Homework - Growth Accounting
Solution

1. Write down the equation for the Solow residual that we derived in class. How can you relate this equation to data? In particular, describe the variables that you would need to compute the Solow residual for a given economy.

   Remember that
   $$Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha} \tag{1}$$
   Hence
   $$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{A}(t)}{A(t)} + \alpha\frac{\dot{K}(t)}{K(t)} + (1-\alpha)\frac{\dot{L}(t)}{L(t)} \tag{2}$$
   Note that :

   - $\frac{\dot{Y}(t)}{Y(t)}$ is the percentage change in GDP
   - $\frac{\dot{K}(t)}{K(t)}$is the percentage change in Capital
   - $\frac{\dot{L}(t)}{L(t)}$is the percentage change in Labor

   If we want to compute $\frac{\dot{A}(t)}{A(t)}$, we can use data for the average growth of GDP, capital and labor. Besides that information, we have to calibrate α . We talk about the calibration in question (3)

2. Download the Penn World Tables.[1] Select three countries, which should include a rich country (choose between Canada, France, Germany, Japan, Italy, U.K., and U.S.), a Latin American country (choose between Argentina, Brazil, Chile, Colombia, Mexico, Peru, and Venezuela), and an Asian growth miracle (choosing between China, Hong Kong, Singapore, South Korea, and Taiwan). Penn World Tables provide data on output, capital, labor, and factor shares to do the exercise. Create a separate sheet for each country with the following variables: capital and output in national constant prices (variables *rkna* and *rgdpna*), employment (*emp*) , average hours worked (*avh*) , labor factor share (*labsh*) and a measure of human capital (*hc*)

   In order to provide guidance in how was expected to solve this homework, we provide an example using US, Colombia and Taiwan. We also used the PWT from the link provided.

3. Compute the average factor shares by country over the accounting period in the exercise. Compare your findings across countries. How can the average share of capital be related to the parameters in the model?

   Remember the firm's problem
   $$\max_{L,K} AK^{\alpha}L^{1-\alpha} - rK - wL$$
   Here w represents wage per hour, r is the interest rate on capital. The FOC with respect to capital is
   $$\alpha\frac{AK^{\alpha}L^{1-\alpha}}{K} - r = 0$$
   Hence
   $$\frac{rK}{Y} = \alpha \tag{3}$$

[1] You can download the database from this link http://www.rug.nl/ggdc/productivity/pwt/

The FOC with respect to labor is

$$(1-\alpha)\frac{AK^{\alpha}L^{1-\alpha}}{L} - w = 0$$

Hence

$$\frac{wL}{Y} = (1-\alpha)$$

Usually, national accounts provide information of total labor payments in the economy and payments to capital. Hence, we can approximate to $1-\alpha$ by taking the average of the ratio $\frac{wL}{Y} = \frac{\text{Total payroll}}{\text{GDP}}$.

4. Compute labor input as the product of employment (emp) and average hours worked (avh). You are required to use this measure to perform the accounting exercise in question 6.

   Remark: We used this measure because the model incorporates hours worked in the economy. Here we have total employees and an average of hours worked. To compute total hours, we calculate the product between these variables.

5. Compute the labor, capital and GDP average growth rates for each country.

   You were required to compute annual growth rates, and then take the average.

6. Using your previous calculations, compute a decomposition of growth for each economy using the Solow's decomposition that you described in question 1. You are required to write down a similar table to the example presented in table Explain your main results. Compare your findings across countries.

   Total points are given if you computed the table as the example and if you also discussed your results. Here there is an example:

   Table 1 presents our results. First, we see that the average GDP growth in Taiwan is 7%, which is by far higher than the growth observed in US and Colombia in the period in consideration. This happens because Taiwan experienced a huge increase in capital after the 60's, and then this country began to catch up. In fact, the capital contribution for the case of Taiwan is very high: I accounts for almost a half the GDP growth. Colombia also experienced a higher GDP growth than US. If we believe in the convergence hypothesis, we can argue that Colombia is also catching up. In the case of Colombia, the higher contribution comes from labor, which is in line with the demographic trends in this country. During this period, working age population increased more than the historical average. In the case of US, the TFP leads the contribution in GDP growth.

Table 1: Growth accounting exercise

	GDP growth	Solow residual (TFP growth)	Contribution from capital	Contribution from labor
Colombia	0.043	0.013	0.013	0.017
$\alpha = 0.31$	% Contribution $\rightarrow$	30%	30%	40%
US	0.032	0.013	0.011	0.007
$\alpha = 0.37$	% Contribution $\rightarrow$	41%	36%	24%
Taiwan	0.074	0.028	0.036	0.010
$\alpha = 0.47$	% Contribution $\rightarrow$	37%	49%	14%

7. The deficiency of the previous labor input measure is that it ignores the changes in worker quality over time. PWT provide a measure of human capital (hc) that can be used to adjust for that. Labor input now can be measured as a product of employment , average hours worked, and human capital. Use the new measure to perform the accounting exercise. How is the result different from the previous one? Do you think it is important to account for quality changes?

   Total points are given if you computed the table as the example and if you also discussed your results. In general, human capital increases in time, so the contribution in labor in our new calculations is higher. In the previous exercise, TFP was picked up because we didn't take into account that there is human capital accumulation over the period. In conclusion, TFP should be lower here. Here there is an example: Table 2 presents our results incorporating human capital in our calculations of labor input. Since human capital is increasing for the countries in our analysis, our new results for TFP are lower than the previous ones. In general, labor contribution increases more than 10 percentage points. This increase reflects quality improvements in

human capital. If we don't control for those quality changes, we are overestimating the technological progress. Consider the case of Colombia: our previous results implied that TFP was responsible of 30% of the GDP growth. This measure overestimates technological progress of this country, which has not experienced huge increases in R&D or Tech investment. However, Colombia has been improving quality of education (specially for those in the right in the distribution of income). As expected, contribution of TFP using the new measure is just 15%.

Table 2: Growth accounting exercise

	GDP growth	Solow residual (TFP growth)	Contribution from capital	Contribution from labor
Colombia	0.043	0.007	0.013	0.023
$\alpha = 0.31$	% Contribution $\rightarrow$	16%	30%	54%
US	0.032	0.009	0.011	0.011
$\alpha = 0.37$	% Contribution $\rightarrow$	29%	36%	34%
Taiwan	0.074	0.021	0.036	0.017
$\alpha = 0.47$	% Contribution $\rightarrow$	28%	49%	23%

8. Now look at different periods for each country. Compare between countries and over time, comment on your findings and try to link them to what the countries actually experienced.

   In this question you are required to perform a comparative analysis between countries and periods, taking into account the economical history of the countries you selected. Here there is an example: We selected two periods : 1980-1999 and 2000-2014. We see that the average GDP growth for Colombia is 4.4% in the 2000's, which is higher than the exhibit between 1980 and 1999. During the 80's and 90's, Colombia experienced a lot of GDP volatility for two reasons: the first one is related to financial crisis in emerging economies; while the second one is related to internal conflict and drug traffic. As we can see in table 3, TFP contribution was very small in this period, which is the consequence of a painful drop in GDP growth rates. On the other hand, Taiwan and US experienced a period of economic prosperity. In the case of US, it was leaded by TFP. In this period there were a lot of technological advances such as the Internet (launched in 1989). In the case of Taiwan, capital contribution is still the key factor for economic development in this decade.
   In the 2000's, Colombia experienced a sustained GDP growth. First, internal conflict reduced in the first part of the decade. Then, there was an increase in investment in Colombia, specially in the Oil sector. We see that the share of capital in this decade increased to 34% from 30%. In fact, oil prices in this decade were very high (it reached 120). It is worth mentioning that oil accounts for 30% of the Colombian exports. So, there was a boom until prices drop at the end of 2013. However, labor contribution still dominates in GDP growth, and TFP rises shyly (16%). The case of US and Taiwan differs from Colombia because GDP growth slows down. As we know, US experienced a dramatic crisis in 2008. This crisis impacted severely unemployment. As the table shows, labor contribution was 15% in this period. For Taiwan, we see that capital investment was not sufficient to sustain GDP growth. So GDP slows down, as well as the contribution of capital.

Table 3: Growth accounting exercise

	GDP growth	Solow residual (TFP growth)	Contribution from capital	Contribution from labor
1980-1999				
Colombia	0.031	0.001	0.012	0.016
$\alpha = 0.30$	% Contribution $\rightarrow$	4%	38%	59%
US	0.040	0.0183	0.011	0.010
$\alpha = 0.38$	% Contribution $\rightarrow$	38%	31%	31%
Taiwan	0.092	0.022	0.043	0.009
$\alpha = 0.46$	% Contribution $\rightarrow$	31%	57%	12%
2000-2014				
Colombia	0.044	0.007	0.014	0.023
$\alpha = 0.34$	% Contribution $\rightarrow$	16%	31%	52%
US	0.017	0.007	0.008	0.003
$\alpha = 0.39$	% Contribution $\rightarrow$	39%	46%	15%
Taiwan	0.043	0.016	0.017	0.010
$\alpha = 0.52$	% Contribution $\rightarrow$	36%	39%	25%