterizes the recovery from a deep banking crisis as particularly slow and ties financial intermediation to economic growth and capital reallocation. In line with our results, Reinhart and Rogoff (2009) calculate that countries where banking crises occurred took up to a decade to recover from banking crises and Cerra and Saxena (2008) document that growths fall to -8% in a cross-



(a) Years to Recovery

(b) Intermediation Volume $Q/(\Delta K)$

(c) Average Quality of Capital $\mathbb{E}_\phi[\Lambda(\overline{\varphi}, \phi)]$

(d) Intermediation Profits $\Pi(Q) \times Q/N$

(e) Equity Injections e

(f) Growth Rate K'/K

Figure 8: Dynamics of the Numerical Solution

country average of financial crises, well beyond the typical recession. Foster, Grim, and Haltiwanger (2016) show that capital reallocation fell dramatically during the Great Recession. All in all, the model produces the rocking-boat dynamics of Fisher (1933) while fitting the narrative of Mishkin (1990) or Calomiris and Gorton (1991), in which asymmetric information plays a critical role.

6 Coordination Failures

A typical response during many crises is a public effort to recapitalize banks. This response is commonly seen as a necessary yet costly bailout of the banking system. An important historical episode suggests another story: a coordination failure. In October 1907, J.P. Morgan summoned New York City's major financial institutions and managed to coordinate a capital injection for several trust and brokerage companies, thus saving them from insolvency (Chernow, 2010). The motivation was not to benefit society, but rather the recognition that a coordinated effort to inject equity would benefit the industry.

In this section, we provide the solution to the coordinated investment problem of bankers to highlight how asymmetric information generates externalities in the decision to recapitalize banks. That is, we solve for the equilibrium in which bankers can coordinate their equity and dividend policies. We write this coordinated investment problem as

$$u^c(\eta) = \max_{e \geq 0, 1 \geq d \geq 0} \left\{ d - e + \theta^c(\eta, e, d)\Big(1 + e - \Gamma(e) - (1+\tau)d\Big) \right\},$$

where

$$\theta^c(\eta, e, d) = \beta \mathbb{E}_\phi \left[u^c(\eta')\right] + \max\left\{ \beta \mathbb{E}_\phi \left[u^c(\eta') \frac{\pi(\overline{\varphi}^c, \phi)}{|\pi(\overline{\varphi}^c, \underline{\phi}) - \varepsilon|} \right], 0 \right\}.$$

Compared to the definition of $u(\eta)$ in equation (3), the value of inside equity, θ^c, is explicitly a function of e and d. Thus, bankers take into account the impact of their decisions on the accumulation of aggregate bankers' wealth, through the law of motion of the state variable η. We label the solution to the coordinated investment problem with a superscript c—e.g., e^c, and d^c for the bankers' decision policies.

In Proposition 8, we formalize an intuitive result: If the marginal utility of wealth is increasing in η, then the coordinated investment problem yields a higher equity injection than the solution of the noncoordinated investment problem. As the value of inside equity inherits the nonmonotonous property of the profit function, the numerical solution of the coordinated investment problem shown in Figure 9 displays a faster recovery due to higher equity injections.

Proposition 8. *If the value of inside equity $\theta(\eta)$ is increasing in the bankers' share of wealth η,*

$$\frac{\partial\theta(\eta)}{\partial\eta} > 0,$$

then

$$e^c(\eta;\theta) > e(\eta;\theta).$$

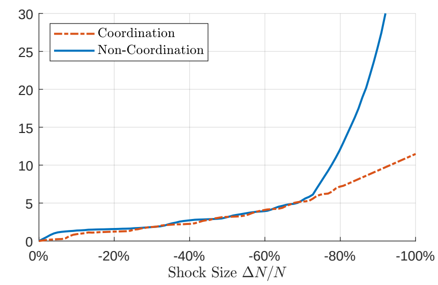
Figure 9 compares the response functions between the equilibrim with and without coordination of bankers' equity injections for a shock that destroys 75% of bankers' net worth. The salient feature is that with coordination, the recovery is much faster. Figure 9a shows the differences in the number of years to recovery. Recall that without coordination, for shocks greater than those that destroy 50% of bank equity, recovery times increase exponentially. With coordination, that slow recovery is no longer present.

Figures 9b to 9f illustrate that feature. Whereas without coordination injections are nil after the shock, with coordination injections are very high (see Figure 9e). With coordination, bankers recognize the impact of their capital choice on on marginal intermediation profits (see Figure 9d). The injection of equity restrains the drop in quality: Intermediation volume and the average quality of intermediated capital fall but not as much as without coordination. Note that adverse selection is still triggering a drop in profitability, but bankers acknowledge that this is only temporary: If they continue to inject equity, profitability will eventually increase (in year 2 in this example). By contrast, profits take up to 7 years to recover without coordination. Naturally, as shown in Figure 9f, the decline in growth only lasts for 3 years under the equilibrium with coordination and it takes up to 8 years without coordination.

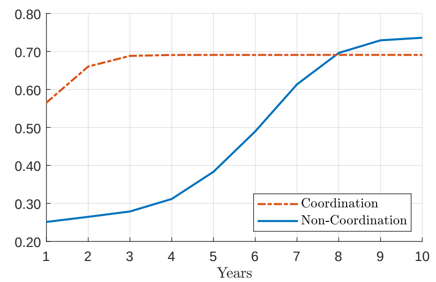
What is striking about banking crises is that they are deep and long-lasting, although human and physical resources remain intact. Fundamentally, when banks lose net worth, the only change for society is the reallocation of property rights over existing resources. That reallocation, however, disrupts economic activity. A typical view among economists is that during financial crises, the private recapitalization of banks is impossible due to various market imperfections. Government bailouts are one way around market imperfections, but these come at a cost for society, either in terms of fiscal resources or future incentives. The view here is more benign. The government can promote equity injections to correct a coordination failure among private investors. Of course, the policy challenge is to weight in moral hazard concerns.

7 Conclusion

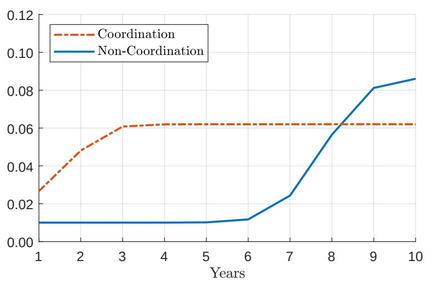
This paper provides a theory of risky financial intermediation under asymmetric information. The central message is that financial markets in which asymmetric information is a first-order friction are likely to be more unstable than otherwise. The source of instability is the decrease in profitability generated by low intermediation volume. This force deters incentives to recapitalize banks after large losses, even when resources are available. The financial crises that emerge are deep and long-lasting.



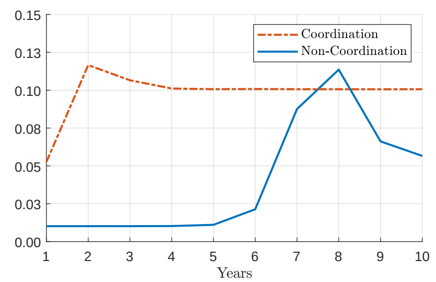
(a) Years to Recovery



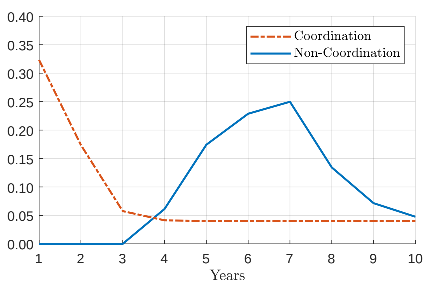
(b) Intermediation Volume $Q/(\Delta K)$



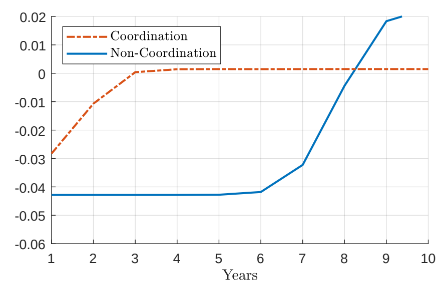
(c) Average Quality of Capital $\mathbb{E}_\phi[\Lambda(\overline{\varphi}, \phi)]$



(d) Intermediation Profits $\Pi(Q) \times Q/N$



(e) Equity Injections e



(f) Growth Rate K'/K

Figure 9: Dynamics of the Numerical Solution with and without Coordination

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Appendices

In these appendices, we prove explicitly every lemma and proposition. We solve the policy functions of producers and bankers. As in the main text, we use a recursive notation.

A Proof of Proposition 1

We define $\mathbb{1}(\varphi)$ as the indicator function that is equal to 1 if the k-producer chooses to sell that φ-unit of capital at price p^s and 0 otherwise. Here, we show that an optimal $\mathbb{1}(\varphi)$ must be monotone decreasing. Thus, choosing $\mathbb{1}(\varphi)$ is identical to choosing a cutoff $\overline{\varphi}$ under which all units of quality lower than this cutoff are sold.

Suppose that $\mathbb{1}'(\varphi)$ is not monotone decreasing. That is, the optimal plan is given by some $\mathbb{1}'(\varphi)$ whose value cannot be attained by any monotone decreasing policy. It is enough to show that the producer can find another candidate $\mathbb{1}(\varphi)$ that integrates to the same number, that is monotone decreasing, and that makes his value weakly greater.

Thus, assume that $\int_0^1 \mathbb{1}(\varphi)d\varphi = \int_0^1 \mathbb{1}'(\varphi)d\varphi$. Since $\mathbb{1}(\varphi)$ is monotone decreasing and $\lambda(\varphi, \phi)$ is monotone increasing in φ,

$$\int_0^1 (1 - \mathbb{1}(\varphi))\lambda(\varphi, \phi)d\varphi \leq \int_0^1 (1 - \mathbb{1}(\varphi))\lambda(\varphi, \phi)d\varphi,$$

implying that any optimal policy can be attained by some $\mathbb{1}^\star(\varphi)$ monotone decreasing.

B Proof of Lemma 1

C-producers The problem of c-producers is given by

$$U^c(k, \eta) = \mathbb{E}_\phi \left[\max_{c^c \geq 0, i^c \geq 0} \left\{ \log(c^c) + \beta U(k', \eta') \right\} \right]$$

subject to their budget constraint:

$$c^c + p^d i^c = ak,$$

and the law of motion for capital:

$$k' = k \int_0^1 \lambda(\varphi, \phi) d\varphi + i^c.$$

First, we guess the functional form for the value function of c-producers as

$$U^c(k, \eta) = \frac{\log(k)}{1-\beta} + \psi^c(\eta),$$

the functional form for the value function of k-producers as

$$U^k(k, \eta) = \frac{\log(k)}{1-\beta} + \psi^k(\eta),$$

which gives us a guess for the value function of producers:

$$U(k, \eta) = \frac{\log(k)}{1-\beta} + \psi(\eta),$$

where

$$\psi(\eta) = (1-\Delta)\psi^c(\eta) + \Delta\psi^k(\eta).$$

With this guess, we can substitute the budget constraint and the law of motion for capital:

$$U^c(k, \eta) = \mathbb{E}_\phi \left[\max_{c^c \geq 0} \left\{ \log(c^c) + \frac{\beta}{1-\beta} \log\left(k\bar{\lambda} + \frac{ak - c^c}{p^d} \right) + \beta\psi(\eta') \right\} \right].$$

Taking the first-order condition with respect to consumption, we get

$$\frac{1}{c^c} = \frac{\beta}{1-\beta} \frac{1}{p^d k \overline{\lambda} + ak - c^c}$$

That is,

$$c^c = (1-\beta)\left(p^d \overline{\lambda} + a\right) k,$$
$$i^c = \beta \frac{ak}{p^d} - (1-\beta)\overline{\lambda} k.$$

Thus,

$$U^c(k,\eta) = \frac{\beta \log(\beta) + (1-\beta)\log(1-\beta)}{1-\beta} + \frac{\log\left(\overline{\lambda} + \frac{a}{p^d}\right)}{1-\beta} + \frac{\log(k)}{1-\beta} + \beta \mathbb{E}_\phi\left[\psi(\eta')\right],$$

and we verified our guess as

$$\psi^c(\eta) = \frac{\beta \log(\beta) + (1-\beta)\log(1-\beta)}{1-\beta} + \frac{\log\left(\overline{\lambda} + \frac{a}{p^d}\right)}{1-\beta} + \beta \mathbb{E}_\phi\left[\psi(\eta')\right].$$

K-producers Similarly, the problem of k-producers is given by

$$U^k(k,\eta) = \max_{\overline{\varphi}} \left\{ \mathbb{E}_\phi \left[\max_{c^k \geq 0, i^k} \left\{ \log(c^k) + \beta U(k', \eta') \right\} \right] \right\}$$

subject to their budget constraint:

$$c^k + \kappa i^k = p^s \overline{\varphi} k,$$

and the law of motion for capital:

$$k' = k \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi) d\varphi + i^k.$$

With this guess, we can substitute the budget constraint and the law of motion for capital:

$$U^k(k,\eta) = \max_{\overline{\varphi}} \left\{ \mathbb{E}_\phi \left[\max_{c^k \geq 0} \left\{ \log(c^k) + \frac{\beta}{1-\beta} \log \left(k \int_{\overline{\varphi}}^1 \lambda(\varphi,\phi) d\varphi + \frac{p^s \overline{\varphi} k - c^k}{\kappa} \right) + \beta \psi(\eta') \right\} \right] \right\}$$

Taking the first-order condition with respect to consumption, we get

$$\frac{1}{c^k} = \frac{\beta}{1-\beta} \frac{1}{\kappa k \int_{\overline{\varphi}}^1 \lambda(\varphi,\phi) d\varphi + p^s \overline{\varphi} k - c^k}.$$

That is,

$$\begin{aligned} c^k &= (1-\beta) \left(\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi,\phi) d\varphi + p^s \overline{\varphi} \right) k, \\ i^k &= \beta \frac{p^s \overline{\varphi}}{\kappa} k - (1-\beta) k \int_{\overline{\varphi}}^1 \lambda(\varphi,\phi) d\varphi. \end{aligned}$$

Thus,

$$U^k(k,\eta) = \frac{\beta \log(\beta) + (1-\beta)\log(1-\beta)}{1-\beta} + \frac{\log \left(\int_{\overline{\varphi}}^1 \lambda(\varphi,\phi) d\varphi + \frac{p^s \overline{\varphi}}{\kappa} \right)}{1-\beta} + \frac{\log(k)}{1-\beta} + \beta \mathbb{E}_\phi \left[\psi(\eta') \right],$$

and we verified our guess as

$$\psi^k(\eta) = \frac{\beta \log(\beta) + (1-\beta)\log(1-\beta)}{1-\beta} + \frac{\log \left(\int_{\overline{\varphi}}^1 \lambda(\varphi,\phi) d\varphi + \frac{p^s \overline{\varphi}}{\kappa} \right)}{1-\beta} + \beta \mathbb{E}_\phi \left[\psi(\eta') \right].$$

Taking the first-order condition with respect to the quality threshold $\overline{\varphi}$, we get

$$\mathbb{E}_\phi \left[\frac{\kappa \lambda(\overline{\varphi},\phi) - p^s}{\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi,\phi) d\varphi + p^s \overline{\varphi}} \right] = 0.$$

Thus, if $p^s = 0$, then $\overline{\varphi} = 0$, and if $p^s \geq \kappa \mathbb{E}_\phi[\lambda(1,\phi)]$, then $\overline{\varphi} = 1$. Let's define the following function:

$$f(\overline{\varphi}, p^s) = \mathbb{E}\left[\frac{-\kappa\lambda(\overline{\varphi}, \phi) + p^s}{\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi)d\varphi + p^s\overline{\varphi}}\right] = 0.$$

Taking the derivative with respect to $\overline{\varphi}$, we get

$$\frac{\partial f(\overline{\varphi}, p^s)}{\partial \overline{\varphi}} = \mathbb{E}\left[\frac{-\kappa\frac{\partial \lambda(\overline{\varphi}, \phi)}{\partial \overline{\varphi}}}{\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi)d\varphi + p^s\overline{\varphi}}\right] - \mathbb{E}\left[\left(\frac{-\kappa\lambda(\overline{\varphi}, \phi) + p^s}{\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi)d\varphi + p^s\overline{\varphi}}\right)^2\right] < 0,$$

since $\frac{\partial \lambda(\overline{\varphi}, \phi)}{\partial \overline{\varphi}} > 0 \ \forall \overline{\varphi} \in [0, 1]$. Similarly,

$$\begin{aligned}
\frac{\partial f(\overline{\varphi}, p^s)}{\partial p^s} &= \mathbb{E}\left[\frac{1}{\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi)d\varphi + p^s\overline{\varphi}}\right] - \mathbb{E}\left[\frac{-\overline{\varphi}\kappa\lambda(\overline{\varphi}, \phi) + p^s\overline{\varphi}}{\left(\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi)d\varphi + p^s\overline{\varphi}\right)^2}\right], \\
&= \mathbb{E}\left[\frac{\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi)d\varphi + p^s\overline{\varphi} + \overline{\varphi}\kappa\lambda(\overline{\varphi}, \phi) - p^s\overline{\varphi}}{\left(\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi)d\varphi + p^s\overline{\varphi}\right)^2}\right], \\
&= \mathbb{E}\left[\frac{\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi)d\varphi + \overline{\varphi}\kappa\lambda(\overline{\varphi}, \phi)}{\left(\kappa \int_{\overline{\varphi}}^1 \lambda(\varphi, \phi)d\varphi + p^s\overline{\varphi}\right)^2}\right] > 0.
\end{aligned}$$

Thus, using the implicit function theorem, we find that

$$\frac{\partial \overline{\varphi}(p^s)}{\partial p^s} = -\left.\frac{\partial f(\overline{\varphi}, p^s)}{\partial p^s}\middle/\frac{\partial f(\overline{\varphi}, p^s)}{\partial \overline{\varphi}}\right. > 0 \qquad \forall p^s \in [0, \kappa\mathbb{E}_\phi[\lambda(1, \phi)]]. \tag{5}$$

C Proof of Lemma 2

A corollary of (5) is that

$$\frac{\partial p^s(\overline{\varphi})}{\partial \overline{\varphi}} > 0 \qquad \forall \overline{\varphi} \in [0, 1].$$

We can define the supply schedule for capital intermediation as the inverse of that function:

$$S(Q) = \overline{\varphi}^{-1}\left(\frac{Q}{\Delta K}\right),$$

where we used the market-clearing condition for capital intermediation:

$$Q = \overline{\varphi}(p^s)\Delta K.$$

Since $\overline{\varphi}(p^s)$ is strictly increasing in p^s, $S(Q)$ is also strictly increasing in Q. Since $p^s(0) = 0$ and $p^s(1) = \kappa\mathbb{E}_\phi[\lambda(1,\phi)]$, then $S(0) = 0$ and $S(\Delta K) = \kappa\,\mathbb{E}_\phi\,[\lambda(1,\phi)]$.

D Proof of Proposition 3 and Proposition 4

Finally, the problem of a banker is given by:

$$U^b(n,\eta) = \max_{q\geq 0, e\geq 0, 1\geq d\geq 0}\left\{\big(d-e\big)n + \mathbb{E}_\phi\Big[\beta U^b(n',\eta')\Big]\right\}$$

subject to the law of motion for wealth:

$$n' = n\Big(1 + e - \Gamma(e) - (1+\tau)d\Big) + q\pi(\overline{\varphi},\phi),$$

and the limited liability constraint:

$$n' \geq \varepsilon q, \qquad \forall\phi, \tag{6}$$

where ε is a small positive number. The intermediation profit $\pi(\overline{\varphi},\phi)$ per unit of capital is the difference between the resale value of capital left after the depreciation shock and the cost of purchasing the pool of φ-units of capital:

$$\pi(\overline{\varphi},\phi) \equiv p^d\,\Lambda(\overline{\varphi},\phi) - p^s,$$

where $\Lambda(\overline{\varphi},\phi) \equiv \frac{\int_0^{\overline{\varphi}} \lambda(\varphi,\phi)d\varphi}{\overline{\varphi}}$ is the average quality of φ-units of capital sold by k-producers.

We can rewrite (6) as

$$n\Big(1+e-\Gamma(e)-(1+\tau)d\Big)+q\Big(\pi(\overline{\varphi},\underline{\phi})-\varepsilon\Big)\geq 0,$$

where $\underline{\phi}$ is defined as the worst realization of the depreciation shock such that $\pi(\overline{\varphi},\underline{\phi}) \leq \pi(\overline{\varphi},\phi)\ \forall\ \phi \in \Phi$. Since bankers are risk-neutral, if expected profits are positive, they will leverage as much as possible. That is,

$$q(n,\eta)=\frac{1+e-\Gamma(e)-(1+\tau)d}{-\pi(\overline{\varphi},\underline{\phi})+\varepsilon}n \qquad \text{if} \ \ \mathbb{E}_\phi\big[\pi(\overline{\varphi},\phi)\big]>0.$$

Given the solution for $q(n,\eta)$ if $\mathbb{E}_\phi\big[\pi(\overline{\varphi},\phi)\big]>0$, we can rewrite the value function as

$$u^b(\eta)=\max_{e\geq 0,1\geq d\geq 0}\left\{d-e+\mathbb{E}_\phi\left[\beta u^b(\eta')\Big(1+e-\Gamma(e)-(1+\tau)d\Big)\left(1+\frac{\pi(\overline{\varphi},\phi)}{|\pi(\overline{\varphi},\underline{\phi})-\varepsilon|}\right)\right]\right\},$$

where $U^b(n,\eta)=u^b(\eta)n$. Therefore the value function is linear in bankers' net worth and the policy function for the quantity of intermediation is given by:

$$\mathfrak{q}(\eta)=\frac{1+e-\Gamma(e)-(1+\tau)d}{|\pi(\overline{\varphi},\underline{\phi})-\varepsilon|} \qquad \text{if} \ \ \mathbb{E}_\phi\big[\pi(\overline{\varphi},\phi)\big]>0,$$

where $q(n,\eta)=\mathfrak{q}(\eta)n$. Note that in equilibrium, it must be the case that $\pi(\overline{\varphi},\underline{\phi})-\varepsilon$ is negative when $\mathbb{E}_\phi\big[\pi(\overline{\varphi},\phi)\big]>0$; otherwise, bankers want to intermediate an infinite quantity of capital.

Furthermore, we can define the value of inside equity as

$$\theta(\eta)\equiv\beta\mathbb{E}_\phi\left[u^b(\eta')\right]+\max\left\{\beta\mathbb{E}_\phi\left[u^b(\eta')\frac{\pi(\overline{\varphi},\phi)}{|\pi(\overline{\varphi},\underline{\phi})-\varepsilon|}\right],0\right\},$$

and rewrite (3) as

$$u^b(\eta) = \max_{e\geq 0, 1\geq d\geq 0} \left\{ d - e + \theta(\eta)\Big(1 + e - \Gamma(e) - (1+\tau)d\Big) \right\}.$$

The first-order condition for dividend payouts is then given by

$$\frac{1}{1+\tau} = \theta(\eta).$$

Thus, the dividend policy is given by

$$d(\eta) = 1 \qquad \text{if} \;\; \theta(\eta) < 1/(1+\tau).$$

The dividend policy is indeterminate at the individual level if $\theta(\eta) = 1/(1+\tau)$ and equal to 0 if $\theta(\eta) > 1/(1+\tau)$.

The first-order condition for equity injections is given by

$$\Gamma_e(e) = \frac{\theta(\eta) - 1}{\theta(\eta)}.$$

Thus, the equity injection policy satisfies

$$e(\eta) = \Gamma_e^{-1}\left(\frac{\theta(\eta) - 1}{\theta(\eta)}\right) \qquad \text{if} \;\; \theta(\eta) \geq 1 \qquad \text{where} \;\; \Gamma_e(e) = \frac{\partial \Gamma(e)}{\partial e}.$$

Since $\Gamma_e(0) = 0$ and $\Gamma_e(e) > 0$, there is always a solution when $\theta(\eta) \geq 1$. The equity injection policy is equal to 0 if $\theta(\eta) < 1$, since $\Gamma_e(e) > 0 \; \forall e \in \mathbb{R}^+$.

E Proof of Proposition 2 and Proposition 5

The market-clearing condition for consumption goods is given by

$$\pi(\overline{\varphi}, \phi)q + c^c + c^k + i^k = a(1-\Delta)K,$$

where K is the aggregate supply of capital. The policy decisions for consumption are given by

$$c^c = (1-\beta)\left(p^d\overline{\lambda} + a\right)(1-\Delta)K,$$

while the budget constraint for k-producers is such that

$$c^k + i^k = p^s\overline{\varphi}\Delta K.$$

Futhermore, the market-clearing condition for intermediated capital is given by

$$q = \overline{\varphi}\Delta K.$$

Combining all of these equations results in

$$(p^d\Lambda(\overline{\varphi},\phi) - p^s)\overline{\varphi}\Delta K + (1-\beta)\left(p^d\overline{\lambda} + a\right)(1-\Delta)K + p^s\overline{\varphi}\Delta K = a(1-\Delta)K.$$

Simplifying, we obtain

$$p^d = \frac{\beta a(1-\Delta)}{\overline{\varphi}\Lambda(\overline{\varphi},\phi)\Delta + (1-\beta)\overline{\lambda}(1-\Delta)}.$$

Note that since $\overline{\varphi}\Lambda(\overline{\varphi},\phi) \leq \overline{\lambda}\ \forall\ \overline{\varphi},\phi$, if

$$\kappa < \frac{\beta a(1-\Delta)}{\Delta\overline{\lambda} + (1-\beta)\overline{\lambda}(1-\Delta)},$$

then in equilibrium, $p^d > \kappa$ is always true and k-producers never purchase intermediated capital from bankers.

Given the depreciation shock ϕ, the intermediation revenue per unit of capital, $d(\overline{\varphi},\phi)$, is the product of the price of intermediated capital sold to c-producers and the average quality of φ-units of capital sold by k-producers:

$$d(\overline{\varphi},\phi) = p^d\Big(\overline{\varphi},\phi\Big)\Lambda\Big(\overline{\varphi},\phi\Big).$$

Thus, we have

$$d(\overline{\varphi},\phi)=\frac{\beta a(1-\Delta)\Lambda(\overline{\varphi},\phi)}{\overline{\varphi}\Lambda(\overline{\varphi},\phi)\Delta+(1-\beta)\overline{\lambda}(1-\Delta)}.$$

The first-order derivative of $d(\overline{\varphi},\phi)$ with respect to ϕ is then given by

$$\begin{aligned}\frac{\partial d(\overline{\varphi},\phi)}{\partial\phi}&=\left(\frac{a\beta(1-\Delta)}{\overline{\varphi}\Lambda(\overline{\varphi},\phi)\Delta+(1-\beta)\overline{\lambda}(1-\Delta)}-\frac{a\beta\overline{\varphi}\Delta(1-\Delta)\Lambda(\overline{\varphi},\phi)}{\left(\overline{\varphi},\Lambda(\overline{\varphi},\phi)\Delta+(1-\beta)\overline{\lambda}(1-\Delta)\right)^2}\right)\frac{\partial\Lambda(\overline{\varphi},\phi)}{\partial\phi}\\&=\frac{\beta a(1-\beta)\overline{\lambda}(1-\Delta)^2}{\left(\overline{\varphi}\Lambda(\overline{\varphi},\phi)\Delta+(1-\beta)\overline{\lambda}(1-\Delta)\right)^2}\frac{\partial\Lambda(\overline{\varphi},\phi)}{\partial\phi}.\end{aligned}$$

Thus, the composition effect always dominates the substitution effect with respect to ϕ.

The first order derivative of $d(\overline{\varphi},\phi)$ with respect to $\overline{\varphi}$ is given by

$$\frac{\partial d(\overline{\varphi},\phi)}{\partial\overline{\varphi}}=\frac{\beta a(1-\beta)\overline{\lambda}(1-\Delta)^2}{\left(\overline{\varphi}\Lambda(\overline{\varphi},\phi)\Delta+(1-\beta)\overline{\lambda}(1-\Delta)\right)^2}\frac{\partial\Lambda(\overline{\varphi},\phi)}{\partial\overline{\varphi}}-\frac{a\beta\Delta(1-\Delta)[\Lambda(\overline{\varphi},\phi)]^2}{\left(\overline{\varphi}\Lambda(\overline{\varphi},\phi)\Delta+(1-\beta)\overline{\lambda}(1-\Delta)\right)^2}$$

This derivative is positive if and only if the following holds:

$$\frac{\partial\Lambda(\overline{\varphi},\phi)}{\partial\overline{\varphi}}>\frac{[\Lambda(\overline{\varphi},\phi)]^2\Delta}{(1-\beta)\overline{\lambda}(1-\Delta)}.$$

Also, we can derive that

$$\begin{aligned}\frac{\partial\Lambda(\overline{\varphi},\phi)}{\partial\overline{\varphi}}&=\frac{\partial}{\partial\overline{\varphi}}\left(\frac{\int_0^{\overline{\varphi}}\lambda(\varphi,\phi)d\varphi}{\overline{\varphi}}\right)\\&=\frac{\lambda(\overline{\varphi},\phi)}{\overline{\varphi}}-\frac{\int_0^{\overline{\varphi}}\lambda(\varphi,\phi)d\varphi}{\overline{\varphi}^2}\\&=\frac{\lambda(\overline{\varphi},\phi)-\Lambda(\varphi,\phi)}{\overline{\varphi}}.\end{aligned}$$

Thus, the previous condition becomes

$$\frac{\lambda(\overline{\varphi},\phi)-\Lambda(\varphi,\phi)}{\overline{\varphi}} > \frac{[\Lambda(\overline{\varphi},\phi)]^2\Delta}{(1-\beta)\overline{\lambda}(1-\Delta)}.$$

F Proof of Proposition 5

The expected intermediation profits are given by

$$\Pi(\overline{\varphi}) = \mathbb{E}_\phi\Big[d\big(\overline{\varphi},\phi\big)\Big] - p^s\big(\overline{\varphi}\big).$$

Profits are increasing in $\overline{\varphi}$ if

$$\frac{\partial \Pi(\overline{\varphi})}{\partial \overline{\varphi}} = \mathbb{E}_\phi\left[\frac{\partial d(\overline{\varphi},\phi)}{\partial \overline{\varphi}}\right] - \frac{\partial p^s(\overline{\varphi})}{\partial \overline{\varphi}} > 0.$$

Using (5), we have that

$$\frac{\partial p^s(\overline{\varphi})}{\partial \overline{\varphi}} = \frac{\mathbb{E}_\phi\left[\frac{\kappa\frac{\partial \lambda(\overline{\varphi},\phi)}{\partial \overline{\varphi}}}{\kappa\int_{\overline{\varphi}}^1 \lambda(\varphi,\phi)d\varphi + p^s\overline{\varphi}}\right] + \mathbb{E}_\phi\left[\left(\frac{-\kappa\lambda(\overline{\varphi},\phi)+p^s}{\kappa\int_{\overline{\varphi}}^1 \lambda(\varphi,\phi)d\varphi + p^s\overline{\varphi}}\right)^2\right]}{\mathbb{E}_\phi\left[\frac{\kappa\int_{\overline{\varphi}}^1 \lambda(\varphi,\phi)d\varphi + \overline{\varphi}\kappa\lambda(\overline{\varphi},\phi)}{\left(\kappa\int_{\overline{\varphi}}^1 \lambda(\varphi,\phi)d\varphi + p^s\overline{\varphi}\right)^2}\right]}.$$

When φ tends to 0, it becomes

$$\lim_{\overline{\varphi}\to 0}\frac{\partial p^s(\overline{\varphi})}{\partial \overline{\varphi}} = \kappa \lim_{\overline{\varphi}\to 0}\mathbb{E}_\phi\left[\frac{\partial \lambda(\overline{\varphi},\phi)}{\partial \overline{\varphi}}\right],$$

since $\lambda(0,\phi)=0$ and $p^s(0)=0$. Furthermore,

$$\lim_{\overline{\varphi}\to 0}\frac{\partial d(\overline{\varphi},\phi)}{\partial \overline{\varphi}} = \frac{\beta a}{(1-\beta)\overline{\lambda}}\lim_{\overline{\varphi}\to 0}\frac{\partial \Lambda(\overline{\varphi},\phi)}{\partial \overline{\varphi}},$$

since

$$\lim_{\overline{\varphi}\to 0} \Lambda(\overline{\varphi}, \phi) = 0.$$

Thus,

$$\lim_{\overline{\varphi}\to 0} \frac{\partial \Pi(\overline{\varphi})}{\partial \overline{\varphi}} = \frac{\beta a}{(1-\beta)\overline{\lambda}} \lim_{\overline{\varphi}\to 0} \mathbb{E}_{\phi}\left[\frac{\lambda(\overline{\varphi}, \phi) - \Lambda(\varphi, \phi)}{\overline{\varphi}}\right] - \kappa \lim_{\overline{\varphi}\to 0} \mathbb{E}_{\phi}\left[\frac{\partial \lambda(\overline{\varphi}, \phi)}{\partial \overline{\varphi}}\right].$$

Using l'Hôpital's rule,

$$\lim_{\overline{\varphi}\to 0} \frac{\partial \Pi(\overline{\varphi})}{\partial \overline{\varphi}} = \frac{\beta a}{(1-\beta)\overline{\lambda}} \frac{1}{2} \lim_{\overline{\varphi}\to 0} \mathbb{E}_{\phi}\left[\frac{\partial \lambda(\overline{\varphi}, \phi)}{\partial \overline{\varphi}}\right] - \kappa \lim_{\overline{\varphi}\to 0} \mathbb{E}_{\phi}\left[\frac{\partial \lambda(\overline{\varphi}, \phi)}{\partial \overline{\varphi}}\right].$$

Thus, if

$$\kappa < \frac{\beta a}{(1-\beta)\overline{\lambda}} \frac{1}{2},$$

then expected intermediation profits are always increasing as the intermediation volume tends to 0.

G Uniqueness of p^s

A sufficient condition for uniqueness of p^s is that

$$\frac{\partial \mathsf{q}\,(\eta, p^s)}{\partial p^s} < 0 \qquad \forall\ p^s \in [0, \overline{p}^s],$$

where

$$\mathsf{q}\,(\eta, p^s) = -\frac{1 + e(\eta) - \Gamma(e(\eta)) - (1-\tau) d(\eta)}{d(\overline{\varphi}(p^s), \underline{\phi}) - p^s - \varepsilon}.$$

Thus,

$$\frac{\partial \mathfrak{q}\left(\eta,p^s\right)}{\partial p^s}=\frac{1+e(\eta)-\Gamma(e(\eta))-(1-\tau)d(\eta)}{d(\overline{\varphi}(p^s),\underline{\phi})-p^s-\varepsilon}\left(\frac{\partial d(\overline{\varphi}(p^s),\underline{\phi})}{\partial p^s}-1\right).$$

Thus, since $\frac{\overline{\varphi}(p^s)}{\delta p^s}>0$, a sufficient condition to guarantee the uniqueness of p^s is to have $\underline{\phi}$ such that

$$\frac{\lambda(\overline{\varphi},\phi)-\Lambda(\overline{\varphi},\phi)}{\overline{\varphi}}<\frac{\left[\Lambda(\overline{\varphi},\phi)\right]^2\Delta}{(1-\beta)\overline{\lambda}(1-\Delta)}\qquad\forall\ p^s\in[0,\overline{p}^s].$$

Indeed, this implies that

$$\frac{\partial d(\overline{\varphi},\underline{\phi})}{\partial\overline{\varphi}}<0.$$

H Proof of Proposition 8

By substituting the definition for $\mathfrak{q}$, we can write the market-learing condition for intermediated capital as

$$\overline{\varphi}(p^s)\Delta=\frac{1+e-\Gamma(e)-(1+\tau)d}{|\pi(\overline{\varphi},\underline{\phi})-\varepsilon|}\eta.$$

Therefore, the state variable that matters for the equilibrium quality threshold $\overline{\varphi}$ and supply price p^s is the quantity of wealth after equity injections and dividends:

$$\overline{\eta}=\Big(1+e-\Gamma(e)-(1+\tau)d\Big)\eta.$$

Thus, given a functional form for the inside value of equity $\theta(\overline{\eta})=\theta^c(\eta,e,d)$, we can rewrite (3) as

$$u^c(\eta)=\max_{e\geq 0,1\geq d\geq 0}\left\{d-e+\theta(\overline{\eta})\Big(1+e-\Gamma(e)-(1+\tau)d\Big)\right\},$$

The first order condition for the injection of equity in the coordinated equilibrium, $e^c(\eta)$, assuming $d^c(\eta) = 0$, is given by:

$$0 = -1 + \theta(\overline{\eta})(1 - \alpha e) + \frac{\partial \theta(\overline{\eta})}{\partial \overline{\eta}} \frac{\partial \overline{\eta}}{\partial e}\Big(1 + e - \Gamma(e)\Big).$$

The first order condition for the injection of equity in the noncoordinated equilibrium, $e(\eta)$, assuming $d(\eta) = 0$, is given by

$$0 = -1 + \theta(\overline{\eta})(1 - \Gamma^e(e)).$$

Therefore, if

$$\frac{\partial \theta(\eta)}{\partial \eta} > 0,$$

then

$$e^c(\eta; \theta) > e(\eta; \theta).$$

Note that we are making a statement conditional on a functional form $\theta(\eta)$. We are not making a statement about the equilibrium value function arising from the numerical solution of the coordinated investment problem.