Homework 2

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ECON 221C MONETARY ECON III

Monetary Economics in Continuous Time.

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For the first part, please consult any textbook or online reference on the method of characteristics. Try to do questions 1 and 2 quickly. Question 3 will take up some time. If you feel assumptions are missing, fill them in.

Excercise 1 - PDE Warm Up

Consider a model of human capital, similar to "Experience vs. Obsolescence: A Vintage-Human-Capital Model" by Matthias Kredler. A level of human capital is denoted by $h \in [0, \bar{h}]$. Human capital depreciates λevery unit of time. For example, absent human capital accumulation, human capital $h(t+\Delta) = h(t) - \lambda\Delta$. Of course, human capital can be accumulated over time. In particular, there's an accumulation rule $u(h)$.

- Write a law of motion of $h(t)$.
- Consider an initial measure (distribution with mass different than 1) of human capital for h, which we denote by $f(h, 0)$. Derive a law of motion for $f(h, t)$.
- Solve $f(h, t)$ using the method of characteristics.
- Describe how a single characteristic would change if there's a positive externality. New to human capital, the externality is only local:
  - In particular, let the accumulation rule now depend on the mass of people with identical human capital, $f(h, t)$. In other words, now capital accumulation is$u(h, f(h, t))$ where now $u_f > 0$.

You may want to code this solution if you are interested in models of inequality. I suspect that $f(h, t)$ may feature a bimodal distribution with the distance in the modes separating over time.

Excercise 2 - Term-Structure

Consider an endowment economy with a deterministic path for the endowment $y(t)$. The path is continuous and almost-everywhere differentiable. Assume there is a representative agent. Define the utility of the agent via: $U(c(t))$ where U is CRRA with coefficient σ.

- Let $y(t)$ follow:

$$\dot{y}(t) = -\alpha(y(t) - \bar{y})$$

- Assume that there's a single traded bond in zero net supply. Assume that the bond is the limit of a one period bond. Please, demonstrates that the instantaneous return of that bond is:

$$r(t) = \rho + \sigma \frac{\dot{y}(t)}{y(t)}.$$

- Now, consider the term premium. Assume that there is a menu indexed by $\tau \in [0, \infty]$ of long-term bonds. Each bond pays one unit of consumption τ periods ahead. Consider a non-arbitrage condition in discrete time. Derive a PDE for $P(t, \tau)$ by taking the continuous time limit of that non-arbitrage condition. Note it should be similar to the one derived in class.

- Solve the PDE using the method of characteristics. Verify your solution by taking derivatives of P with respect to t and τ.

- Construct a corresponding yield curve as the **constant** discount rate $r(t, \tau)$, such that discounting the unit payoff at that rate, we obtain the price $P(t, \tau)$.

  – What is the shape of the yield curve as a function of $\dot{y}(0)$. Which rates are more sensitive, long-term rates or short-term rates?

- Now assume that with Poisson intensity θ, the value of output can drop to y_d, where y_d is a disaster value. The shock can happen only once —after the first shock, the intensity vanishes to zero. How does the PDE for $P(t, \tau)$ change?

Excercise 3

A Housing Model with Monetary Policy

This question is related to the paper by Guren and Krishnamurthy presented this week in the macro seminar. I want to take a different angle though. I'm interested in the distribution effects of monetary policy, as in particular, I believed that the low-rate policy of the FED after the housing crash favored people a generation older than me. This effect will last through my lifetime. To study the distributional effects of the FED consider an economy populated by two types of agents:

- **An overlapping generation of agents.** Agents live for T periods. There age is given by some $a \in [0, T]$. The wage profile is some $y(a)$ with $y''(a)$ and $y'(a) < 0$ if and only if $a > \bar{a}$. Agents die at age T. The mass of living agents is 1.

- **A set of international investors.** International investors lend elastically at an international rate $r*$.

- **The Housing Stock.** There's unit measure of houses. Homes can be owned or rented. Thus, the indicator for an agent of age a of home ownership is $h(a) \in \{0, 1\}$.

A Benchmark Economy. Assume that agents do not save. Furthermore, they borrow, but only through their home-mortgages. The instantaneous utility of the agent is:

$$U(c(t) + Ah$$

where A is a non-pecuniary value to owning a house. Their time discount is:

- Consider the price of homes to be $p(t)$ —at a stationary equilibrium, it's a constant price.
- If homes are rented, it's because they are owned by the international investors, so the rental flow is $z(t) = r * \cdot p(t)$.
- If homes are owned, it's because the funds are owned by the international investors, and the mortgage payments are $m(t) = (1 + \sigma r*)p(t)$. If an agent dies with a mortgage, he pays the principal selling the home. Here, σ is an intermediation spread.

Every newborn is born a renter. However, renters have a Poisson intensity θ of having the option to switch from home renter to home ownership —and vice-versa. Let the value function be denoted by:

$$V(a, h)$$

If an agent switches from h to h' it draws two shocks, $\epsilon^{h,h}$ and $\epsilon^{h,h'}$. The first shock, $\epsilon^{h,h}$ is the psychological cost of staying at state h. The second $\epsilon^{h,h}$ is the psychological cost of moving. Each shock is measured in utiles. Furthermore, these shocks are drawn from a type-I extreme value distribution, both are i.i.d shocks. The parameter of that distribution is ν. For the rest of the exercise, we study a stationary equilibrium.

[1.a] Show that the HJB equation can be written like this:

$$(\theta + \rho)V(a, h) = U(y(a) - (hm + (1 - h)z) + Ah + \theta\nu log(\sum_{k\in\{h,h'\}} exp(V(a, k))^{1/\nu}).$$

Given a price p, can you solve for $V(a, h)$?

[1.b] Let $f(a, h)$ be the mass of agents age a and state h. Can you please derive a system of ODE's that solve $f(a, h)$. Express transitions from h to h' as functions that depend on:

$$\mu_{hh'} = \frac{exp(V(a, h'))^{1/\nu}}{\sum_{k\in\{h,h'\}} exp(V(a, k))^{1/\nu}}.$$

Can you solve $\mu_{hh'}$?

[1.c] Obtain a market clearing condition for home-purchases at steady-state. Note that at steady state, the mass of renters and home owners is constant. Can we express that condition as a single equation in p? Explain the economics of why a price is determined or not.

An Economy with Savings.

[1.d] How would you adapt the problem to incorporate Andy Atkeson's criticism that the model didn't account for savings? What would be the appropriate market clearing condition for the interest rate r^*? Provide the sketch of an algorithm to solve the model. [Pretend you know the methods to solve PDE's and ODE's without saying what they are.

Planner Solution. [1.e] Setup the planner's problem in the closed economy version of the model. Explain how you would adapt Bigio, Nuno and Passadore to solve for the planner solution.

Pecuniary Externality. [1.f] Assume that in oder to buy a house, there's an loan-to-income constraint: you can't take a mortgage if the mortgage payment is greater than κ times income. What inefficiencies would emerge as a consequence. Would you introduce taxes in that case?

Note (the solutions are commented out)