Lecture 2: Growth Accounting

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"Everything reminds Milton Friedman of the money supply. Well, everything reminds me of sex, but I keep it out of the paper."
—Robert Solow—

1 The Decomposition of Growth

From Levels to Rates of Change. In the previous lecture, we briefly discussed that one of the virtues of the Solow model is that we can use it to determine the sources of growth. We also studied the continues-time version of the model. We argued that we could write the model as if all variables were functions of time. Recall our production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where now we have included t-subscripts. If we had in our hands, a time series Y_t, K_t and L_t we could certainly determine what the change in productivity has been provided that we have the value of α. Taking derivatives with respect to time of equation (1):

$$\begin{aligned}
\dot{Y}_t &= \dot{A}_t K_t^{\alpha} L_t^{1-\alpha} + \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} \dot{K}_t + (1-\alpha) K_t^{\alpha} L_t^{-\alpha} \dot{L}_t \\
&= \dot{A}_t K_t^{\alpha} L_t^{1-\alpha} + \alpha A_t K_t^{\alpha} L_t^{1-\alpha} \frac{\dot{K}_t}{K_t} + (1-\alpha) K_t^{\alpha} L_t^{1-\alpha} \frac{\dot{L}_t}{L_t} \\
&= \dot{A}_t \frac{A_t}{A_t} K_t^{\alpha} L_t^{1-\alpha} + \alpha A_t K_t^{\alpha} L_t^{1-\alpha} \frac{\dot{K}_t}{K_t} + (1-\alpha) A_t K_t^{\alpha} L_t^{1-\alpha} \frac{\dot{L}_t}{L_t} \\
&= Y_t \frac{\dot{A}_t}{A_t} + \alpha Y_t \frac{\dot{K}_t}{K_t} + (1-\alpha) Y_t \frac{\dot{L}_t}{L_t},
\end{aligned}$$

where I have used the standard notation of $\dot{Y}_t$ to refer to the derivative: $\partial Y_t / \partial t$. We can divide both sides by Y_t and express this equation as:

$$\frac{\dot{Y}_t}{Y_t} = \alpha \frac{\dot{K}}{K} + (1-\alpha) \frac{\dot{L}}{L} + \frac{\dot{A}_t}{A}$$

Note that variables expressed as $\dot{x}/x$ are approximately the per cent growth of that variable. Technology improvements are not observed, but we do have data on $\frac{\dot{Y}}{Y}$, $\frac{\dot{K}}{K}$ and $\frac{\dot{L}}{L}$ such that we can actually compute $\frac{\dot{A}_t}{A}$ in the data. The main question now is to determine what α is.

Determination of the Capital Share. So far we talked only about how output is produced. We didn't mention anything about the determination of wages or the return on capital. Suppose now that there is a representative competitive firm that solves:

$$\max_{K,L} Y - r_t K - w_t L \tag{2}$$

where r is the cost of capital, and w is the cost of labor. The firm rents capital at rate r. The solution to the firm's problem gives the first-order conditions (FOC):

$$\frac{\partial Y}{\partial K} = r \tag{3}$$

$$\frac{\partial Y}{\partial L} = w \tag{4}$$

These conditions say that the marginal product of labor (capital) equals the cost of using labor (capital). Since people own capital and labor, their total income is:

$$\begin{aligned} Y_t &= \frac{\partial Y}{\partial K} K_t + \frac{\partial Y}{\partial L} L_t \\ &= r_t K_t + w_t L_t. \end{aligned}$$

Using 3 and 4 gives the **factor income shares**:

$$\frac{w_t L_t}{Y_t} = \frac{\partial Y_t}{\partial L_t} \frac{L_t}{Y_t} \tag{5}$$

$$\frac{r_t K_t}{Y_t} = \frac{\partial Y_t}{\partial K_t} \frac{K_t}{Y_t}. \tag{6}$$

where $\frac{wL}{Y}$ is the income share of labor and $\frac{rK}{Y}$ is the income share of capital.

Question. Show that with the Cobb-Douglas technology, we obtain:

$$\begin{aligned} \frac{\partial Y_t}{\partial K_t} \frac{K_t}{Y_t} &= \alpha = r_t \frac{Y_t}{K_t} \text{ and} \\ \frac{\partial Y_t}{\partial L_t} \frac{L_t}{Y_t} &= 1 - \alpha = \frac{w_t L_t}{Y_t}. \end{aligned}$$

This assumes that, overall, the economy behaves competitively. However, even if there was some monopoly power in the economy, the results we derive are still valid. The important part is that if we have data on profits plus rents we can obtain r_t and if we have data on wages w_t we can obtain α. Most countries in the world report official statistics on GDP and it's distribution between wages and rental rates —this data is typically constructed out of individual tax filings. Obviously, α won't be a constant, but typically it's a stable number, at least in the long run.

Determination of Capital Stock. We usually lack reliable data on K_t because it is an imputed time series: we need an initial stock of capital at some point. Regardless of this downside, good approximations of today's capital may be obtained from data on investment and using the fact that depreciation would do the job of making any initial capital stock negligible in determining today's capital. Thus, official statistics use versions of the capital accumulation equation:

$$K_{t+1} - \delta K_t = \underbrace{sA_t K_t^{\alpha} L_t^{1-\alpha}}_{I_t} - \delta K_t.$$

using information on investment, I_t, also computed from tax filings. A classic problem is that there are multiple forms of capital. Thus, typically, we construct capital from using a investment-good index. Similarly, workers are paid very differently, and a series for hours is obtained from a weighted average of wages.

Solow's Decomposition. In his classic 1957 paper, Solow came up with this decomposition. The term $\frac{\dot{A}_t}{A}$ is the so called Solow residual, and is obtained as:

$$\frac{\dot{A}_t}{A} = \frac{\dot{Y}}{Y_t} - \left(\alpha \frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}\right)$$

or rather simply in words:

$$\text{Solow Residual} = \Delta\% \ \cdot \text{GDP} \ - \alpha\Delta\% \ \cdot \text{Capital Growth-} (1-\alpha)\,\Delta\% \ \cdot \ \text{Labor Growth.}$$

When analysing this decomposition for the U.S. economy, Solow found a surprising result: 70% of the increase in output in the U.S. output could be attributed to an increase in productivity. This result is remarkable because it suggested that the U.S. economy was in a very healthy shape. Interestingly, 1957 was a time in which the Soviet Union seemed to rapidly catch up with the U.S. The atomic bomb was already in their hands and they were about to Launch the Sputnik. Many economists and politicians in the U.S. were confident that this was a real trend. Nikita Krushev, the famous soviet leader was even more confident. He would remark "We will soon crush you". Turning to the facts Paul Krugman claimed that most of the Soviet increase in output was due to a forced industrialization. Big capital increases were due to increments in the savings rate and mass forced exodus from rural areas to industrial clusters. The difference between the U.S.'s substantial growth and the U.S.S.R.'s impressive growth were the sources. Without a substantial increase in productivity, the soviets would inevitably face the law of diminishing returns to scale, and apparently so they did.

2 Some Discussions

What can we say about the Solow residual today. The next subsections present diverse evidence from different regions and different areas around the world. Some discussion is worthwhile.

2.1 U.S. vs. Europe

Table 1: Growth Accounting: U.S. and Europe

Country	Growth Rate of GDP	Contribution from Capital	Contribution from Labor	TFP Growth Rate
		Panel A: 1950-1973		
Canada	0.0479	0.0171	0.0271	0.0126
($\alpha = 0.33$)		(37%)	(38%)	(26%)
France	0.0515	0.0172	0.0034	0.0308
($\alpha = 0.31$)		(33%)	(6.6%)	(60%)
Germany	0.0584	0.0197	0.0017	0.0370
($\alpha = 0.31$)		(34%)	(2.9%)	(63%)
Italy	0.0560	0.0216	0.0039	0.0305
($\alpha = 0.36$)		(39%)	(6.9%)	(55%)
Japan	0.0859	0.0386	0.0152	0.0321
($\alpha = 0.40$)		(45%)	(18%)	(37%)
U.K.	0.0284	0.0111	0.0048	0.0125
($\alpha = 0.40$)		(39%)	(17%)	(44%)
U.S.	0.0392	0.0135	0.0139	0.0117
($\alpha = 0.34$)		(34%)	(36%)	(30%)
		Panel B: 1973-2011		
Canada	0.0287	0.0164	0.0120	-0.0018
($\alpha = 0.39$)		(62%)	(45%)	(-6.7%)
France	0.0199	0.0088	0.0062	0.0049
($\alpha = 0.34$)		(44%)	(31%)	(25%)
Germany	0.0188	0.0081	0.0066	0.0041
($\alpha = 0.33$)		(43%)	(35%)	(22%)
Italy	0.0181	0.0118	0.0063	0.0001
($\alpha = 0.41$)		(65%)	(35%)	(0.4%)
Japan	0.0228	0.0177	0.0012	0.0038
($\alpha = 0.44$)		(78%)	(5.4%)	(16.8%)
U.K.	0.0202	0.0086	0.0018	0.0097
($\alpha = 0.36$)		(43%)	(9.0%)	(48%)
U.S.	0.0263	0.0090	0.0092	0.0081
($\alpha = 0.35$)		(34%)	(35%)	(31%)

Data Source: Penn World Tables 8.1.

Table 1 presents the growth accounting exercise for several rich countries using the data from Penn World Tables. The table is divided into two panels which correspond to the periods 1950-1973 and 1973-2011 respectively. Labor input is adjusted for differences in human capital. Looking at the upper panel, all countries grow fast, with the lowest annual growth rate of GDP equal to 2.84% (U.K.). Both capital and labor contribute to GDP growth with labor has a much smaller role for most countries. The greatest contribution comes from TFP growth. Interestingly, except for U.K., all other countries achieved faster growth than U.S., which largely comes from differences in TFP growth. The lower panel shows a different picture. GDP growth is much lower in all countries. This fits the description of a *productivity slowdown* since the early 1970s, compared to the previous period. Even though both capital and labor grow slower, the largest drop comes from TFP growth. Canada even had a negative TFP growth over the period. Naturally, the contribution of TFP to GDP growth drops. Comparing U.S. to the other countries also shows that the faster growers in the previous period also experienced larger drop in the latter period.

2.2 Is there a Asian Growth Miracle?

Table 2: Growth Accounting: East Asian Miracles

Country	Growth Rate of GDP	Contribution from Capital	Contribution from Labor	TFP Growth Rate
		Panel A: 1960-1990		
Hong Kong	0.0776	0.0372	0.0284	0.0119
($\alpha = 0.51$)		(48%)	(37%)	(15%)
Singapore	0.0828	0.0492	0.0187	0.0148
($\alpha = 0.58$)		(59%)	(23%)	(18%)
South Korea	0.0885	0.0385	0.0324	0.0176
($\alpha = 0.38$)		(43%)	(37%)	(20%)
Taiwan	0.0853	0.0473	0.0204	0.0176
($\alpha = 0.45$)		(55%)	(24%)	(21%)
		Panel B: 1990-2011		
Hong Kong	0.0395	0.0237	0.0085	0.0073
($\alpha = 0.49$)		(60%)	(22%)	(18%)
Singapore	0.0612	0.0358	0.0195	0.0059
($\alpha = 0.55$)		(59%)	(32%)	(9.7%)
South Korea	0.0512	0.0748	0.0124	0.0109
($\alpha = 0.45$)		(65%)	(13%)	(21%)
Taiwan	0.0487	0.0313	0.0082	0.0093
($\alpha = 0.49$)		(64%)	(17%)	(19%)
		Panel C: China 1978-2011		
China	0.0943	0.0445	0.0145	0.0354
($\alpha = 0.49$)		(47%)	(15%)	(36%)

a. Data Source: Penn World Tables 8.1. b. For South Korea, the first period is 1963 1990.

Several East Asian countries (economies) started grow dramatically during the 1960s for around 30 years. Annual rates of output growth in Hong Kong, Singapore, South Korea, Taiwan were all around 8 percent, well above what the OECD countries had experienced in the fast-growth period.[1] These countries are thus dubbed the ”East Asian Miracles” for their miraculous growth experience. What drives these growth miracles? Panel A of Table 2 shows that the growth miracles are mainly about input growth, especially capital growth. Capital accumulation contributes to around half of the output growth.[2] The contribution of TFP growth is much smaller compared to rich OECD countries while the growth rate of TFP is not that exceptional at all.

The results in Table 2 confirms Alwyn Young's original findings. In a series of papers, Young (1992,1994,1995) performed careful growth accounting for these countries, finding it was capital deepening driving the East Asian growth miracles. This led Paul Krugman (1994) to conclude that the growth in these countries cannot be continued, largely based on the logic of the Solow model. Was Krugman prediction right? Panel B of Table 3 shows that output growth in these countries all slowed down during 1990-2011. Nevertheless, the growth rates are still high under normal standard, with Singapore achieved the highest rate of 6.12 percent among the four. What's interesting is that capital accumulation still plays a even big role during this period. The impact of growth in labor input was lowered. TFP growth also slowed down, making a similar contribution as in the previous period. Taking together, it seems Krugman was right that growth in these economies would slowed down. However, their experience doesn't really fit the Solow model as growth was still driven by capital accumulation.

As a latecomer, China started its reform in the late 1970s and early 1980s and its economy similarly took off since 1978 as her East Asian neighbours. Panel C of Table 2 looks at China's experience during 1978-2011. Output growth is even faster than the neighbours and capital accumulation is still the important source of growth. What's different is that China does show a above normal TFP growth, contributing to 36% of total output growth. These numbers could be off as people suspect economic growth can be exaggerated.[3] Alwyn Young (2003) shows that output growth in China is inflated by around 2 percent, which directly translates into a 2 percent reduction in TFP growth. With this revision, the Solow model again leads to the prediction that China's economic growth cannot be sustained. How the history unfolds however might not be as simple as the Solow model predicts.

2.3 The “lost” Latin America decade

Many Latin American countries experienced a debt crisis during the 1980s as they were unable to service their foreign debt. The period is often referred to as the ”lost decade” as economic growth was stopped or became negative. What does a growth accountant have to say about the ”lost decade” in Latin America? Table 3 presents the accounting exercise for seven Latin American

[1] The only exception is Japan, which grew at 8.59 percent annually and is also in East Asia.

[2] These countries also had higher capital shares in general.

[3] This could be due to the fact that Chinese government promotes officials according to economic performance of their administrated area, which provides incentives for them to fake data.

Table 3: Growth Accounting: Latin America 1980 1990

Country	Growth Rate of GDP	Contribution from Capital	Contribution from Labor	TFP Growth Rate
Argentina	-0.0112	0.0072	0.0112	-0.0295
($\alpha = 0.49$)		(64%)	(100%)	(-264%)
Brazil	0.0156	0.0183	0.0276	-0.0302
($\alpha = 0.45$)		(117%)	(176%)	(-193%)
Chile	0.0288	0.0147	0.0232	-0.0091
($\alpha = 0.56$)		(51%)	(81%)	(-32%)
Colombia	0.0335	0.0197	0.0130	0.0009
($\alpha = 0.49$)		(59%)	(39%)	(2.6%)
Mexico	0.0179	0.0181	0.0205	-0.0207
($\alpha = 0.57$)		(101%)	(115%)	(-116%)
Peru	-0.0117	0.0132	0.0258	-0.0507
($\alpha = 0.48$)		(113%)	(220%)	(-433%)
Venezuela	0.0062	0.0121	0.0073	-0.0132
($\alpha = 0.59$)		(195%)	(118%)	(-213%)

a. Data Source: Penn World Tables 8.1. b. For Argentina and Peru, the percentage contribution of all three sources add up to -100% to reflect negative output growth these two countries experienced.

countries. A first thing to notice is that output growth is really low compared to the other country experiences we've seen in this section. For Argentina and Mexico, it is even negative. The picture for growth of output per capita is even dimmer where we take population growth into account. Growth in capital and labor inputs was not great, but should not be held accountable for the sluggish output growth. For most countries, growth in capital and labor inputs were well above that output growth. On the other hand, output growth suffered a lot from negative TFP growth in all countries but Colombia, which achieved a trivial annual TFP growth of 0.09%. The negative TFP growth is not to say these countries experienced technical regress during this period. It just shows how large a drop in economic efficiency these countries have had. It is important to bear in mind that the TFP measure we calculated is a residual, consisting all the effects not captured by measures of input growth.

3 Convergence Implications

The main implication of Solow's model is the convergence among regions. Here are some other examples. We can test this hypothesis visually, by comparing the growth rates for countries were poorer in say, 1960, with countries that were richer. The neoclassical growth model predicts that countries that begin poorer will grow faster and catch up with richer ones. Figure 1 does precisely that. We can't find any patterns visually nor in regressions. Does this mean that the theory is wrong?

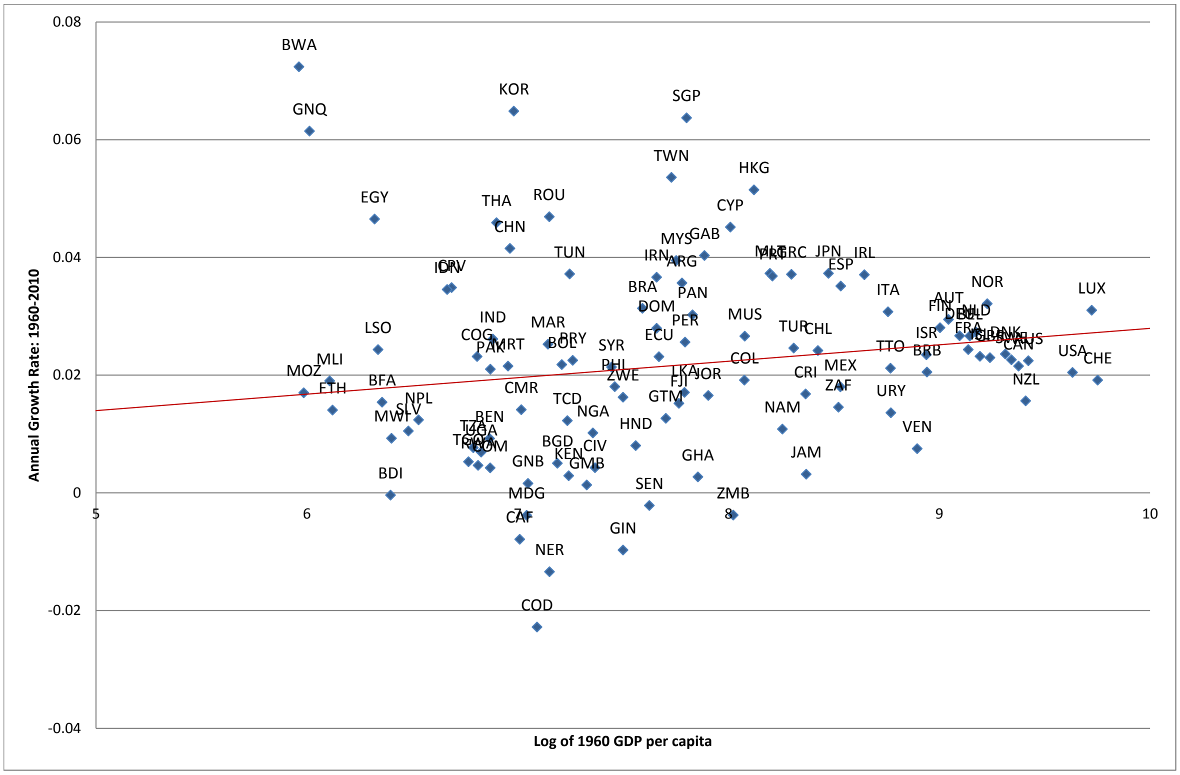

Figure 1: No convergence for all countries. Source: PWT 8.0.

We look now at a time series for the OECD countries and the rest of the economies. The motivation is to show that the dynamics that the Solow model predicts are pretty good for the OECD economies. Countries with higher GDP levels tend to grow at slower rates than countries with smaller GDP levels.

4 Some Examples from GapMinder.Org

As an exercise, please log on to GapMinder.Org and build your own examples on convergence. Study the comparison between the country of your choice and the countries you would like to study..

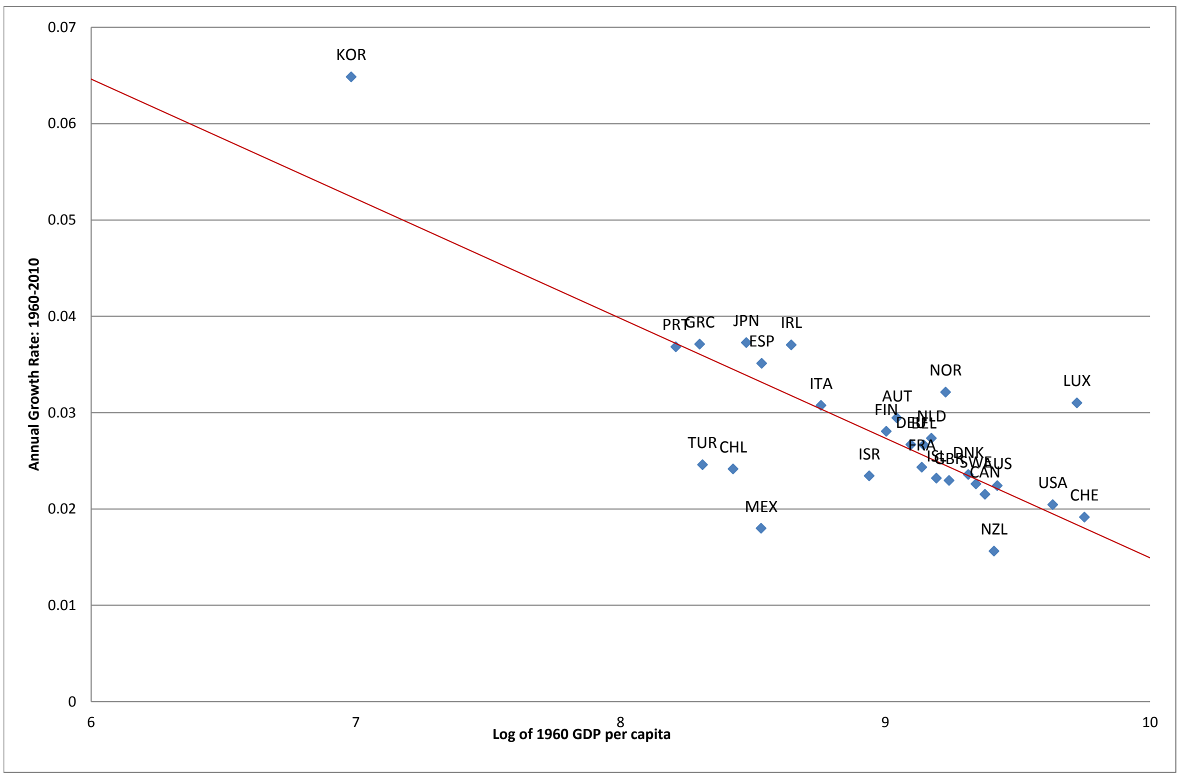

Figure 2: Convergence among OECD countries. Source: PWT 8.0.