Discussion of "Business Cycle Asymmetry and Input-Output Structure: The Role of Firm-to-Firm Networks" by Miranda-Pinto, Silva and Young

Saki Bigio (UCLA)

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Overview. The paper Miranda-Pinto et al. (forthcoming), henceforth MSY, explores the relationship between network density and skewness in economic growth rates, at both the cross-country and firm levels. The paper argues that countries and firms with denser supplier networks tend to exhibit greater skewness in the distribution of their output growth. Network density, is measured through measured through the count of suppliers in the case of firms or, the average count, in the case of countries.

The paper's cross-country analysis finds that countries with denser networks exhibit more skewed output, suggesting a positive correlation between network density and economic growth. Likewise, using Chilean firm-network data, the authors find that a similar pattern is observed at the firm level, where firms with denser supplier networks feature a greater amount of skewness. One of the key figures in the paper shows that firms with denser networks performed worse during the Covid-=\19 crisis in Chile.

To rationalize this pattern, the paper builds on the theoretical foundation laid by Baqaee and Farhi's Second-Order paper, Baqaee and Farhi (2019), henceforth BF, focusing on i.i.d. Total Factor Productivity (TFP) shocks. The model in the paper is used to argue that there is more skew in the output distribution in denser networks, indicating a correlation that verifies the facts. A critical factor in this relationship is the presence of elasticities of input substitution with values less than 1.

In this discussion paper, I want to reflect on a number of issues that this paper made me think about. First, I want to argue that understanding growth skewness is important. Second, I argue that that from BF's formula, it is not evident that denser networks produce more skewness. However, I want to elucidate, by means two examples, why low elasticities of substitution lead to the predictions that a greater number of interconnections produces growth skewness. I then argue that the evidence presented in the paper is not necessarily a validation of the theory as it may also speak about the nature of shocks.

Why is the question important? If we look into firm-level data on earnings and employment growth, as for example in Bigio et al. (2023), a striking pattern emerges: earnings and employment growth data often display a skewed distribution. The same is verified about labor earnings. The data is characterized by frequent stable growth periods and infrequent slumps with partial recoveries. They are also typically followed by partial reversions to the mean. Similar patterns are observed in business cycles, where drops in growth infrequent, but large relative to expansions. Work by Dew-Becker and Vedolin (2021) discuss this subject in substantial detail.

The novelty in the MSY paper is a derivative. The authors argue that fluctuations become more asymmetric as economies become increasingly interconnected. This is not a result I would have expected. The result is surprising because I would have predicted that emerging economies, being more volatile and with less denser networks than richer economies, would show greater skewness. It turns out that skewness and volatility don't necessarily go hand to hand.

If the predictions of the paper is right, it has important implications. If we think that growth is driven the accumulation of more complex technologies that require greater mixes of inputs, the theory predicts that as economies grow, negative tail events will become more likely. In other words, the left skewness of growth will become increasingly important.

The observed patterns raise several important questions regarding the future:

a. Should we expect more asymmetric business cycles in the future, given the increasing complexity and interconnectedness of economies?

b. Will there be greater risk premia in sectors that lead growth, as they may be more susceptible to fluctuations and downturns?

If the answer is yes, then this prediction should be considered in the design of future social insurance policies.

Understanding the role of network density. MSY builds on BF's second-order formula which is surveyed in Baqaee and Rubbo (forthcoming). Take equation 21 in Baqaee and Rubbo (forthcoming). They present the following formula for the first- and second-order terms of the impact of sector $i's$ shock on TFP:

$$\Delta \ln GDP \approx \sum_i \lambda_i \Delta \ln A_i - \overbrace{\sum_j \lambda_j \times (1-\theta_j)\, \frac{1}{2} VAR_{\Omega_{(j,:)}} \left[\sum_i \Psi_{(:,i)} \Delta \ln A_i \right]}^{\text{expenditure swithching effect}} . \tag{1}$$

Here, A_i it's total factor productivity, the fundamental shocks. Then, λ_i is the ratio of firm $i's$ sales relative to total output, also known as the Domar weight. In turn, Ω is the cost-basis input-output matrix and Ψ its corresponding Leontief inverse: $\Psi \equiv (1-\Omega)^{-1}$. The term θ_j is the elasticity input substitution of sector j. The first term in the expression is a linear term, associated with Hulten's Theorem. Clearly, if shocks are symmetric, then skewness cannot be produced from this term because symmetry is preserved by linear transformations.

Any skewness must follow from the second-order term, the expenditure switching effect, or from higher order terms not present in the approximation. As explained by BR, the term $\sum_i \Psi_{(:,i)} \Delta \ln A_i$ corresponds to the vector of price changes of all goods given a TFP shock to firm i.[1] Thus, $VAR_{\Omega_{(j,:)}} \left[\sum_i \Psi_{(:,i)} \Delta \ln A_i\right]$ measures the response of the variance of input prices of firm j, where the variance operator is weighted by the input shares: the operator $VAR_{\Omega_{(j,:)}}$ measures the variance of the responses to the shock under the probability measure corresponding to the entries of the j-th row of the input-output matrix. The expenditure switching effect captures the change in production costs caused by the price dispersion.

The sign of the second-order terms in equation (21) actually depends on whether the microeconomic elasticities of substitution θ_j is above or below one. If $\theta_j > 1$, dispersion is actually beneficial because goods are easy to substitute, thereby dampening the effect. We can see this effect clearly because as $\theta_j > 1$, the term becomes positive. To the linear term associated with Hulten's Theorem, we add the quadratic term $\Delta \ln A_i$ that enters in the variance operator, making the map a convex function. If, by contrast, $\theta_j < 1$, the map becomes concave. In terms of skewness, an increasing concave map will generate a left skew on the left distribution.

MSY stresses the role of low elasticities of substitution θ_j, but also emphasizes density. That is the novelty of the paper, say relative DB. However, it is not immediately obvious from the formula above why density should contribute to greater skewness. Holding fixed the Domar weights, setting all $\theta_j < 1$, we should ask why density increases the coefficients associated with the operator $VAR_{\Omega_{(j,:)}} \left[\sum_i \Psi_{(:,i)} \Delta \ln A_i\right]$. This is not obvious: Ψ is an inverse Leontief matrix whose entries enter at a square rate (and some may be less than one, while others not) while the terms corresponding to the entries of Ω enter linearly. The paper provides numerical demonstrations. It is not clear that density as measured in the paper should always contribute to greater skewness, even conditional on θ_j. Intuitively, there is a race between diversification and substituability associated with prices, not with the substitution within inputs. I suspect that it is possible to construct counterexamples but these would be rather exceptional.

Instead of providing a general result, I can try to explain an example that compares two Leontief production functions. The example should illustrate how a denser network can result in greater skewness in economic growth rates. Both production functions feature two sectors, 1 and 2. A planner allocates labor across sectors with total labor being a fixed factor: $L_1 + L_2 = \bar{L}$. Corresponding TFP's are A_1 and A_2. The consumer only consumes the good of sector 2. I use the planner's allocation throughout.

Proposition 1. *Let the production functions of Economy I be given by:*

$$Y_1 = A_1 L_1 \text{ and } Y_2 = A_2 \min\{Y_1, L_2\},$$

[1] Why is this given by the Leontief inverse? This results from an application of the Envelope theorem on the planner's resources constraint.

and the resource constraint given by $C = Y_2$. Then, output is:

$$C = A_2 \frac{A_1}{1 + A_1}.$$

Proof. Aggregate production is:

$$C = A_2 \min \{A_1 L_1, L_2\}$$

which implies that $A_1 L_1 = L_2$. Therefore,

$$\frac{1 + A_1}{A_1} L_2 = \bar{L} \to C = \frac{A_2 A_1}{1 + A_1}.$$

□

Next, I consider an economy with one additional link.

Proposition 2. *Let the production functions of Economy II be given by:*

$$Y_1 = A_1 \min \{L_1, X_{12}\} \text{ and } Y_2 = A_2 \min \{Y_1, L_2\},$$

and the resource constraint given by $C + X_{12} = Y_2$. Then, output is:

$$C = \frac{A_2 A_1 - 1}{1 + A_1} L.$$

Proof. Optimality requires to equate input uses in both sectors: $L_1 = X_{12}$ and $L_2 = A_1 X_{12}$. Thus, we have that the labor market can be solved in terms of the intermediate input: $X_{12}(1 + A_1) = L$. Therefore, we have that:

$$Y_2 = A_2 \frac{A_1}{A_1 + 1} L$$

and thus:

$$C = A_2 \frac{A_1 - A_2^{-1}}{A_1 + 1} L.$$

□

Then, from both propositions, we have the following Corollary.

Corollary 1. *Let $\ln A_1$ and $\ln A_2$ be i.i.d. and symmetrically distributed. Considering only the second-order terms, the second economy will feature greater skewness in the distribution of GDP (in levels).*

Proof. The first and second derivatives of $\theta = \frac{a+b}{a+1}$ are:

$$\frac{\partial \theta}{\partial a} = \theta \left[\frac{1}{a+b} - \frac{1}{a+1} \right].$$

$$\frac{\partial^2 \theta}{\partial a^2} = \theta \left[\frac{b-1}{(a+b)(a+1)^2} \right]$$

Thus, in Economy I, the second-order expansion of changes in GDP is:

$$\Delta \log C = \left[\frac{1}{A_1} - \frac{1}{1+A_1} \right] \Delta \log A_1 - \left[\frac{1}{A_1 (1+A_1)^2} \right] (\Delta \log A_1)^2 + \mathcal{O}\left((\Delta \log A_1)^3 \right).$$

In the second economy, the second-order expansion for changes in GDP is:

$$\Delta \log C = \left[\frac{1}{A_1 - A_2^{-1}} - \frac{1}{1+A_1} \right] \Delta \log A_1 - \left[\frac{1 + A_2^{-1}}{\left(A_1 - A_2^{-1}\right)(1+A_1)^2} \right] (\Delta \log A_1)^2 + \mathcal{O}\left((\Delta \log A_1)^3 \right).$$

The coefficient in the quadratic term is larger in the second economy, which make the function even more concave in $\Delta \log A_1$ around the inflection point of zero. This provokes greater downwards skewness. □

The key to understanding why interconnectedness provokes more skewness in this example is to understand the economics of why the coefficients that map TFP to output are more sensitive in the second economy. That is, what is the economic intuition behind the result that the quadratic coefficient is higher in the second economy:

$$\frac{1}{A_1} < \frac{1 + A_2^{-1}}{A_1 - A_2^{-1}}.$$

The intuition is a roundabout logic: In both economies, an decrease in TFP in Sector 1 decreases its output. Then sector 2 must release labor resources toward Sector 1, because sector 2 has to process less resources. Hence, there's a dampening effect of the TFP loss because labor is reallocated to the sector that experiences the TFP loss. In Economy II, which has one intermediate input link, there is a second round effect. Because Sector 1 less productive, Sector 2 releases labor, but it must also produce more intermediate inputs so that labor can be used in sector I. Thus, labor reallocation is accompanied by more waste in intermediate inputs. This is the sense in which denser networks coupled with complementarities provoke higher non-linear effects. The lesson is that economies with more interconnections require will require more input and labor towards the sectors that have the negative TFP shocks. These effects are only captured by the non-linear terms. Density in a network with high complementarity will provoke greater waste in resources to maintain production in the sector.

But What is TFP and what is Density? Next, I want to show another sense in which density adds to asymmetric cycles. In this case, I will not increase the intermediate inputs, but I will add more sectors. We can think of aggregate production or a specific sector. While the paper's case study is focused on the Covid-19 pandemic, interpreting the crisis in terms of productivity seems inadequate. It is more appropriate to consider changes in labor availability L_i rather than focusing on productivity itself. At high frequencies, labor is in any case, a fixed factor. Without labor reallocation or input reallocation. Yet, the effects on skewness are still there, but this time given by a logic of lack of diversification. Here, I want to compare two different aggregate production functions, the Cobb-Douglas and Leontief, to emphasize the importance of understanding the connection between skewness and the number of inputs.

Consider a symmetric Cobb-Douglas production function with n nodes that only use labor,

$$\ln Y_n^c = \frac{1}{n}\left(\sum_i \ln L_i\right).$$

In turn, consider the Leontief technology:

$$\ln Y_n^l = \min_i \{\ln L_i\}.$$

Assume that $\ln L_i$ is distributed according to some symmetric distribution such that $F(\ln L) = 1 - F(-\ln L)$ with density f. By linearity and symmetry of shocks, we have that the mean and skewness of the Cobb-Douglas are zero, for any n. By the law of large numbers, the variance of output should decline with n:

$$\frac{\partial\left[\mathbb{E}\left[\ln Y_n^c\right]\right]}{\partial n} = 0,\ \frac{\partial\left[\mathbb{V}\left[\ln Y_n^c\right]\right]}{\partial n} < 0,\ \frac{\partial\left[Skew\left[\ln Y_n^c\right]\right]}{\partial n} = 0.$$

The behavior of the Leontief will be rather different. Whereas the variance will drop, again, by the law of large numbers, this is not true about the mean and the skewness. To see this, notice that for the Leontief, the CDF is the $n-th$ power of F. The CDF of the distribution of log output under the Leontief with n inputs can be easily calculated. The probability the min among multiple draws, requires that n independent draws exceed the value

$$F^n(x) = 1 - [1 - F(\ln L)]^n = 1 - F(-\ln L)^n,$$

where the second equality follows by symmetry. The corresponding p.d.f. f^n of the distribution of log output under the Leontief with n inputs is:

$$f^n(\ln L) = \frac{\partial}{\partial L}\left[F(\ln L)^n\right] = nf(-\ln L)\,F(-\ln L)^{n-1} = nF(-\ln L)^{n-1} f(\ln L).$$

Thus, the original density is distorted, but more so for higher values; the weights $nF(-\ln L)^{n-1}$ are lower for higher values. I conclude the following:

$$\frac{\partial\left[\mathbb{E}\left[\ln Y_n^l\right]\right]}{\partial n} < 0,\ \frac{\partial\left[\mathbb{V}\left[\ln Y_n^l\right]\right]}{\partial n} < 0,\ \frac{\partial\left[Skew\left[\ln Y_n^l\right]\right]}{\partial n} < 0.$$

In other words, in a Leontief production function, we should expect skewness to increase as we add more products. As we increase n, the mean drifts downward as we increase the number of inputs in the case of a Leontief. But also, lower tail events are sampled more often thereby adding more and more skewness. This result helps me address my original puzzle: how come more developed economies feature more lower-tail events when we know they are less volatile. The answer is that while risk can fall, left tail events can become more frequent!

To provide a further insight, let me establish a connection between the Leontief production function and Extreme Value Theory (EVT). Specifically, let me draw on the Fisher-Tippett-Gnedenko (FTG) theorem which is discussed in Embrechts et al. (2001). The FTG theorems tells us that

Proposition 3. *Let A_i be drawn from any i.i.d. distribution. Then, if there exists sequences $\{m_n\}$ and $\{a_n\}$ such that*

$$\lim_{n\to\infty} P\left[\frac{\ln Y_n^l + m_n}{a_n} \leq x\right] = \lim_{n\to\infty} 1 - \left[1 - F\left(a_n x + m_n\right)\right]^n = 1 - H\left(-x\right)$$

for some H, then H is an extreme value distribution (either a Frechet, Weibull, or Gumbel).

What this result tells us is that the (normalized) output of a Leontief production function with i.i.d. shocks to inputs will converge to a distribution associated with an extreme value distribution H. For any distribution where the convergence is guaranteed, the there is a domain of attraction toward some Extreme Value Distribution. Extreme value distributions, exhibit right tails, but if we rotate the axis, as we should when we consider a minimum among random variables, we find the desired left skewness.

In economics, it is interesting to draw a parallel between the Cobb-Douglas and the Leontief. Like the law of large numbers, more inputs diversifies risks, in the sense that the standard deviation disappears. The Central Limit Theorem tells us that starting from any skewed distribution, by appropriately normalizing output, we approach a symmetric distribution. The Leontief works in the opposite way. The law of large numbers still tells us that we diversify risks with more inputs, but the FTG theorem tells us that starting from a symmetric distribution, by appropriately normalizing output, we converge to asymmetric distributions with fatter left tails! The implication for economics is that if growth is produced by inventing more inputs, we should expect more diversification and less volatility, but bad news will become more frequent.

A Point of Caution. I criticized the original version of the paper along the following lines. In the original formulation, the data showed skewness in growth. The calculations, however, exhibits skewness in levels. This observation is important because of the following Proposition:

Proposition 4. *Let A_i be drawn from any i.i.d. distribution. Then,*

$$Skewness(\Delta \ln GDP) = 0.$$

The proof is immediate. We know that $\ln Y_t \sim F$ where F is a distribution generated by the mapping of the vector of A_i. Since the values of A_i are i.i.d., so is the distribution F. Regardless of whether F is skewed,

$$\Pr\left[\log\left[Y_t\right] - \log\left[Y_{t-1}\right]\right] = \Pr\left[\log\left[Y_{t-1}\right] - \log\left[Y_t\right]\right],$$

meaning that we cannot have skewed growth from i.i.d. shocks. What is going on? The Baqaee and Farhi (2019) formula in (1) is a derivative in space, not in time. It is a comparison across levels, not growth rates! The paper subsequently carried out the cross-country analysis by looking at the cyclical component of GDP, instead of growth rates relative to a Hodrick-Prescott filter. While the correlation of density and the skewness of the distribution of deviations from trend is re-assuring, it is not entirely clear that the Hodrick-Prescott filter does not generate a bias, when trends are not exactly linear. After all, the data must be produced by shocks to levels and shocks to trends, so it is hard to know if shocks themselves exhibit skewness—although I believe Dew-Becker and Vedolin (2021) show that firm-level shocks are approximately symmetric.

All in all, I hope that all readers, find this paper to be topical, interesting, and stimulating, as I did.

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