A Theory of Bank Balance Sheets*

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Abstract

We present a theory of the asset and liability composition of bank balance sheets. Trade requires the use of assets as means of payments. Because of information asymmetries, some assets are poor means of payment: they are illiquid and do not circulate. Banks swap those assets for their liabilities. Banks' liabilities can be designed to circulate although fully backed by assets that do not. Liquid assets, that could circulate on their own, are brought to the banks' balance sheet to enhance liquidity creation. Hence, liquid and illiquid assets are complementary factors for liquidity creation. We study banks' asset choice and liability design and implement it with standard instruments, such as saving deposits, time deposits, and bank loans. We argue that the optimal asset and liability composition resembles bank balance sheets in practice.

*All errors are ours.

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1 Introduction

In its essence, banking is about the swap of assets for bank liabilities. When it purchases a security or when it makes a loan to a business owner, a bank acquires an asset. In exchange, the seller of the asset or the borrower receive a deposit, a bank liability. The deposit is liquid because it circulates: for example, the business owner can use it to pay his workers, workers can use it to make further purchases, and so on. Is the liability creation process pure accounting, or does it promote economic activity? Why and how do banks create circulating liabilities? Which assets should they swap for liabilities? Views on these classical questions shape monetary policy and financial regulation.

In this paper, we develop a theory of bank balance sheets. We start from the premise that some assets are illiquid because of asymmetric information. Banks purchase these illiquid assets in exchange for liabilities. In principle, since bank liabilities are backed by illiquid assets, they could also suffer from asymmetric information. However, banks can structure the payoff of the liabilities so as to make them liquid, as in Gorton and Pennacchi (1990). This explains why banks purchase illiquid assets in exchange for liquid liabilities. In practice banks also purchase many liquid assets, i.e., assets for which asymmetric information is hardly a problem. Our model is consistent with this observation, because it predicts that that liquid and illiquid assets are complementary input for liquidity creation. A theory of bank balance sheets emerges. We find that banks hold a larger fraction of the total supply of illiquid assets, e.g., loans to households or corporation, but a smaller fraction of the total supply of liquid assets, e.g., treasuries or corporate bonds. Within each asset class, illiquid and liquid, we find that banks hold safer assets in larger proportion than riskier assets.

We consider a three-period two-state economy with many types of assets in positive supply. Assets differ in terms of risk exposures: think of cash, fixed income securities, equities, mortgages, consumer or business loans. There is a continuum of producers who each own one asset, and a continuum of workers who supply labor. Producers and workers are matched bilaterally and anonymously to trade labor before production. As in the literature following Lagos and Wright (2005), both producers and workers lack commitment. This precludes the use of credit, and implies that producers and workers must use assets as means of payment. As in Rocheteau (2011), we assume that producers receive private information about their asset payoffs before matching with workers. This creates an asymmetric information problem, and implies that some producers who hold risky assets fail to trade with workers. In this context, we introduce an agent who can commit, called the banker. The banker purchases heterogeneous assets from producers in exchange for liabilities, before the arrival of private information. As in the security design literature (for example DeMarzo and Duffie, 1999; Biais and Mariotti, 2005, among many others), the banker chooses the state contingent payoffs of the liabilities he issues.

To explain the liability design problem, consider the simplest case first: when the banker makes an offer to a single producer holding some illiquid asset. The offer is a swap of the illiquid asset for a liquid liability. The banker profits from this swap because it issues a liability with lower average payoff than the asset. In agreeing to this swap, the producer looses payoffs, but he gains liquidity. Namely, the banker can structure the state-contingent payoffs of the liability so as to make it safer than the underlying asset. This promotes liquidity: the producer can now use the banker's liability to pay workers in bilateral trades. The amount of liquidity that the banker can create depends on the riskiness of the underlying asset. Indeed, to be liquid in bilateral trades, the banker's liability must have payoffs that do not vary too much across states. Since the liability is fully backed by the underlying asset, the value of the asset in the worst state limits the quantity of liquid liabilities that the banker can create. The riskier the asset, the lower is this limit.

The optimal liability design in the single asset case works as follow. First, the banker issues as many liquid liabilities as possible out of a given asset. When the asset is very risky, the liquidity the banker can create is insufficient to compensate the producer for surrendering his asset. In that case, the banker has to compensate those producers by issuing additional liabilities that do not circulate in bilateral trades. We call these very risk assets *fundamentally illiquid*. For safer assets, the banker can create more liquid securities than the amount needed to compensate the producer. For those assets, the banker issues just enough liquid securities to compensate the producer. We call these assets *structurally illiquid* because they are illiquid in their primitive form, but their payoff can be restructured so as to make them liquid. Finally, in the single asset case, the banker's liability design has no added value for producers holding *liquid assets*. Indeed, these producers can already use their assets as means of payment in bilateral trades.

Next, we turn to the multi assets problem: that is, the problem of a banker facing multiple producers with heterogeneous assets. We find that the optimal liability design works as if the banker treats his entire portfolio of assets as a single asset and then solves the optimal liability design for that single asset. As in the single asset case, the banker issues as many liquid liabilities as allowed by the asset portfolio, and issues illiquid liabilities if this is not enough to compensate all the producers he trades with. Importantly, the banker achieves a higher profit by dealing with all producer's simultaneously instead of separately. Put differently, the multi-asset liability design is more than a simple sum of single-asset liability designs. This is because different assets turn out to be complementary inputs in the process of liquidity creation. To understand the source of complementarity, consider the banker's problem with a single asset. If the asset is fundamentally illiquid the banker cannot create much liquidity and so must also issue an illiquid security to compensate the producer. If the asset is structurally illiquid, the banker can create liquidity in excess of what is needed to compensate its producer. Hence, in a multi-asset liability design, in which the structurally liquid and fundamentally illiquid asset are pooled together, the banker can use some of the excess liquidity created from the structurally illiquid asset to compensate the owner of the fundamentally illiquid asset. This reduces the issuance of illiquid liabilities and increases the banker's profit. Hence, structurally illiquid assets are valuable not only because they can be restructured, but also because they enhance the liquidity of fundamentally illiquid assets. Notice that the same complementarity arises with liquid assets. Hence, in the multi assets setting, there is complementarity between two broad asset classes: liquid and structurally illiquid assets on the one hand, and illiquid assets on the other.

We study the implementation of the optimal design with realistic bank contracts and liabilities. We show that, for the safest trees, the liability design can be implemented using a combination of checking deposit, which can be used for bilateral trades, and time deposit, which cannot. For the riskiest trees, the liability design can be implemented using an over-collateralized non-recourse loan: the banker gives the producer a given quantity of deposits, the loan size, and the producer promises to repay a given quantity of consumption good, the face value, leaving his tree to the banker as collateral. The loan is over-collateralized, in that the face value is lower than the payoff of the tree in the high state. By making the loan over collateralized, the banker effectively gives the producers an illiquid liability: an option to repurchase the tree at a strike price equal to the loan face value.

We go on to study an optimal asset choice problem. To make that problem non-trivial, we assume that the banker faces convex intermediation costs to attract producers. This could represent for example, the brick and mortar cost of establishing a branch in a particular neighborhood, the cost of acquiring expertise in a particular asset class, marketing or search costs for customers, or costs incurred in the process of screening the quality of customers' asset. Our main finding are the following. First, it is never optimal to choose a balance sheet that is only made of liquid assets: liquid assets are always held together with illiquid assets. Second, if intermediation costs are the same for all assets, our model predicts that banks will hold a small fraction of the total supply of liquid assets, and a larger fraction of the total supply of illiquid assets. One may argue that this corresponds to the observation that banks do hold some liquid and tradeable assets, such as treasuries, corporate bonds, municipal bonds, but in much smaller proportion than illiquid and non-tradeable securities, such as loan to households and businesses. Fourth, within each asset class, liquid and illiquid, banks will hold safest assets in larger proportion than riskier assets. That is, within liquid and tradeable assets, banks will hold more fixed income than equity. Within illiquid and non-tradeable assets, banks will hold more mortgages and loans than equity shares in non-publicly traded corporations.

Finally, the optimal asset choice problem delivers further insights about the complementarity and substitutability of assets for liquidity creation. Assets are complement between two asset classes: liquid and structurally illiquid on the one hand, and fundamentally illiquid on the other hand. They are substitute within each class. Hence, a decrease in intermediation costs for some fundamentally illiquid asset, say mortgage loans, will have two effects. It will reduce the banker's holdings of other fundamentally illiquid assets, say business loans, but it will increase the banker's holdings of liquid of structurally liquid assets, say treasury or corporate bonds.

Some Empirical Evidence about Bank Balance Sheets

Clearly, many factors that may affect balance sheet composition in practice are left out of our theory, such as banking regulation, maturity composition, or expertise. Yet, we argue that our theory goes in the direction of explaining qualitative observations about bank balance sheets along two dimensions: the asset and liability composition, and the holding shares by asset classes, out of the total supply outstanding.

We use data corresponding to all Chartered Depositary Institutions in the United States for two particular quarters and total supply outstanding for several asset classes. The data is obtained from the Flow of Funds, for the last quarters of 1985 and 2005.[1]

Figure 1 reveals that banks' liabilities are, for the most part, made up of checking deposits, time deposits and equity. Notice that checking deposits correspond to over 50-60% of the liabilities, while time deposits only represent 10-20%. Hence, checking deposits are much larger than time deposits. In the context of our model, this corresponds to the finding that bankers strive to create liquid liabilities rather than illiquid ones. Interbank liabilities are also important, but we argue that they would arise in our model if we introduced heterogenous banks.

Figure 2 shows the asset composition. It reveals that over 70% of the assets on the left-side of the balance sheets are loans. The rest are tradable assets, such as Treasuries, Corporate Bonds, or Equities. In the context of our model, this corresponds to the observation that the banker finds it optimal to hold more illiquid assets than illiquid ones. Figure 3 shows the *fraction* of the outstanding supply held by banks, by asset classes. It shows, as suggested

[1] We choose the same quarter because it is a simple way to control for seasonality. We focus on years that pre-date the 2007-2009 financial crisis, since monetary policy has changed dramatically since then, with substantial impact on banks' behavior.

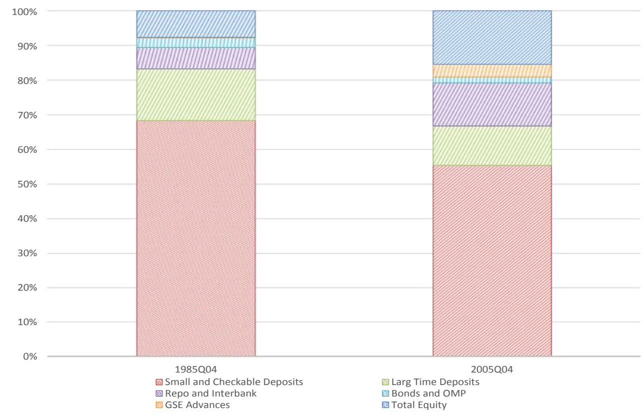

Figure 1: The liability composition.

by the model, that banks hold a larger fraction of the total supply of illiquid than of liquid assets. Within liquid assets, banks hold a larger fraction of the supply of safer assets (fixed income) and a much smaller faction of the supply of riskier assets (equities)

Literature Review (Incomplete)

Because we focus on the design of means of payment, our work is related to the money and payment literature following Lagos and Wright (2005). Berentsen, Camera, and Waller (2007) have studied the role of banks in helping agents insure against preference shocks: banks reallocate idle balance from agents who want to consume, towards agents who do not. In contrast, our model does not generate any such demand for insurance. Another closely related paper is Rocheteau (2011) who study bilateral trades with multiple assets in the presence of asymmetric information. We complement Rocheteau's work by adding a security design problem. We find that agents do not trade with assets directly. Instead, they find it optimal to deposit these assets in banks, and they trade by exchanging bank liabilities.

Farhi and Tirole (2015) consider the trade-off between tranching bundling an asset in a model of bilateral asset trade with asymmetric information, with a focus on endogenous information acquisition. Tranching means re-structuring the payoff of some underlying asset so as to create two assets for bilateral trade, a safe asset and a maximally risky asset. Bundling means keeping the underlying asset for bilateral trade. They derive conditions under which bundling leads to more trade than tranching. In contrast, we introduce multiple assets and study optimal asset choice and liability design. In that context, there is no

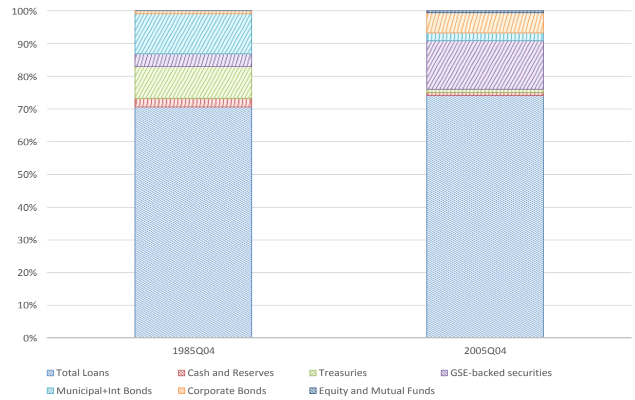

Figure 2: The asset composition.

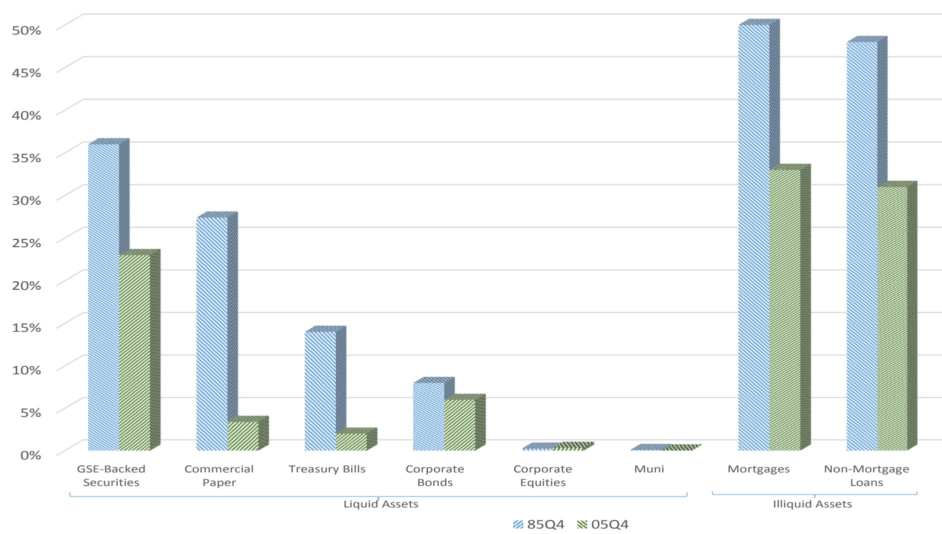

Figure 3: Asset composition relative to aggregate supply outstanding, by asset class.

binary choice between bundling and tranching: instead, the optimal balance sheet involves a combination of both. Bankers bundle multiple assets on the left side of their balance sheet, and find the optimal way to tranche the bundle to induce trade.

Our work complements the recent paper of Dang, Gorton, Holmström, and Ordoñez (2014). They show that banks are optimally opaque about their balance sheet and design debt-like liabilities which reduce agent's incentive to acquire private information. They show that banks seek to hold safe assets so as to design securities minimizing incentives to acquire private information. We develop a different model in which we abstract from bank opacity and private information acquisition. This leads to a different liability design problem, and a different demand for safe assets. We derive predictions for the optimal design of a bank's balance sheet, in particular for banks' optimal asset holdings at all points of the liquidity and safety spectrum.

Finally, our model is related to the vast security design literature in corporate finance, such as DeMarzo and Duffie (1999) and Biais and Mariotti (2005), just to name a few. Our contribution relative to this literature is to study the design of means of payments backed by an optimally chosen portfolio of assets.

2 Model

Section 2.1 lays out the economic environment. Section 2.2 studies a building block of our model: bilateral trade under asymmetric information about the payoff of the means of payment.

2.1 The economic environment

There are three dates, $t \in \{0, 1, 2\}$, and two states, $\omega \in \{\omega_\ell, \omega_h\}$, with probability $\pi(\omega)$.

Risky trees. There is a continuum of risky assets called "trees", in positive supply. Trees come in finitely many types, indexed by $j \in J$. The payoff of a type $j \in J$ tree is $R_j(\omega)$ in state $\omega \in \{\omega_\ell, \omega_h\}$ at $t = 2$. The expected payoff of each tree is normalized to one, that is, $\mathbb{E}[R_j(\tilde{\omega})] = 1$ for all $j \in J$. Trees represent broad asset classes, such as cash, treasuries, fixed income securities, equity, future human capital or business income.

Agents. The economy is populated by three types of agents: a continuum of producers, a continuum of workers, and one banker. All agents are risk neutral and only enjoy consumption at $t = 2$.

At $t = 0$, each producer is endowed with one tree. At $t = 1$, each producer operates a linear production technology, whereby q units of labor yield $r(\omega)q$ units of output at time $t = 2$. Labor is supplied by workers at some constant marginal cost, which we normalize to one. As is common in the monetary literature following Lagos and Wright (2005), we assume that the producer and the worker cannot commit. This precludes the use of credit in bilateral trades, even using the tree as collateral,[2] and promotes the use of assets as means of payment.

The banker is endowed with $\mu_0 \geq 0$ shares of some risk-free asset, with payoff normalized to one, interpreted as his net worth. The banker cannot operate a production technology nor supply labor. But the banker is special: unlike producers and workers, he can commit to make future payments.

Three stages. The timing of the model is as follows:

- $t = 0$: contracting stage. The banker approaches producers and makes a take-it-or-leave-it offer. In doing so, he chooses which assets to bring on its balance sheet, and what type of liability to give in exchange for these assets.

- $t = 1$: trading stage. Each producer learns the state, $\omega \in \{\omega_h, \omega_\ell\}$, and is matched bilaterally and anonymously with a worker. Unlike the producer, the worker does not know the state, which creates asymmetric information. Since the producer and the worker cannot commit, they cannot use credit. Instead, labor must be traded in exchange for some means of payment: the tree endowment of the producer, or the liability issued by the banker to the producer.

- $t = 2$: payoff, settlement, and consumptions. Output is produced, all trees pay off, contracts with the banker are settled, and all agents consume.

Two maintained parametric assumptions. For all what follows we assume that

$$r(\omega_h) > r(\omega_\ell) = 1 \tag{1}$$

$$\pi(\omega_h) r(\omega_h) < 1. \tag{2}$$

The first assumption means that there are only gains from bilateral trade in the high state. The second assumption ensures that some assets suffer from a lemon problem in bilateral

[2] Using the tree as collateral for state-contingent promises requires commitment on part of the worker. Indeed, the worker must commit to return the tree to the producer in exchange for making the promised payment.

trade. As will become clear shortly, this creates gains from trade between the producers and the banker.

2.2 Bilateral trade between the producer and the worker

In this section, we study the bilateral trade between the producer and the worker, when the producer makes payment using a state contingent security. This is a key building block of our model, as it determines the value of using alternative means of payments in order to purchase the labor supplied by the worker.

Consider a producer who owns some security with state-contingent payoff $S(\omega)$. This security can be either a liability issued by the banker, or the initial tree endowment of the producer. At time $t = 1$, the producer learns the realization of the state ω and is matched bilaterally with some worker. He makes a take-it-or-leave-it offer, to purchase a quantity q of labor input in exchange for a quantity $n \in [0, 1]$ of the security. Crucially, the producer knows the realization of the state, ω, but the worker does not. As is typical, asymmetric information makes it possible to sustain many equilibria, depending on the specification of the worker's beliefs upon observing an out-of-equilibrium offer. As Farhi and Tirole (2015), we focus on bilateral trades $q(\omega)$ and $n(\omega)$ maximizing the amount of labor input supplied by the worker when there are gains from trades, in the high state. Clearly, given asymmetric information, the bilateral trades maximizing labor supplied in the high state solve the following optimization problem:

$$\max q(\omega_h) \tag{3}$$

with respect to $q(\omega)$ and $n(\omega)$, and subject to:

$$\begin{aligned}
\forall \omega \quad & q(\omega) \geq 0 \\
\forall \omega \quad & 0 \leq n(\omega) \leq 1 \\
\forall \omega \quad & q(\omega) r(\omega) + [1 - n(\omega)]\, S(\omega) \geq S(\omega) \\
\forall \omega \quad & \mathbb{E}\left[n(\tilde{\omega}) S(\tilde{\omega}) \,|\, q(\omega), n(\omega) \right] - q(\omega) \geq 0 \\
\forall (\omega, \tilde{\omega}) \quad & q(\omega) r(\omega) + [1 - n(\omega)]\, S(\omega) \geq q(\tilde{\omega}) r(\omega) + [1 - n(\tilde{\omega})]\, S(\omega).
\end{aligned}$$

The first two constraints are feasibility constraints. The third constraint is the participation constraint of the producer: in all states, the producer must prefer the bilateral trade to holding on to his security. The fourth constraint is the participation constraint of the worker: the worker expected value from the bilateral trade, conditional on the offer, must be positive.

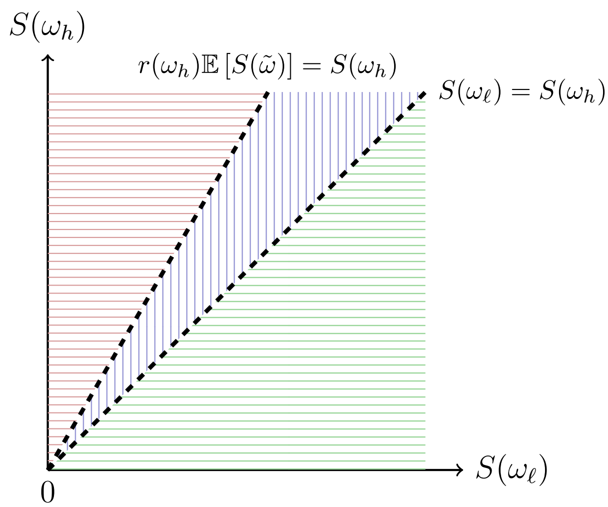

Figure 4: Three regions of the security payoff space for $\mathbb{U}[S(\tilde{\omega})]$.

The last constraint is the incentive constraint of the producer: he must prefer the offer recommended in the realized state, ω, to the offer recommended in the other state.

In the Appendix, we solve the optimization problem (3). We show that the corresponding optimal bilateral trades are the outcome of a Perfect Bayesian Equilibrium (PBE) of the bargaining game, given an appropriate specification of workers' out-of-equilibrium beliefs. For all what follows, however, the details of the optimal bilateral trade do not matter. All we need to know is the ex ante value of the producer for the security:

$$\mathbb{U}[S(\tilde{\omega})] = \sum_{\omega} \pi(\omega) \{r(\omega)q(\omega) + [1 - n(\omega)] S(\omega)\}, \tag{4}$$

where the the bilateral trade $q(\omega)$ and $n(\omega)$ solves the optimization problem (3).

Proposition 1. *For a producer, the ex ante value of the security, $\mathbb{U}[S(\tilde{\omega})]$, is the same for all optimal bilateral trades and is equal to:*

$$\text{If } S(\omega_h) \leq r(\omega_h)\mathbb{E}[S(\tilde{\omega})]: \quad \mathbb{U}[S(\tilde{\omega})] \equiv \min\{\mathbb{E}[r(\tilde{\omega})S(\tilde{\omega})], \mathbb{E}[r(\tilde{\omega})]\,\mathbb{E}[S(\tilde{\omega})]\};$$
$$\text{If } S(\omega_h) > r(\omega_h)\mathbb{E}[S(\tilde{\omega})]: \quad \mathbb{U}[S(\tilde{\omega})] = \mathbb{E}[S(\tilde{\omega})].$$

Consider first a security such that $S(\omega_h) \leq S(\omega_\ell)$. In Figure 4, this corresponds to the bottom region marked with green horizontal lines. In this region, output is first best. Indeed, the optimal bilateral trade prescribes that in the high state, when there are strict gains from trade, the producer sells the security for its "true value", $S(\omega_h)$ units of labor.

Next, consider a security such that $S(\omega_h) \geq S(\omega_\ell)$ but $S(\omega_h) \leq r(\omega_h)\mathbb{E}[S(\tilde{\omega})]$. In Figure

4, this corresponds to the middle region marked with blue vertical lines. In this region, the optimal bilateral trade can be shown to be pooling: in both states, the producer buys $\mathbb{E}\left[S(\tilde{\omega})\right]$ units of labor from the worker in exchange for one unit of the security. Notice that, in the high state, the price of the security is depressed. The producer can only purchase $\mathbb{E}\left[S(\tilde{\omega})\right]$ units of labor, which is less than the true value of the security, $S(\omega_h)$. Output is second best: it is less than what would be produced with symmetric information about ω.

Finally, consider a security such that $S(\omega_h) > r(\omega_h)\mathbb{E}\left[S(\tilde{\omega})\right]$, corresponding to the top region marked with red horizontal lines in Figure 4. In this region, bilateral trade breaks down in the high state. Indeed, after learning that $\omega = \omega_h$, the producer's value of holding on to the security, $S(\omega_h)$, is larger than the value of purchasing the worker's labor at the pooling price, $r(\omega_h)\mathbb{E}\left[S(\tilde{\omega})\right]$. In the low state, bilateral trade does not break down, but there are no gains from trade anyway. Taken together, this implies that $\mathbb{U}\left[S(\tilde{\omega})\right]$ coincides with $\mathbb{E}\left[S(\tilde{\omega})\right]$, the value of not trading at all and simply holding on to the security until $t = 2$.

The above discussion reveals that, for a producer, the ex ante value of the security, $\mathbb{U}\left[S(\tilde{\omega})\right]$, depends on the severity of the asymmetric information problem. Namely, $\mathbb{U}\left[S(\omega)\right]$ is increasing and concave in payoffs when $S(\omega_h) \leq r(\omega_h)\mathbb{E}\left[S(\tilde{\omega})\right]$. In Figure 4, this corresponds to green region marked with horizontal lines and the blue region marked with vertical lines. Then, when $S(\omega_h) > r(\omega_h)\mathbb{E}\left[S(\tilde{\omega})\right]$, trade breaks down and $\mathbb{U}\left[S(\tilde{\omega})\right]$ jumps down discretely. In the figure, this corresponds to the red region marked with horizontal lines. The concavity and the discontinuity both arise because the asymmetric information problem becomes more and more severe as the gap between $S(\omega_\ell)$ and $S(\omega_h)$ increases. Without asymmetric information, by contrast, the ex ante value of the security would be linear and continuous in payoffs, and equal to $\mathbb{E}\left[r(\omega)S(\omega)\right]$.

3 Monopolistic liability design given assets

In this section, we study the choice of the liability side of the balance sheet, given assets. In our model, this corresponds to choice made by the banker at $t = 0$, the contracting stage. We show that, given the pool of assets on the left-side of his balance sheet, the banker finds it optimal to issue one single liquid liability that producers use for bilateral trades, and possibly some illiquid liability that cannot be used in bilateral trade.

3.1 The banker's problem with heterogenous producers

At $t = 0$, the banker, with net worth μ_0, approaches a measure μ_j of type $j \in J$ producers, for a grand liability design problem. The banker offers each producer of type j to bring its tree in exchange for two liabilities. A liquid liability, $D_j(\omega)$, which a producer of type j can use for bilateral trade with workers, and an illiquid liability, $I_j(\omega)$, which the producer has to hold to maturity on the banker's balance sheet. The liquid liability could represent a checking deposit, and the illiquid liability a time deposit. We assume that the security $D_j(\omega)$ must satisfy the constraint $r(\omega_h)\mathbb{E}\left[D_j(\tilde{\omega})\right] \geq D_j(\omega_h)$, i.e., it must be liquid in bilateral trade. This is without loss of generality since, when $D_j(\omega)$ is illiquid in bilateral trade, it is equivalent to the illiquid security $I_j(\omega)$. With this in mind, the banker's problem given $\mu = \{\mu_j\}_{j \in J}$ is:

$$V(\mu) = \max \mathbb{E}\left[c(\omega)\right], \tag{5}$$

with respect to positive $c(\omega)$, $D_j(\omega)$, and $I_j(\omega)$, and subject to:

$$\begin{aligned}
\forall \omega \quad & c(\omega) + \sum_{j \in J} \mu_j \left[D_j(\omega) + I_j(\omega)\right] \leq \mu_0 + \sum_{j \in J} \mu_j R_j(\omega) \\
\forall j \quad & r(\omega_h)\mathbb{E}\left[D_j(\tilde{\omega})\right] \geq D_j(\omega_h) \\
\forall j \quad & \mathbb{U}\left[D_j(\tilde{\omega})\right] + \mathbb{E}\left[I_j(\tilde{\omega})\right] \geq \mathbb{U}\left[R_j(\tilde{\omega})\right].
\end{aligned}$$

There are three constraints. The first is a resource constraint. The second is a liquidity constraint, which ensures that the liquid liability circulates in subsequent bilateral trades. The third is a participation constraint, ensuring that producers of type j are weakly better off trading with the banker than going on to trade with their tree on their own. An immediate result is:

Lemma 2. *The banker's problem has a solution. The banker's value, $V(\mu)$, is weakly increasing, homogenous of degree one, and concave.*

Monotonicity arises because it is always feasible for the banker to issue liabilities that replicate the payoffs of the trees brought by agents. Homogeneity and concavity arises because the objective is linear and the constraint set is homogenous and convex.

3.2 The banker's problem with a representative producer

Next, we show that the banker's problem can be simplified: it can be solved "as if" the banker approaches a representative producer, with an appropriately defined representative tree and outside option.

An equivalence result. To state the result leading to this simplification, let

$$\bar{R}(\omega) \equiv \mu_0 + \sum_{j \in J} \mu_j R_j(\omega) \tag{6}$$

$$\bar{U}(\omega) \equiv \sum_{j \in J} \mu_j \mathbb{U}\left[R_j(\tilde{\omega})\right] \tag{7}$$

denote, respectively, the total resources and outside option of the producers approached by the banker. Then, let us define the banker's problem with a representative producer:

$$\max \mathbb{E}\left[c(\omega)\right] \tag{8}$$

with respect to positive $c(\omega)$, $D(\omega)$, and $I(\omega)$ and subject to:

$$\begin{aligned} \forall \omega \quad & c(\omega) + D(\omega) + I(\omega) \leq \bar{R}(\omega) \\ & r(\omega_h)\mathbb{E}\left[D(\tilde{\omega})\right] \geq D(\omega_h) \\ & \mathbb{U}\left[D(\tilde{\omega})\right] + \mathbb{E}\left[I(\tilde{\omega})\right] \geq \bar{U}. \end{aligned}$$

We then obtain:

Lemma 3. *If the banker's problem with heterogenous producers has a solution, then so does the banker's problem with a representative producer. Moreover, consider any solution* $D^\star(\omega)$, $I^\star(\omega)$ *and* $c^\star(\omega)$ *of the banker's problem with a representative producer. Then, a solution of the banker's problem with heterogenous producers is:*

$$D_j^\star(\omega) = \alpha_j D^\star(\omega), I_j^\star(\omega) = \alpha_j I^\star(\omega), \text{ where } \alpha_j = \frac{\mathbb{U}\left[R_j(\tilde{\omega})\right]}{\sum_{k \in J} \mu_k \mathbb{U}\left[R_k(\tilde{\omega})\right]}. \tag{9}$$

This result arises because of concavity and homogeneity. Just as in Pareto problems with identical homogenous utility functions, there exists an optimum in which the banker gives the same liability to all producers. Some producers receive more liabilities than others, depending on their outside option.

A two-steps solution. Given Lemma 3, our next task is to solve the banker's problem with a representative producer. To that end, we fix some $\bar{U} > 0$ and we consider the following auxiliary optimization problems.

1. The *deposit minimization* problem:

$$\min \mathbb{E}\left[D(\tilde{\omega})\right],$$

with respect to $D(\omega) \geq 0$, subject to:

$$D(\omega) \leq \bar{R}(\omega)$$
$$r(\omega_h)\mathbb{E}\left[D(\tilde{\omega})\right] \geq D(\omega_h)$$
$$\mathbb{U}\left[D(\tilde{\omega})\right] \geq \bar{U}.$$

2. The *deposit maximization* problem:

$$\max \mathbb{U}\left[D(\tilde{\omega})\right] + \mathbb{E}\left[R(\tilde{\omega})\right] - \mathbb{E}\left[D(\tilde{\omega})\right],$$

with respect to $D(\omega) \geq 0$, subject to

$$D(\omega) \leq \bar{R}(\omega)$$
$$r(\omega_h)\mathbb{E}\left[D(\tilde{\omega})\right] \geq D(\omega_h).$$

The deposit minimization problem is to find the smallest liquid liability that is consistent with participation. The deposit maximization problem is to maximize the value of the representative producer, with respect to some liquid liability, $D(\omega)$, and some residual illiquid liability, $\bar{R}(\omega) - D(\omega)$. Based on these auxiliary optimization problems, we obtain a simple two-steps procedure for solving the bank's problem.

Proposition 4. *Given some arbitrary value $\bar{U}$, the banker's problem with a representative producer has a solution if and only if*

$$\mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right] - \mathbb{E}\left[D^\star_{\max}(\tilde{\omega})\right] + \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] \geq \bar{U}, \tag{10}$$

where $D^\star_{\max}(\omega)$ denotes any solution of the deposit maximization problem. If (10) holds, then the banker's problem can be solved in two steps.

- *If the deposit minimization problem has a solution, then it is also a solution of the banker's problem with $I^\star(\omega) = 0$.*

- *Otherwise, $D^\star(\omega)$ and $I^\star(\omega)$ solve the banker's problem if and only if $D^\star(\omega)$ solves the deposit maximization problem with $I^\star(\omega) = \alpha\left[\bar{R}(\omega) - D^\star(\omega)\right]$, and α is chosen so that the participation constraint binds.*

In both cases, the banker's consumption is $c^\star(\omega) = \bar{R}(\omega) - D^\star(\omega) - I^\star(\omega)$.

To understand the first point of the proposition, recall that the deposit maximization problem solves for the maximum value that can be delivered to the representative producer.

If that value is less than the outside option, $\bar{U}$, then it is clear that the constraint set of the banker's problem is empty.

To understand the rest of the proposition, recall that a producer always prefer liquid over illiquid liabilities: liquid liabilities allow to purchase labor services from workers. This leads to the first bullet point of the proposition: if it is possible to satisfy the producer's participation constraint with a liquid liability only, this liquid liability must solve the banker's problem. Otherwise, the banker issues the largest liquid liability possible, creates an illiquid security so as to satisfy the participation constraint, and consumes the rest.

3.3 Positively correlated trees

To simplify the exposition in what follows, we solve the banker's problem when all trees' payoffs are positively correlated with productivity (a treatment of other cases can be found in the Appendix.) Then, we have:

Lemma 5. *Suppose that $R_j(\omega_\ell) < R_j(\omega_h)$ for all $j \in J$. Then the deposit minimization problem has a solution if and only of $\mathbb{U}\left[D^\star_{\max}(\omega)\right] \geq \bar{U}$.*

With this in mind, we turn to the deposit maximization problem:

Proposition 6. *Suppose that $R_j(\omega_\ell) < R_j(\omega_h)$ for all $j \in J$. Then, the solution of the deposit maximization problem is*

$$D^\star_{\max}(\omega_\ell) = \bar{R}(\omega_\ell) \text{ and } D^\star_{\max}(\omega_h) = \min\left\{\bar{R}(\omega_h), \frac{r(\omega_h)\left[1-\pi(\omega_h)\right]}{1-r(\omega_h)\pi(\omega_h)}\bar{R}(\omega_\ell)\right\}. \tag{11}$$

The Proposition is illustrated in Figure 5. The region enclosed by the blue box is the set of all resource-feasible security payoffs. The regions marked with green horizontal lines and blue vertical lines is the set of liquid securities. And the region marked with red horizontal lines is the set of illiquid securities. The concave green curves are iso-value curves for the deposit maximization problem, with a highest curve corresponding to a highest value. Clearly, the optimum is to pick a liability on the highest possible iso-value curve, subject to the liquidity constraint that this liability belongs to the green or the blue region. This optimum is represented by the green dot on the figure.

The solution of the deposit maximization problem, $D^\star_{\max}(\omega)$ less information sensitive than the underlying pool of trees, $\bar{R}(\omega)$, in the sense that its payoff does not vary as much between the low and the high state. In this sense, it is debt-like. However, $D^\star_{\max}(\omega)$ is not riskless. Indeed, designing a riskless liability would sacrifice purchasing power in the high state, without enhancing liquidity.

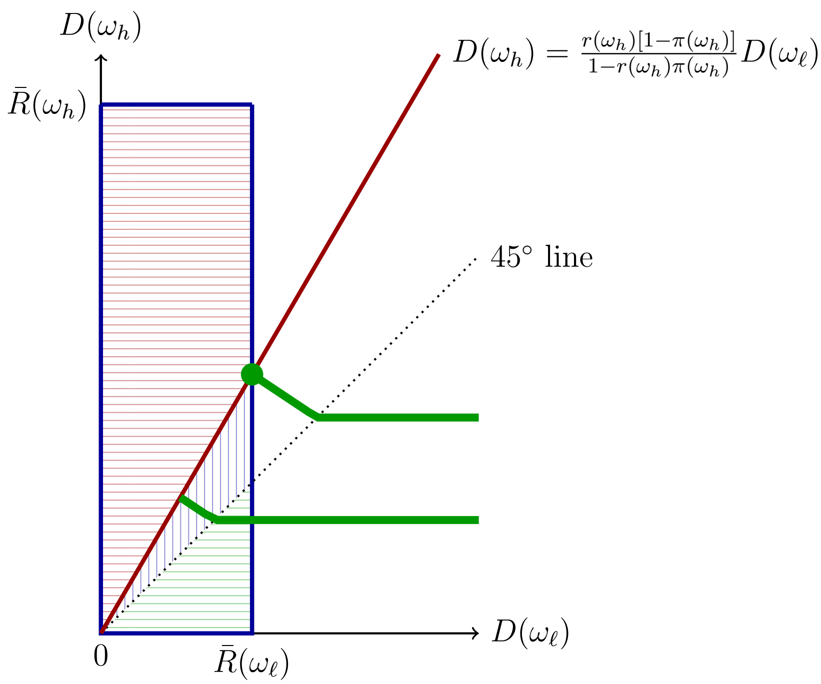

Figure 5: The deposit maximization problem.

Notice that the size of $D^\star_{\max}(\omega)$ depends on the riskiness of the underlying pool of assets. The riskier the pool, the lower is the size of $D^\star_{\max}(\omega)$. For example, when the pool is maximally risky and $\bar{R}(\omega_\ell) = 0$, then $D^\star_{\max}(\omega) = 0$ as well. This means in particular that, if the pool of asset is sufficiently risky, $D^\star_{\max}(\omega)$ will not be large enough to convince the producer to surrender his asset. This explains why, in Proposition 4, the banker may have to issue an illiquid liability.

A solution of the banker's problem. We now turn to the banker's problem. We already know from Proposition 4 that, when there is an illiquid liability, $I^\star(\omega) > 0$, the liquid liability solving the banker's problem is equal to $D^\star_{\max}(\omega)$, the solution of the deposit maximization problem. This is not true in general, however, because $D^\star_{\max}(\omega)$ may give too much utility to producers. The banker can increase its value by reducing the size of $D^\star_{\max}(\omega)$ and transfer some state-contingent payoff to itself. A general solution of the banker's problem is thus as follows:

Proposition 7. *Suppose that $R_j(\omega_h) \geq R_j(\omega_\ell)$ for all $j \in J$. Then, a solution to the bank's problem takes the following form. First, the banker contributes an amount $\mu_0^\star \in [0, \mu_0]$ of his net worth to the the pool of asset, so that the payoff of the pool is $R^\star(\omega) = \mu_0^\star + \sum_{j \in J} \mu_j R_j(\omega)$.*

Second, the banker issues an aggregate liquid liability with payoff:

$$D^\star(\omega_\ell) = R^\star(\omega_\ell) \text{ and } R^\star(\omega_\ell) \le D^\star(\omega_h) \le \min\left\{ R^\star(\omega_h), \frac{r(\omega_h)(1-\pi(\omega_h))}{1-r(\omega_h)\pi(\omega_h)} R^\star(\omega_\ell) \right\}.$$

The illiquid liability is of the form $I^\star(\omega) = \alpha\,[R^\star(\omega) - D^\star(\omega)]$, for some $\alpha \in [0,1]$. The banker's payoff is $c^\star(\omega_\ell) = \mu_0 - \mu_0^\star$ and $c^\star(\omega_h) = \mu_0 - \mu_0^\star + R^\star(\omega_h) - D^\star(\omega_h) - I^\star(\omega_h)$.

Although $D^\star(\omega)$ differs from $D^\star_{\max}(\omega)$, it retains its main properties. Namely, after the banker has contributed some net worth to the pool, the liquid liability $D^\star(\omega)$ pays off the entire value of the asset pool in the low state, and at least the same amount in the high state.

As mentioned above, when $I^\star(\omega) > 0$, there is a unique solution to the banker's problem, in which $D^\star(\omega) = D^\star_{\max}(\omega)$. When $I^\star(\omega) = 0$, however, there are in general multiple solutions. For example, an optimal solution is to offer a liquid liability of the form $\lambda D^\star_{\max}(\omega)$, for some $\lambda \in [0,1]$. The proposition proposes another design, which has the property of giving the banker an equity share in the bank. We argue that this is a desirable property, because of the commonly held view that bankers add value by exerting unobservable monitoring effort: that is, bankers have a special role in making sure that assets on their balance sheet have high payoff. Giving banker an equity share provides maximum incentives to exert monitoring effort.[3]

Taking stock, the solution of the optimal liability design problem has the following features. There is a single liquid liability issued to all producers in different quantities, in accordance to the producer's outside option. This liability has a debt-like payoff. In particular, it may be more risky than some other assets on the balance sheet. This means that some producers may contribute very safe assets, and receive in exchange a more risky liability. Think for example of bank depositors bringing cash to the bank, and coming out of the bank with a deposit, which is more risky because it is backed in part by mortgage loans on the bank's balance sheet. Vice versa, other producers contributes very risky and illiquid trees, and leave the banker with a much safer liability. As we argue below in Section 4, this can represent deposits that the bank creates after making a loan collateralized by a risky tree.

[3] To be more precise, imagine that exerting monitoring effort means that, conditional on $\omega = \omega_h$, the pool of assets pays off $R^\star(\omega_h)$ with probability one. Not exerting monitoring effort means that, conditional on $\omega = \omega_h$, the pool of asset pays off $R^\star(\omega_\ell) < R^\star(\omega_h)$. Then, following standard arguments, one sees that the banker's has maximum incentives to exert effort when he only receives payment in the high state.

3.4 Balance sheet composition and liability design

In this section we show how the liability design, in particular whether or not the banker issues an illiquid liability, depends on two characteristics of the asset pool: the aggregate payoff of the pool, $\bar{R}(\omega)$, and the outside option of the representative producer, $\bar{U}$.

Liquid balance sheet. The first case is when the aggregate payoff is liquid, i.e. $\bar{R}(\omega_h)$ is not too large:

$$\bar{R}(\omega_h) \leq r(\omega_h)\mathbb{E}\left[\bar{R}(\tilde{\omega})\right].$$

In that case, $D^\star_{\max}(\omega) = \bar{R}(\omega)$: simply pooling all of the producers' assets eliminates the lemon problem. For example, in that case, an optimal liability design would be for the bank to let producer trade with a portfolio of asset, $D^\star(\omega) = \lambda D^\star_{\max}(\omega) = \lambda \bar{R}(\omega)$.

Structurally illiquid balance sheet. The second case is when

$$\bar{R}(\omega_h) > r(\omega_h)\mathbb{E}\left[R(\tilde{\omega})\right] \text{ and } \mathbb{U}\left[D^\star_{\max}(\omega)\right] \geq \bar{U}.$$

That is, the pool of assets is illiquid, but the solution of the deposit maximization problem delivers enough value so as to satisfy producers' participation constraints. Hence, by Proposition 4 and Lemma 5, we know that the banker only issues a liquid liability, and does not issue any illiquid liability. In contrast with the previous case, the optimal liquid liability cannot be the portfolio of of all trees on the bank's balance sheet, $\bar{R}(\omega)$. Indeed, this portfolio is illiquid. Hence, the banker must restructure the payoff of the portfolio in order to satisfy the liquidity constraint. For example, Proposition 7 indicates that one natural way to restructure the payoff is to create a debt liability backed by the pool of asset. This is why, in this case, we call the balance sheet "structurally illiquid": an optimal liability design just involves restructuring, and does not require the issuance of an illiquid liability, i.e., $I^\star(\omega) = 0$.

Fundamentally illiquid balance sheet. The third case is when

$$\bar{R}(\omega_h) > r(\omega_h)\mathbb{E}\left[R(\tilde{\omega})\right] \text{ and } \mathbb{U}\left[D^\star_{\max}(\omega)\right] < \bar{U}.$$

In this case, the solution of the deposit maximization problem does not delivers enough value to satisfy producers' participation constraints. To satisfy the constraint, the banker must create an illiquid liability in addition to the liquid liability.

Taking stock, with a figure. Figure **??** illustrates the regions of the parameter spaces where the banker balance sheet is liquid, structurally illiquid, or fundamentally illiquid. To construct this figure, we set net worth to zero for simplicity, $\mu_0 = 0$, and we assume that $R_j(\omega_\ell) \leq R_j(\omega_h)$ for all $j \in J$. We make two normalizations. First, we let the expected payoff of every tree be $\mathbb{E}\left[R_j(\tilde{\omega})\right] = 1$. Second, we let the total measure of tree be $\sum_{j \in J} \mu_j = 1$. In the figure, $\mu_I = \sum_{j \in I} \mu_j$ is the fraction of illiquid assets on the bank's balance sheet, and $\bar{R}(\omega_\ell)$ is the payoff of the asset pool in the low state. The details of the construction are explained in Appendix C. The figure reveals that the liquidity status of the pool only depends on μ_I and $\bar{R}(\omega_\ell)$.

Namely, given that $\mathbb{E}\left[\bar{R}(\tilde{\omega})\right] = 1$, a lower payoff in the low state, $\bar{R}(\omega_\ell)$, translates into a higher payoff in the high state. Hence the pool of assets is more likely to be illiquid. Holding μ_I constant, one sees from the figure that the pool is liquid when $\bar{R}(\omega_\ell)$ is large enough, structurally illiquid in an intermediate range, and fundamentally illiquid when $\bar{R}(\omega_\ell)$ is small enough.

The fraction of illiquid asset, μ_I, also matters. Holding $\bar{R}(\omega_\ell)$ constant, an increase in μ_I relaxes the participation constraint of the representative producer. Indeed, illiquid trees have no value for bilateral trades, implying that holders of illiquid trees have a lower outside option. This makes it easier to satisfy the representative producer's participation constraint, hence easier to solve the bank's problem with a liquid liability only. That is, when μ_I increases, $\bar{U}$ decreases and the pool of assets is more likely to be structurally illiquid.

4 An implementation with standard debt

This section describes how we can implement the solution to the banker's problem with two types of transactions. The first transaction is a loan contract backed by risky trees, and the second is an outright purchase of safer trees. We argue that the balance sheet emerging from this implementation resembles those that we see in practice.

Recall that the solution to the banker's problem involves the transfer of trees to the banker in exchange for a mix of liquid and illiquid securities, fully backed by trees on the asset side of the banker's balance sheet. These securities are state-contingent. The implementation must replicate those state-contingent payoffs. We focus on an implementation with realistic debt-like instruments, loans and bank deposits. In this case, state-contingency arises because the banker defaults in the low state.

4.1 Bank liabilities and assets

In this implementation, banks issue two debt instruments: checking deposits and savings deposits. Both deposits are standard-debt contracts. Checking deposits are promises to pay the ultimate holders one unit of consumption at $t = 2$. They are means of payments producers use to purchase the labor of workers. Checking deposits mimic the payoffs of the liquid liability, $D(\omega)$, in the primitive contract because the bank can default and pay less than one consumption good per checking deposit. We use one unit of checking deposit as the unit of account.

Savings deposits are similar to checking deposits except for two differences. First, savings deposits do not circulate because the are an entitlement to a specific producer. Second, they are junior to checking deposits. Savings deposits mimic the payoffs of the illiquid liability, $I(\omega)$, in the primitive banker problem.

In addition to issuing both types of deposits, the banker holds equity, the most junior claim. The equity payoff mimics the consumption of the banker, $c(\omega)$, in the primitive contract. Finally, the banker is insolvent if the number of outstanding deposits at $t = 2$ is below the value of goods it collects by $t = 2$.

Finally, Bankers hold two types of securities on the asset side of their balance sheet. They either hold trees directly or hold loan contracts collateralized by trees. We now describe the transactions that lead to those balance sheets.

4.2 Transactions

Collateralized loans and outright purchases. We first describe the collateralized loans and then turn to outright purchases. A collateralized loan for tree j specifies the following:

- Loan size. The loan size, denoted by L_j, is the number of checking deposits that the banker gives the producer. Note that L_j represents the amount of consumption goods promised by the banker.

- Face Value. The face value for the loan, denoted by F_j, is the amount of consumption goods that the producer promises to repay to the banker at $t = 2$.

- Default and Liquidation. The producer has the option to default in any state. A liquidation procedure allows the bank to seize the proceeds of the tree if the bank is not repaid F_j. In case of default, only the collateral asset can be seized by the bank.

From $\{L_j, F_j\}$ we can compute additional characteristics of the loan. One is the interest $i_j \equiv F_j/L_j - 1$. Another characteristic is the value of the collateral in the high state in excess of the face value of debt:

$$H_j \equiv R_j(\omega_h) - F_j.$$

The second type of transaction is an outright purchase. Outright purchases are characterized by a price p_j that determines a number of checking deposits given to the producer in exchange for tree of type j. Checking deposits can immediately be transferred one-for-one into time deposits account that earns an interest i. This interest rate scales the amount of promises by $1+i$ from $t=1$ to $t=2$.[4]

Billateral trade. At $t=1$ producers trade with workers. Workers transfer one unit of hours in exchange for a wage w.

Payoffs, settlements and consumptions. After the realization of the state, producers decide to repay their debts or default. If, after collecting fruits obtained from debt repayments and the liquidation of collateral, the bank is insolvent, the bank defaults on its deposits respecting their seniority status.

Trees used as collateral. In the implementation, trees with risk above a certain cutoff are used as collateral. We introduce some expressions to economize notations. Let $\rho \equiv \mathbb{E}[r(\tilde{\omega})]$ denote the expected output per hour worked. Also, let $\psi \equiv (r(\omega_h)-1)/(1-r(\omega_h)\pi(\omega_h))$, the threshold level of risk that makes a security illiquid. In addition, let η be the fraction of the representative producer's outside option delivered with illiquid securities in the primitive problem $\eta \equiv \bar{I}(\omega_h)/\bar{U}$. Finally, let $\chi_l \equiv \eta/(1-\eta\pi(\omega_h))$ and $\chi_i \equiv \eta/(1-\eta\rho\pi(\omega_h))$. With this notation, we can define the risk cutoff $\chi \equiv \min\{\chi_i, \max\{\chi_l, \psi\}\}$. Specifically, trees whose high-state payoff satisfy:

$$R_j(\omega_h) \geq (1+\chi)R_j(\omega_\ell), \tag{12}$$

are used as collateral. Next, we now conjecture the terms of all transactions.

Conjectured Transaction Terms. We present a conjecture for the terms at which all trades occur under this implementation. The wage is:

$$w = \frac{\bar{D}(\omega_h)}{\mathbb{E}\left[\bar{D}(\tilde{\omega})\right]}.$$

The loan size for the tree j is $L_j = D_j(\omega_h) = \alpha_j \bar{D}(\omega_h)$. The face value of debt is such that $H_j = I_j(\omega_h) = \alpha_j \bar{I}(\omega_h)$ —we deduce F_j via $F_j = R_j(\omega_h) - H_j$. For outright purchases,

[4]Bear in mind that i_j is the interest on a loan, a lending rate, and i the interest on a time deposit, a borrowing rate.

the price is

$$p_j = w\frac{\mathbb{U}\left[R_j\left(\tilde{\omega}\right)\right]}{\rho}.$$

Finally, the interest on the savings accounts is

$$i = \frac{\rho\mathbb{E}\left[\bar{D}\left(\tilde{\omega}\right)\right]}{\pi\left(\omega_h\right)\bar{D}\left(\omega_h\right)} - 1.$$

Under these conjectured terms, we obtain the following proposition:

Proposition 8. *The allocation in the banker's primitive problem can be implemented by offers of collateralized loan contracts, outright purchases, and time-deposit accounts under the conjectured terms. In the implementation, trees that satisfy condition 12 are used for collateralized loans and the rest are purchased directly by the banker. All producers default on their loans only when $\omega = \omega_\ell$ and the banker is insolvent only when $\omega = \omega_\ell$.*

Some features of this implementation are worth noting. First, for any measure of trees M_j held by producers, there is a cutoff of risk such that the safest trees are brought to the bank as an outright purchase. Riskier trees are used as collateral. Second, liquid trees may also be used as collateral if the aggregate tree is sufficiently risky. Third, all collateralized loans feature the same spread between the highest state payoff and the face value debt within the groups of liquid and illiquid securities.

The intuition for outright purchases is clear. Savings deposits are just like the illiquid securities in the primitive contract. The intuition for collateralized loans is that we can interpret a collateralized loans as an exchange of a security in exchange for a deposits plus an option contract. For the producer, the option to default or repay and retain the asset the same as an option that allows him to to repurchase the tree at a strike price F_j. This option is illiquid, because only the producer is able to exercise it. Naturally, an alternative implementation would be to label this option as a repurchase agreement.

5 Asset choice

In this section we study the optimal composition of assets, on the left-side of the banker's balance sheet.

5.1 The banker's value

We start by analyzing $V(\mu)$, the maximum value of the banker given assets μ. This value can be interpreted as a liquidity creation function: it measures the value created by the banker when he uses a collection μ of assets as input into the creation of optimal liquid and illiquid liabilities. Much of our insights about asset choice ultimately rely on understanding the basic properties of this function, in particular, which assets are substitute or complementary inputs for liquidity creation.

To derive $V(\mu)$, recall that the participation constraint of the representative producer is always binding:

$$\mathbb{U}\left[D^\star(\tilde{\omega})\right] + \mathbb{E}\left[I^\star(\tilde{\omega})\right] = \bar{U} \Leftrightarrow \mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D^\star(\tilde{\omega})\right] + \mathbb{E}\left[I^\star(\tilde{\omega})\right] = \bar{U},$$

where we used that, in an optimal liability design, $D^\star(\omega_h) \geq D^\star(\omega_\ell)$. Solving for $\mathbb{E}\left[D^\star(\omega)\right]$ and substituting in the banker's profit, we obtain:

$$\begin{aligned} V(\mu) &= \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \mathbb{E}\left[D^\star(\tilde{\omega})\right] - \mathbb{E}\left[I^\star(\tilde{\omega})\right] \\ &= \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \frac{\bar{U}}{\mathbb{E}\left[r(\tilde{\omega})\right]} - \mathbb{E}\left[I^\star(\tilde{\omega})\right]\left(1 - \frac{1}{\mathbb{E}\left[r(\tilde{\omega})\right]}\right). \end{aligned}$$

The formula shows that an illiquid liability, $I^\star(\omega) > 0$, always reduces the value of the banker. This is expected since the banker only adds value by creating liquid liabilities.

When the asset pool is either liquid or structurally illiquid, then the banker optimal liability design is such that $I^\star(\omega) = 0$. Using the above formula, it follows that the value of the banker is:

$$V(\mu) = \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \frac{\bar{U}}{\mathbb{E}\left[r(\tilde{\omega})\right]} = \mu_0 + \left(1 - \frac{1}{\mathbb{E}\left[r(\tilde{\omega})\right]}\right)\sum_{j \in I} \mu_j. \tag{13}$$

where the second equality follows after substituting in the formula (7) for the aggregate outside option, $\bar{U}$.

In contrast, when the pool of asset is fundamentally illiquid, the deposit maximizing liability is too small to meet the producer's outside option. As a result the banker must create an illiquid liability as well, that is, $I^\star(\omega) > 0$. To calculate the exact formula for $V(\mu)$, we use that the optimal liquid liability is the solution of the deposit maximization problem: $D^\star(\omega) = D^\star_{\max}(\omega)$. The value of the banker is, then

$$\begin{aligned} V(\mu) &= \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \mathbb{E}\left[D^\star_{\max}(\tilde{\omega})\right] - \mathbb{E}\left[I^\star(\tilde{\omega})\right] \\ &= \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \mathbb{E}\left[D^\star_{\max}(\tilde{\omega})\right] - \bar{U} + \mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right], \end{aligned} \tag{14}$$

where we used the participation constraint of the representative producer to state that $\mathbb{E}\left[I^\star(\tilde{\omega})\right] = \bar{U} - \mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right]$. Using the formula for $D^\star_{\max}(\omega)$ shown in Proposition 6, we obtain:

Proposition 9. *Suppose that $R_j(\omega_h) \geq R_j(\omega_\ell)$ for all $j \in J$ and normalize $\mathbb{E}\left[R_j(\tilde{\omega})\right] = 1$ for all $j \in J$. Let L denote the subset of liquid trees, and I the subset of illiquid trees. Then the value of the banker is $V(\mu) = \min\left\{V^{LS}(\mu), V^F(\mu)\right\}$, where*

$$\begin{aligned} V^{LS}(\mu) &= \mu_0 + \left(1 - \frac{1}{\mathbb{E}\left[r(\tilde{\omega})\right]}\right)\sum_{j\in I}\mu_j \\ V^F(\mu) &= \mu_0 + \left(\mathbb{E}\left[r(\tilde{\omega})\right] - 1\right)\left(-\sum_{j\in L}\mu_j + \frac{1-\pi(\omega_h)}{1 - r(\omega_h)\pi(\omega_h)}\left[\mu_0 + \sum_{j\in J} R_j(\omega_\ell)\right]\right) \end{aligned}$$

In the proposition $V^{LS}(\mu)$ is (13), the value of the banker when the asset pool is either liquid or structurally illiquid. Likewise, $V^F(\mu)$ is (14), the value of the banker when the asset pool is fundamentally illiquid. We also learn from the proposition that the value of the banker can be expressed as the minimum between $V^{LS}(\mu)$ and $V^F(\mu)$. This formulation makes it clear that the value of the banker is indeed increasing, concave, and homogenous of degree one, as expected from Lemma 2.

Complementarity between assets in a special case. We first observe that some assets are complementary inputs for liquidity creation. To see this point clearly, consider the following illustrative example. Suppose that there are only two types of trees, $J = \{1, 2\}$, and that the banker has no net worth. Tree $j = 1$ is risk free, i.e., $R_1(\omega_h) = R_1(\omega_\ell) = 1$, and tree $j = 2$ is maximally risky, i.e., $R_2(\omega_\ell) = 0$ and $R_2(\omega_h) = 1/\pi(\omega_h)$. Then, direct calculation shows that the value of the banker is

$$V(\mu_1, \mu_2) = \left(\mathbb{E}\left[r(\tilde{\omega})\right] - 1\right)\min\left\{\mu_1 \frac{\pi(\omega_h)\left[r(\omega_h) - 1\right]}{1 - r(\omega_h)\pi(\omega_h)}, \frac{\mu_2}{\mathbb{E}\left[r(\tilde{\omega})\right]},\right\}. \tag{15}$$

The expression reveals a clear complementarity. The reason is intuitive. If there are only liquid trees, i.e., $\mu_1 > 0$ and $\mu_2 = 0$, then the banker creates no value, $V(\mu_1, \mu_2) = 0$. This is because producers who hold liquid trees can just go ahead and trade bilaterally with their

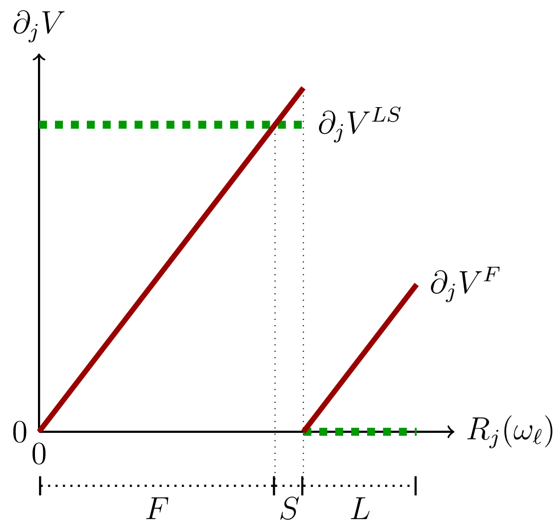

Figure 6: The marginal value of trees, in the LS vs. F regions. On the x axis, "F" indicates the location of fundamentally illiquid trees, "S" the location of structurally illiquid trees, and "L" the location of liquid trees.

liquid asset on their own. If there are only illiquid trees, $\mu_1 = 0$ and $\mu_2 > 0$, then the banker has no value either. The reason is that a maximally risky tree, such that $R_j(\omega_\ell) = 0$, cannot be productively restructured: after carving an equity tranche and reducing the payoff in the high state, the tree always remains maximally risky and illiquid. Therefore, in this special case, the complementarity is extreme: the only way for the banker to add value is to combine together liquid and illiquid trees on the left side of his balance sheet.

Complementarity and substitutability in the general case. We turn to the general case. Consider the partial derivative of $V^{LS}(\mu)$ with respect to each μ_j:

$$\partial_j V^{LS} \equiv \begin{cases} 1 & \text{if } j = 0 \\ 0 & \text{if } j \in L \\ 1 - \frac{1}{\mathbb{E}[r(\tilde{\omega})]} & \text{if } j \in I, \end{cases}$$

and, likewise, the partial derivatives of $V^F(\mu)$ with respect to each μ_j:

$$\partial_j V^F \equiv \begin{cases} 1 + \left(\mathbb{E}\left[r(\tilde{\omega})\right] - 1\right) \frac{\pi(\omega_h)[r(\omega_h)-1]}{1-r(\omega_h)\pi(\omega_h)} & \text{if } j = 0 \\ \left(\mathbb{E}\left[r(\tilde{\omega})\right] - 1\right) \left(-1 + \frac{1-\pi(\omega_h)}{1-r(\omega_h)\pi(\omega_h)} R_j(\omega_\ell)\right) & \text{if } j \in L \\ \left(\mathbb{E}\left[r(\tilde{\omega})\right] - 1\right) \frac{1-\pi(\omega_h)}{1-r(\omega_h)\pi(\omega_h)} R_j(\omega_\ell) & \text{if } j \in I. \end{cases}$$

For the next Lemma below, we call a tree *structurally illiquid* if it is illiquid, but its associated deposit maximizing liability,[5] $D^{\star}_{\max,j}(\omega)$, is such that $\mathbb{U}\left[D^{\star}_{\max,j}(\tilde{\omega})\right] > \mathbb{U}\left[R_j(\tilde{\omega})\right]$. Likewise, we call a tree *fundamentally illiquid* if it is illiquid, but its associated deposit maximizing liability is such that $\mathbb{U}\left[D_{\max,j}(\tilde{\omega})\right] < \mathbb{U}\left[R_j(\tilde{\omega})\right]$. These are the same concepts we introduced earlier to describe asset pools, but now applied to single tree. With this in mind, we obtain by direct computations:

Lemma 10. *In the cross-section of trees:*

- *liquid and structurally illiquid trees are such that* $\partial_j V^F > \partial_j V^{LS}$*;*
- *fundamentally illiquid trees are such that* $\partial_j V^F < \partial_j V^{LS}$*.*

The Lemma partitions the universe of trees in two broad classes: liquid and structurally illiquid ("LS"), and fundamentally illiquid ("F"). It suggests that assets will be substitutes within each class, and complement across classes. For example, as one increases the amount of liquid or structurally illiquid trees, the pool of assets moves from the F to the LS region. As a result, the marginal value of all LS trees drops. At the same time, the marginal value of all F trees goes up. We formalize this intuition in the next Section, where we study the banker's optimal asset choice problem.

5.2 Optimal asset choice subject to intermediation costs

To make the asset choice problem not trivial, we assume that the banker incurs costs to attract producers. This could represent for example, the cost of establishing a branch in a particular city, the cost of acquiring expertise in a particular asset class, marketing or search costs for customers, or costs incurred in the process of screening the quality of customers' assets.

Assume that there is an exogenously given measure M_j of type j producers in the economy at large. But reaching producers is costly, and some producers are more costly to reach than others. Formally, the cumulative measure of producers of type j who can be reached at cost less than c is $M_j\Phi(c)$, for some continuous and strictly increasing function with full support, $[0,\infty)$. Now suppose that the banker chooses to attract $\mu_j \in [0, M_j]$ producers of type j. It is obvious that the banker will attract low cost producers first. More precisely, for any given $\mu_j \in [0, M_k]$, it is optimal for the banker to attract all producers below percentile $\frac{\mu_j}{M_j}$ in the cost distribution, and none of the producers above that percentile. Hence, the cumulative

[5] The deposit maximizing liability for asset j has the payoff shown in equation (11) of Proposition 6, with $R_j(\omega)$ in place of $\bar{R}(\omega)$.

cost that the banker must incur to reach a collection $\{\mu_j\}_{j\in J}$ of producers is:

$$\Gamma(\mu) = \sum_{j\in J} M_j \int_0^{\frac{\mu_j}{M_j}} \Phi^{-1}(x)\, dx.$$

In the integrand $\Phi^{-1}(x)$ is the cost of attracting a producer located at percentile $x \in [0,1]$ in the cost distribution. With this definition in mind, we define the banker's *asset choice problem* to be:

$$\max V(\mu) - \Gamma(\mu),$$

with respect to $\mu_j \in [0, M_j]$ for all $j \in J$.

Solving the optimal asset choice problem. The optimal asset choice problem has a strictly concave objective and a convex and bounded constraint set. Hence, it has a unique solution, which we denote by $\mu^\star$. If $V^{LS}(\mu^\star) < V^F(\mu^\star)$, then the objective is differentiable at the optimum. Keeping in mind that the cost distribution, $\Phi(c)$, has full support, we obtain the first-order condition:

$$\partial_j V^{LS} - \Phi^{-1}\left(\frac{\mu_j^\star}{M_j}\right) \Rightarrow \mu_j^\star = \mu_j^{LS}, \text{ where } \mu_j^{LS} \equiv M_j \Phi\left(\partial_j V^{LS}\right).$$

That is, the optimal measure of type j producers is simply the total measure, M_j, multiplied by the fraction of type j producers with cost below the marginal value, $\partial_j V^{LS}$. Notice that, since the first-order conditions are sufficient, the converse is true: if $V^{LS}(\mu^{LS}) < V^F(\mu^{LS})$, then $\mu^\star = \mu^{LS}$. A similar reasoning applies in the F region. If $V^F(\mu^\star) < V^{LS}(\mu^\star)$, then

$$\mu_j^\star = \mu_j^F, \text{ where } \mu_j^F \equiv M_j \Phi\left(\partial_j V^F\right).$$

But there is a third case. It can be that:

$$V^{LS}(\mu^{LS}) \geq V^F(\mu^{LS}) \text{ and } V^F(\mu^F) \geq V^{LS}(\mu^F). \tag{16}$$

This is, in fact, a natural case. For example, it always arises in the two-type example of the previous section. Recall that, in this example, there are two assets, $J = \{1,2\}$, assets of type $j = 1$ are risk-free, $R_1(\omega_\ell) = R_1(\omega_h) = 1$, while assets of type $j = 2$ are maximally risky, $R_2(\omega_\ell) = 0$ and $R_2(\omega_h) = 1/\pi(\omega_h)$. Then, the optimum cannot lie in the interior of the LS region. If it did, then the marginal value of the liquid tree $j = 1$ would be $\partial_1 V^{LS} = 0$ and that of the maximally illiquid tree would be $\partial_2 V^{LS} > 0$, implying that $\mu_1^\star = 0$ and $\mu_2^\star > 0$.

This is a contradiction because it implies that the asset pool $\mu^\star$ is in fact fundamentally illiquid. The same reasoning shows that the optimum cannot lie in the interior of the F region, because in this case $\partial_1 V^F > 0$ and $\partial_2 V^F = 0$. Hence, the optimum has to be at the boundary, i.e., $V^{LS}(\mu^\star) = V^F(\mu^\star)$.

The general characterization of the optimum is:

Proposition 11. *Suppose that $R_j(\omega_h) \geq R_j(\omega_\ell)$ for all $j \in J$. Then, the solution of the optimal asset choice problem is*

$$\mu_j^\star = M_j \Phi \left[\lambda^\star \partial_j V^F + (1-\lambda^\star) \partial_j V^{LS}\right],$$

for any $\lambda^\star$ such that:

- $\Delta(\lambda) > 0$ *and* $\lambda = 0$*;*
- $\Delta(\lambda) = 0$ *and* $\lambda \in [0,1]$*;*
- $\Delta(\lambda) < 0$ *and* $\lambda = 1$*,*

where $\Delta(\lambda) \equiv \left(\partial_0 V^F - \partial_0 V^{LS}\right) \mu_0 + \sum_{j \in J} \left(\partial_j V^F - \partial_j V^{LS}\right) M_j \Phi \left[\lambda \partial_j V^F + (1-\lambda) \partial_j V^{LS}\right]$.

In the first case, $\Delta(0) > 0$, the optimal asset pool lies in the interior of the LS region. In the second case, $\Delta(\lambda^\star) = 0$ and $\lambda^\star \in [0,1]$, the optimal asset pool lies at the boundary between the LS and the F region. Finally, in the third case, $\Delta(1) < 0$, the optimal asset pool lies in the interior of the F region.

The Proposition shows that the measure of type j assets is determined by a convex combination of the marginal values $\partial_j V^{LS}$ and $\partial_j V^F$, with a convex weight which is the same for all assets. It is obvious that the convex weight is identical for all assets in the interior of the LS and F region. To see why this should be the case at the boundary between the LS and the F region, consider the choice of an optimal asset pool at the boundary between the two regions. Formally, the banker must be maximizing $V^{LS}(\mu) - \Gamma(\mu)$, subject to making a choice such that $V^F(\mu) = V^{LS}(\mu)$. The constraint of making choices within the boundary is akin to a budget constraint, in which differences in marginal values, $\partial_j V^F - \partial_j V^{LS}$ play the role of prices. In an optimum, the marginal value of increasing the measure of type-j asset must be equal to the price, multiplied by the Lagrange multiplier on the constraint:

$$\partial_j V^{LS} - \Phi^{-1}\left(\frac{\mu_j^\star}{M_j}\right) = \lambda \left(\partial_j V^F - \partial_j V^{LS}\right),$$

which can be rearranged into the formula of the Proposition.

The main implications of the Proposition are the following. First, it is never optimal to choose a balance sheet that is only made of liquid assets: liquid assets are always held together with illiquid assets. Second, as long as $\lambda^\star > 0$, it is optimal to bring some liquid assets to complement holding of illiquid assets. Third, liquid assets have lower marginal value to the banker because holders of these assets have a high outside option. If the cost distribution is the same for all assets, our model thus predicts that banks will hold a small fraction of the total supply of liquid assets, and a larger fraction of the total supply of illiquid assets. One may argue that this corresponds to the observation that banks do hold some liquid and tradeable assets, such as treasuries, corporate bonds, municipal bonds, but in much smaller proportion than illiquid and non-tradeable securities, such as loan to households and businesses. Fourth, within each asset class, liquid and illiquid, our model predicts that banks will hold safer assets in larger proportion than riskier assets. That is, within liquid and tradeable assets, banks will hold more fixed income than equity. Within illiquid and non-tradeable assets, banks will hold more mortgages and loans than equity shares in non-publicly traded corporations.

Complementarity and substitutability in liquidity creation. To analyze complementarity and substitutability between assets, one must focus on parameters such that the optimal asset choice lies at the boundary between the LS and the F regions.[6] This leads to:

Proposition 12. *Suppose that $R_j(\omega_h) \geq R_j(\omega_\ell)$, $\partial_j V^F - \partial_j V^{LS} \neq 0$ for all $j \in J$ and that the optimal asset choice is associated with some $\lambda^\star \in (0,1)$. Consider a marginal increases in the supply of the asset, M_j. Then:*

- *<u>for $k = j$</u>: $\mu_k^\star$ increases;*
- *<u>for $k \neq j$, if $j \in LS$</u>: $\mu_k^\star$ decreases for $k \in LS$, and increases for $k \in F$;*
- *<u>for $k \neq j$, if $j \in F$</u>: $\mu_k^\star$ increases for $k \in LS$ and decreases for $k \in F$.*

Consider increase the supply M_j of some asset j. Then, given our specification of the cost function, this amounts to reduce the cost of attracting producers holding type j assets. Quite intuitively, the Proposition shows that the banker now finds it optimal to attract more producers holding type j trees. Whether this decreases the measure of type $k \neq j$ trees depends on the asset class, LS or F. If $j \in LS$, then the banker will attract less of other

[6] Within the interior of the LS and the F regions, when $V^{LS}(\mu^\star) \neq V^F(\mu^\star)$, complementarities between assets are irrelevant. This is because the objective function is locally separable between assets. As a result, optimal asset choices are independent: for example, changing the supply M_j of asset j only changes the the optimal choice for asset j, but nothing else.

LS trees, and more of F trees. The opposite is true if $j \in F$. This shows that trees are substitute within each asset classes, LS and F, and complement between.

Hence, one prediction of our model is that, a reduction in the supply of very safe assets (Treasuries) will have two effects. First, banks will increase their holding of riskier tradeable and liquid assets: for instance, they will increase their holdings of AAA Bonds or Municipal Bonds. Second, banks will reduce their holding of non-tradeable and illiquid assets: it will make less loans to households and businesses.

6 Conclusion

In conclusion, we discuss several potential applications of our theory. In each application, there is a tension between the liquidity creation process and some agency friction.

Agency Frictions between Banks and the Public. A critical aspect of our model is that producers have information about the assets on the banker's balance sheet. That is a strong assumption. A natural extension of our theory would allow the banker to deceive workers about their choice of assets. What mechanisms could be employed to guarantee incentives and what are the costs for liquidity creation?

Agency Frictions in the Banking System. An important institutional aspect of deposits left out of this paper is that they are joint liability. As an example, consider a business that obtains a loan from Citbank (Citi) and to buy supplies from a supplier with an account in J.P. Morgan Chase (Chase). When the transaction takes place, there is a transfer of deposits from Citi to Chase. This means that Chase has a liability against the supplier, although it was Citi who made a loan. Chase in turn has a claim against Citi registered clearing system. The takeaway observation is that Citi may issue a deposit that at least for a moment, is a claim on Chase.

In our model, there only a single bank, so there are no cross-bank claims. Imagine now that there are multiple banks who have access to different customer bases. For example Wells Fargo is better at making mortgage loans on the West Coast, and Citi Bank on the East Coast. Then, according to the model, banks will find it optimal to pool their assets to improve liquidity creation. This is also a force for joint-liability, which is a form of pooling. However, a joint-liability system is naturally prone to moral hazard: a bank could potentially issue deposits to acquire bad assets, but deposits may become a liability of the banking system. In practice, banks clear those positions daily, transferring reserves at a central bank, or interbank loans. Cavalcanti, Erosa, and Temzelides (2005) and Cavalcanti and Wallace (1999) argue that this is a disciplining device to cope with moral hazard. We would like to use this model to study the design of a clearing system that trades-off liquidity creation with the provision of incentives.

Financial Regulation. Policies such as capital requirements, reserve requirements, central bank discounts, deposit insurance, last-resort lending, separation of banking activities, etc., are all about regulating the balance sheet of banks. Good policy prescriptions identify an externality that cannot be corrected by markets. Extensions to our model should speak to those externalities and the best way to correct them. We leave this for later work.

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Appendix

A Detail of Figures 1 to 3

The data on asset and liability composition corresponds to US Chartered Depositary Institutions presented in Table L.110 of the Flow of Funds Tables. Figure1 is constructed as follows. We use several series for liabilities in Table L.110 to construct the 6 series of in Figure 1. For the category Small and Checkable Deposits we use the the sum of the data series checkable deposits and small time deposits. For Large Time Deposits we use the series for large time deposits. For REPO and Interbank we use the the sum of the series net interbank transactions, federal funds, and security repurchase agreements (net). For the category Bonds and OMP we use the sum of the series Corporate Bonds, Foreign Bonds and Open-Market Paper. The category GSE advances is the series for FHLB advances and and Sallie Mae loans. The category Equity is the sum of total financial assets minus total financial liabilities plus equity investment by bank holding companies (subcategory of miscellaneous liabilities) minus taxes payable minus unidentified miscellaneous liabilities.

Figure 2 presents seven categories. The category for Total Loans corresponds to the series for total loans. Cash and reserves is the sum of the series of cash assets, Fed Funds and reverse RP's with banks and other assets. Treasuries is the sum of the series treasury and agency securities. GSE-Backed Securities is the series for GSE-backed securities. Municipal+Int Bonds is the series for municipal bonds and foreign issued bonds. Corporate bonds is the series for corporate bonds. Equity and Mutual funds is the sum of the series equity shares and mutual fund shares.

Figure 3 presents seven asset classes. Total outstanding amounts and holdings by depositary institutions of each class is found in a different table of the flow of funds. We selected the following categories that are prevalent among the asset holdings of US Depositary Institutions. Data for the Commercial Paper category is found in the table for Open Market paper, table L.209. The category Treasury Bills corresponds to the series on Treasury securities from table L.210. The category GSE-backed is the series for securities Agency- and GSE-backed securities, table L.211. The category Muni in the figure corresponds to the table for Municipal Bonds, table L.212. The category Corporate Bonds is the series Table L.213 in the figure. The category for Total Loans corresponds to the series for total loans. Cash and reserves is the sum of the series of cash assets, Fed Funds, and reverse RP's with banks and other assets. Treasuries is the sum of the series treasury and agency securities. GSE-Backed Securities is the series for GSE-backed securities. The category Municipal+Int Bonds is the series for municipal bonds and foreign issued bonds. Corporate bonds is the series for corporate and foreign bonds, table L.213. The category Corporate securities corresponds to the series in table L.223. For the category Mortgages we use the series found in table L.217 corresponding to total mortgages. For the category Non-Mortgage loans, we take the sum of total loans in table L.214 and subtract the entries in table L.217 to get series for non-mortgage loans in outstanding amounts and held by depositary institutions.

B Omitted proofs

B.1 Proof of Proposition 1

B.1.1 Optimal bilateral trades

We first solve the optimization problem (3). In the proposition that follows, a bilateral trade is said to be separating if $(n(\omega_\ell), q(\omega_\ell)) \neq (n(\omega_h), q(\omega_h))$, and pooling otherwise.

Proposition B1. *In the optimization problem (3):*

- *If $S(\omega_h) < S(\omega_\ell)$: all optimal bilateral trades are separating, with $n(\omega_h) = 1$, $q(\omega_h) = S(\omega_h)$, and $q(\omega_\ell) = n(\omega_\ell)S(\omega_\ell)$;*
- *If $S(\omega_h) = S(\omega_\ell)$: there exists optimal bilateral trades which are pooling or separating. The optimal separating trades are such that $n(\omega_h) = 1$, $q(\omega_h) = S(\omega_h)$, and $q(\omega_\ell) = n(\omega_\ell)S(\omega_\ell)$. The optimal pooling trades are such that $n = 1$ and $q = \mathbb{E}[S(\omega)]$;*
- *If $S(\omega_\ell) < S(\omega_h) \leq r(\omega_h)\mathbb{E}[S(\tilde{\omega})]$: then the unique optimal bilateral trade is pooling, with $n(\omega_h) = n(\omega_\ell) = 1$, and $q(\omega_h) = q(\omega_\ell) = \mathbb{E}[S(\tilde{\omega})]$;*
- *If $r(\omega_h)\mathbb{E}[S(\tilde{\omega})] < S(\omega_h)$: all optimal bilateral trades are separating, such that $q(\omega_h) = n(\omega_h) = 0$, and $q(\omega_\ell) = n(\omega_\ell)S(\omega_\ell)$.*

Optimal pooling trades. We first restrict attention to trades such that $q(\omega_h) = q(\omega_\ell) = q$, $n(\omega_h) = n(\omega_\ell) = n$. The incentive compatibility constraints in this case hold trivially. The optimization problem becomes

$$\begin{aligned} &\max q \\ &s.t. \quad q \geq 0 \text{ and } 0 \leq n \leq 1 \\ &\forall\omega \quad qr(\omega) + (1-n)S(\omega) \geq S(\omega) \\ &\qquad n\mathbb{E}[S(\tilde{\omega})] \geq S. \end{aligned}$$

The second constraint simplifies to $qr(\omega) \geq nS(\omega)$. Clearly it is best to pick $q = n\mathbb{E}[S]$. Hence, the problem simplifies to:

$$\begin{aligned} &\max n \\ &s.t. \quad 0 \leq n \leq 1 \\ &\forall\omega \quad r(\omega)\mathbb{E}[S(\tilde{\omega})] \geq S(\omega) \text{ if } n > 0. \end{aligned}$$

The constraints $r(\omega)\mathbb{E}[S(\tilde{\omega})] \geq S(\omega)$ simplify to:

$$S(\omega_\ell) \leq S(\omega_h) \leq \frac{r(\omega_h)[1-\pi(\omega_h)]}{1-r(\omega_h)\pi(\omega_h)} S(\omega_\ell). \tag{B.1}$$

We thus obtain two cases:

- If $S(\omega)$ is such that (B.1) holds, then the optimal pooling trade is $n = 1$ and $q = \mathbb{E}[S(\tilde{\omega})]$.
- If $S(\omega)$ is such that (B.1) do not hold, then the only feasible and hence optimal pooling trade is $n = q = 0$.

Optimal separating trades. Consider the maximization problem restricted to separating trades:

$$\begin{aligned}
& \max q(\omega_h) \\
w.r.t. \quad & q(\omega) \text{ and } n(\omega), \text{ and } s.t. \\
\forall \omega, \quad & q(\omega) \geq 0 \\
\forall \omega, \quad & 0 \leq n(\omega) \leq 1 \\
\forall \omega, \quad & q(\omega)r(\omega) + [1 - n(\omega)]\, S(\omega) \geq S(\omega) \\
\forall \omega, \quad & n(\omega)S(\omega) \geq q(\omega) \\
\forall (\omega, \tilde{\omega}), \quad & q(\omega)r(\omega) + [1 - n(\omega)]\, S(\omega) \geq q(\tilde{\omega})r(\omega) + [1 - n(\tilde{\omega})]\, S(\omega) \\
& (q(\omega_h), n(\omega_h)) \neq (q(\omega_\ell), n(\omega_\ell)).
\end{aligned}$$

Since $r(\omega_\ell) = 1$, the individual rationality constraints for the producer and the worker in the low state imply that both $q(\omega_\ell) \geq n(\omega_\ell)S(\omega_\ell)$ and $n(\omega_\ell)S(\omega_\ell) \geq S(\omega_\ell)$, so that

$$q(\omega_\ell) = n(\omega_\ell)S(\omega_\ell). \tag{B.2}$$

Similarly, the individual rationality constraints or the producer and the worker in the high state are equivalent to

$$q(\omega_h) \leq n(\omega_h)S(\omega_h) \leq q(\omega_h)r(\omega_h). \tag{B.3}$$

Using (B.2), the incentive constraint in the high and low state can be rearranged as:

$$q(\omega_h)r(\omega_h) \geq n(\omega_h)S(\omega_h) + n(\omega_\ell)\left[r(\omega_h)S(\omega_\ell) - S(\omega_h)\right] \tag{B.4}$$

$$n(\omega_h)S(\omega_\ell) \geq q(\omega_h). \tag{B.5}$$

We consider two cases:

- If $r(\omega_h)S(\omega_\ell) - S(\omega_h) \geq 0$, then we can let $n(\omega_\ell) = 0$ without loss of generality: formally, any trade that is feasible with $n(\omega_\ell) > 0$ remains feasible after setting $n(\omega_\ell) = q(\omega_\ell) = 0$, and achieves the same value. Then, we are left with the following constraints:

  $$\begin{aligned}
  q(\omega_h) \leq n(\omega_h)S(\omega_h) &\leq r(\omega_h)q(\omega_h) \\
  q(\omega_h) &\leq n(\omega_h)S(\omega_\ell) \\
  (q(\omega_h), n(\omega_h)) &\neq (0, 0).
  \end{aligned}$$

  The best trade satisfying the constraints $q(\omega_h) \leq n(\omega_h)S(\omega_h)$ and $q(\omega_h) \leq n(\omega_h)S(\omega_\ell)$ is clearly $n(\omega_h) = 1$ and $q(\omega_h) = \min\{S(\omega_h), S(\omega_\ell)\}$. Since $r(\omega_h)S(\omega_\ell) \geq S(\omega_h)$, this trade also satisfies the constraint $q(\omega_h)r(\omega_h) \geq n(\omega_h)S(\omega_h)$. Finally, since $n(\omega_h) = 1$, it is clear that $(q(\omega_h), n(\omega_h)) \neq (0, 0)$. Taken together, these argument show that this bilateral trade must be an optimum.

- If $r(\omega_h)S(\omega_\ell) - S(\omega_h) < 0$, then the individual rationality constraint of the producer in the high state, $r(\omega_h)q(\omega_h) \geq n(\omega_h)S(\omega_h)$, implies that incentive constraint in the high state. Hence, we can ignore the incentive constraint in the high state and we are left with the same constraints as for the previous bullet point. However, they now imply that $q(\omega_h) \leq$

$n(\omega_h)\min\{S(\omega_h), S(\omega_\ell)\}$ and $r(\omega_h)q(\omega_h) \geq n(\omega_h)S(\omega_h)$, so that

$$n(\omega_h)r(\omega_h)\min\{S(\omega_h), S(\omega_\ell)\} \geq n(\omega_h)S(\omega_h).$$

But one sees that the only way this inequality can be satisfied is if $n(\omega_h) = 0$ implying that $q(\omega_h) = 0$.

Taking stock. We now compare the optimal pooling and the separating trades we derived above:

- If $S(\omega_h) < S(\omega_\ell)$. Then there exists no pooling trade that is incentive feasible. The maximization problem is solved by separating trades such that $n(\omega_h) = 1$ and $q(\omega_h) = S(\omega_h)$, $n(\omega_\ell)S(\omega_\ell) = q(\omega_\ell)$, and it achieves the value $S(\omega_h)$.
- If $S(\omega_h) = S(\omega_\ell)$. Then, the optimal pooling trade is such that $n = 1$ and $q = \mathbb{E}[S(\tilde{\omega})]$, while the all optimal separating trades are such that $q(\omega_h) = S(\omega_h)$, $n(\omega_h) = 1$, and $q(\omega_\ell) = n(\omega_\ell)S(\omega_\ell)$. Clearly, all these optimal trades, pooling or separating, achieve the same value.
- If $S(\omega_h) > S(\omega_\ell)$ but $S(\omega_h) \leq r(\omega_h)\mathbb{E}[S(\tilde{\omega})]$. The optimal pooling trade is $n = 1$ and $q = \mathbb{E}[S(\tilde{\omega})]$, and it achieves a value equal to $\mathbb{E}[S(\tilde{\omega})]$. The optimal separating trade achieves the value $n(\omega_h)S(\omega_\ell)$, which is lower.
- If $r(\omega_h)\mathbb{E}[S(\tilde{\omega})] < S(\omega_h)$. Then there exists no incentive feasible pooling trade. Since this inequality implies that $r(\omega_h)S(\omega_\ell) < S(\omega_h)$, we know that the best separating trade is such that $n(\omega_h) = q(\omega_h) = 0$ and $q(\omega_\ell) = n(\omega_\ell)S(\omega_\ell)$, and so it achieves the value of zero.

B.1.2 Implementing an optimal bilateral trade in a PBE

We now establish that an optimal bilateral trade is the outcome of a PBE, in which the producer makes a take-it-or-leave-it offer. The strategy and the beliefs of the players are as follows:

- If $S(\omega_h) < S(\omega_\ell)$: the producer of type $\omega = \omega_\ell$ makes the offer $q(\omega_\ell) = n(\omega_\ell) = 0$, and that the producer of type $\omega = \omega_h$ makes the offer $q(\omega_h) = S(\omega_h)$, $n(\omega_h) = 1$. If $(q, n) = (0, 0)$, then the worker believes that the producer is of type $\omega = \omega_\ell$. If $(q, n) \neq (0, 0)$, then the worker believes that the producer is of type $\omega = \omega_h$. The worker always accepts the offer $(q, n) = (0, 0)$ and he accepts $(q, n) \neq 0$ offer as long as $q \leq S(\omega_\ell)$.
- If $S(\omega_h) \geq S(\omega_\ell)$ and $S(\omega_h) \leq r(\omega_h)\mathbb{E}[S(\tilde{\omega})]$: producers of type $\omega \in \{\omega_\ell, \omega_h\}$ make the same offer, $q = \mathbb{E}[S(\tilde{\omega})]$, and that, for all offers, the worker believes that $\omega = \omega_h$ with the true probability $\pi(\omega_h)$. The worker accepts an offer as long as $q \leq n\mathbb{E}[S(\tilde{\omega})]$.
- If $S(\omega_h) > r(\omega_h)\mathbb{E}[S(\tilde{\omega})]$: the producer of type $\omega = \omega_\ell$ makes the offer $n(\omega_\ell) = S(\omega_\ell)$ and $q(\omega_\ell) = 1$. The producer of type $\omega = \omega_h$ makes the offer $n(\omega_h) = q(\omega_h) = 0$. If $(q, n) = (0, 0)$, the worker believes that the producer is of type $\omega = \omega_h$, and otherwise if $(q, n) \neq (0, 0)$ he believes that the producer is of type $\omega = \omega_\ell$. The worker always accept a $(q, n) = (0, 0)$ offer, and he accepts a $(q, n) \neq 0$ offer as long as $q \leq S(\omega_\ell)$.

One can then directly verify that the strategies of the players are optimal, and that beliefs satisfy Bayes' rule whenever possible.

B.1.3 The ex ante value of the producer

The value of a producer holding $S(\omega)$, before learning the realization of the state ω, is:

$$\pi(\omega_h)\left\{r(\omega_h)q(\omega_h)+\left[1-n(\omega_h)\right]S(\omega_h)\right\}+\pi(\omega_\ell)\left\{q(\omega_\ell)+\left[1-n(\omega_\ell)\right]S(\omega_\ell)\right\}. \tag{B.6}$$

Let us evaluate this expression at any optimal bilateral trade. According to Proposition B1, we have four cases to consider.

- If $S(\omega_\ell) > S(\omega_h)$, then all optimal bilateral trades are separating, with $n(\omega_h) = 1$, $q(\omega_h) = S(\omega_h)$ and $q(\omega_\ell) = n(\omega_\ell)S(\omega_\ell)$. Plugging this into (B.6), we obtain that the ex ante value is equal to $\mathbb{E}\left[r(\tilde{\omega})S(\tilde{\omega})\right]$. Notice that, since $S(\omega_\ell) > S(\omega_h)$, the security payoff is negatively correlated with productivity, $r(\omega)$, implying that:

  $$\mathbb{E}\left[r(\tilde{\omega})S(\tilde{\omega})\right] < \mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[S(\tilde{\omega})\right].$$

- If $S(\omega_\ell) = S(\omega_h)$, then optimal bilateral trades can be separating or pooling. For optimal separating trades, the same calculation as above shows that the ex ante value is $\mathbb{E}\left[r(\tilde{\omega})S(\tilde{\omega})\right]$. For optimal pooling trades, we have $q = \mathbb{E}E\left[S(\tilde{\omega})\right]$ and $n = 1$, which leads to an ex ante value of $\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[S(\tilde{\omega})\right]$. Since $S(\omega_\ell) = S(\omega_h)$, this is the same as for an optimal separating trade.

- If $S(\omega_\ell) < S(\omega_h) \leq r(\omega_h)\mathbb{E}\left[S(\tilde{\omega})\right]$, the optimal bilateral trade is pooling, with $n = 1$ and $q = \mathbb{E}\left[S(\tilde{\omega})\right]$. The ex ante value of the producer is $\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[S(\tilde{\omega})\right]$. Notice that, sine $S(\omega_h) > S(\omega_\ell)$, the security payoff is positively correlated with productivity, $r(\omega)$, implying that:

  $$\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[S(\tilde{\omega})\right] < \mathbb{E}\left[r(\tilde{\omega})S(\tilde{\omega})\right].$$

- If $S(\omega_\ell) < S(\omega_h)$ and $r(\omega_h)\mathbb{E}\left[S(\tilde{\omega})\right] < S(\omega_h)$, then the optimal bilateral trade is separating, with $n(\omega_h) = q(\omega_h) = 0$, and $n(\omega_\ell)S(\omega_\ell) = q(\omega_\ell)$. In that case, the ex ante value is simply $\mathbb{E}\left[S(\tilde{\omega})\right]$, the value of holding on to the security and not trading.

B.2 Proof of Lemma 2

Since $\mathbb{U}\left[D_j(\tilde{\omega})\right]$ is continuous over the set of deposit securities such that $r(\omega_h)\mathbb{E}\left[D_j(\tilde{\omega})\right] \geq D_j(\omega_h)$, the constraint set is closed. It is clearly bounded. It is non empty since it is always feasible for the banker to issue liabilities that allow agents to merely replicate the trades they would have done on their own, by setting $D_j(\omega) = R_j(\omega)$ and $I_j(\omega)$ for liquid trees, while setting $D_j(\omega) = 0$ and $I_j(\omega) = R_j(\omega)$ for illiquid trees. Hence, the bank's problem has a solution.

Next, we turn to the properties of the value function, $V(\mu)$. It is clearly weakly increasing in net worth, since any contract that is feasible with a low net worth is also feasible with a higher net worth. To show that it is weakly increasing in μ_j, $j \in J$, consider first $\mu_j' > \mu_j$ for some security j that is either separating or pooling (i.e., for which there is no lemon problem). Then consider the liquid and illiquid securities:

$$D_j'(\omega) = \frac{\mu_j}{\mu_j'}D_j(\omega) + \left(1 - \frac{\mu_j}{\mu_j'}\right)R_j(\omega) \text{ and } I_j'(\omega) = \frac{\mu_j}{\mu_j'}I_j(\omega),$$

and keep everything else the same. The resource constraint is satisfied by construction. The liquidity constraint is satisfied because both $D_j(\omega)$ and $R_j(\omega)$ satisfy it by construction. The participation constraint is satisfied because it is satisfied for D_j and I_j and because $\mathbb{U}\left[D(\tilde{\omega})\right]$ is concave. Since the value of the objective stays the same, the maximum attainable value must increase weakly.

Now consider some security j that is a lemon. Now consider the liquid and illiquid securities:

$$D'_j(\omega) = \frac{\mu_j}{\mu'_j} D_j(\omega) \text{ and } I'_j(\omega) = \frac{\mu_j}{\mu'_j} I_j(\omega) + \left(1 - \frac{\mu'_j}{\mu'_j}\right) R_j(\omega),$$

and keep everything else the same. As before the resource constraint is satisfied by construction. The liquidity constraint is satisfied because it is linear. The participation constraint is also satisfied because $\mathbb{U}(D)$ is homogenous of degree one, because it is satisfied for D_j and I_j, and because $\mathbb{U}\left[R_j(\tilde{\omega})\right] = \mathbb{E}\left[R_j(\tilde{\omega}\right]$ for a lemon security.

For homogeneity let us denote a generic choice for the bank by $x = (c, D, I)$ and the constraint set by $X(\mu)$. It is clear that, for any $\lambda \geq 0$, $(c, D, I) \in X(\mu)$ if and only if $(\lambda c, D, I) \in X(\lambda\mu)$. Since the objective of the bank is linear, it immediately implies that $V(\lambda\mu) = \lambda V(\mu)$.

For concavity consider any two μ and μ', with associated $x \in X(\mu)$ and $x' \in X(\mu')$. Now consider some $\lambda \in [0, 1]$ as well as $\hat{x} = (\hat{c}, \hat{D}, \hat{I})$:

$$\begin{aligned}
\hat{c}(\omega) &= \lambda c(\omega) + (1-\lambda) c'(\omega) \\
\hat{D}_j(\omega) &= \frac{\lambda\mu_j}{\lambda\mu_j + (1-\lambda)\mu'_j} D_j(\omega) + \frac{(1-\lambda)\mu'_j}{\lambda\mu_j + (1-\lambda)\mu'_j} D'_j(\omega) \\
\hat{I}_j(\omega) &= \frac{\lambda\mu_j}{\lambda\mu_j + (1-\lambda)\mu'_j} I_j(\omega) + \frac{(1-\lambda)\mu'_j}{\lambda\mu_j + (1-\lambda)\mu'_j} I'_j(\omega).
\end{aligned}$$

In the above, $\hat{D}_j$ is the convex combination of the D_j and D'_j deposit, per type-j capita, and similarly for $\hat{I}_j$. One can then directly verify that $\hat{x}$ satisfies the resource constraint. It also satisfies the liquidity constraint because this constraint is linear. Finally, it also satisfies the participation constraint for each type j, because $\mathbb{U}[D]$ is concave and $\mathbb{E}[I]$ is linear. We conclude that $\hat{x} \in X[\lambda\mu + (1-\lambda)\mu']$. It generates a value $\lambda V(\mu) + (1-\lambda)V(\mu')$, which must be less than the maximum attainable value for the bank, $V[\lambda\mu + (1-\lambda)\mu']$.

B.3 Proof of Lemma 3

Consider any feasible $D_j(\omega)$ and $I_j(\omega)$ for the bank's problem with heterogenous depositors. Let

$$\bar{D}(\omega) = \sum_{j \in J} \mu_j D_j(\omega) \text{ and } \bar{I}(\omega) = \sum_{j \in J} \mu_j I_j(\omega).$$

Clearly, $\bar{D}(\omega)$ and $\bar{I}(\omega)$ is resource feasible for the bank's problem with a representative depositor. By linearity, since each $D_j(\omega)$ satisfies the liquidity constraint, then $\bar{D}(\omega)$ also satisfies it. Finally, because $\mathbb{U}\left[D(\tilde{\omega})\right]$ is concave and homogenous of degree one in $D(\tilde{\omega})$ and since $\mathbb{E}\left[I(\tilde{\omega})\right]$ is linear in $I(\tilde{\omega})$, we have that

$$\mathbb{U}\left[\bar{D}(\tilde{\omega})\right] + \mathbb{E}\left[\bar{I}(\tilde{\omega})\right] \geq \sum_{j \in J} \mu_j \left\{\mathbb{U}\left[D_j(\tilde{\omega})\right] + \mathbb{E}\left[I_j(\tilde{\omega})\right]\right\} \geq \sum_{j \in J} \mu_j \mathbb{U}\left[R_j(\tilde{\omega})\right].$$

Hence, $\bar{D}(\tilde{\omega})$ and $\bar{I}(\tilde{\omega})$ satisfies the participation constraint of the representative depositor. We conclude from this that the constraint set of the bank's problem with a representative depositor is not empty. Moreover, any solution of the bank problem with a representative depositor is an upper bound for the solution of the bank problem with heterogenous depositor.

Next, we show that this upper bound is attained. To see this, let $\bar{D}(\tilde{\omega})$ and $\bar{I}(\tilde{\omega})$ denote some solution to the bank's problem with a representative depositor. Now consider the offer for the bank problem with heterogenous depositors given by (9). By construction the offers are resource feasible and they satisfy the liquidity constraint. To see that they also satisfy the participation constraint, note that:

$$\begin{aligned}\mathbb{U}\left[D_j(\tilde{\omega})\right]+\mathbb{E}\left[I_j(\tilde{\omega})\right]=\alpha_j\left\{\mathbb{U}\left[\bar{D}(\tilde{\omega})\right]+\mathbb{E}\left[\bar{I}(\tilde{\omega})\right]\right\}&=\mathbb{U}\left[R_j(\tilde{\omega})\right]\frac{\mathbb{U}\left[\bar{D}(\tilde{\omega})\right]+\mathbb{E}\left[\bar{I}(\tilde{\omega})\right]}{\sum_{k\in J}\mu_k\mathbb{U}\left[R_k(\tilde{\omega})\right]}\\&\geq\mathbb{U}\left[R_j(\tilde{\omega})\right],\end{aligned}$$

where the first equality follows because $\mathbb{U}\left[D(\tilde{\omega})\right]$ and $\mathbb{E}\left[D(\tilde{\omega})\right]$ are homogenous of degree one, the second equality follows by substituting in the expression for α_j, and the last inequality follows because $\bar{D}(\tilde{\omega})$ and $\bar{I}(\tilde{\omega})$ satisfy the participation constraint for the bank problem with a representative investor.

B.4 Proof of Lemmas and Propositions 2 through 9

For what follows we note that

$$D(\omega_h)\leq r(\omega_h)\mathbb{E}\left[D(\tilde{\omega})\right]\Leftrightarrow D(\omega_h)\leq\frac{r(\omega_h)\left[1-\pi(\omega_h)\right]}{1-r(\omega_h)\pi(\omega_h)}D(\omega_\ell). \tag{B.7}$$

That is, the liquidity constraint is equivalent to the requirement that the deposit payoff does not vary too much in the high vs. the low state. Also, in all what follows, we will denote by $\mathbb{U}_h^-$ and $\mathbb{U}_h^+$ the left- and right- partial derivative of $\mathbb{U}$ with respect to $D(\omega_h)$, and similarly for $\mathbb{U}_\ell^-$ and $\mathbb{U}_\ell^+$. We note that:

$$\begin{aligned}
\text{if } D(\omega_h)<D(\omega_\ell) \quad &: \quad \mathbb{U}_h^-=\mathbb{U}_h^+=\pi(\omega_h)r(\omega_h)\\
&\quad\;\; \mathbb{U}_\ell^-=\mathbb{U}_\ell^+=\pi(\omega_\ell)\\
\text{if } D(\omega_h)=D(\omega_\ell) \quad &: \quad \mathbb{U}_h^-=\pi(\omega_h)r(\omega_h),\mathbb{U}_h^+=\pi(\omega_h)\mathbb{E}\left[r(\tilde{\omega})\right]\\
&\quad\;\; \mathbb{U}_\ell^-=\pi(\omega_\ell),\mathbb{U}_\ell^+=\pi(\omega_\ell)\mathbb{E}\left[r(\tilde{\omega})\right]\\
\text{if } D(\omega_h)>D(\omega_\ell) \quad &: \quad \mathbb{U}_h^-=\mathbb{U}_h^+=\pi(\omega_h)\mathbb{E}\left[r(\tilde{\omega})\right]\\
&\quad\;\; \mathbb{U}_\ell^-=\mathbb{U}_\ell^+=\pi(\omega_\ell)\mathbb{E}\left[r(\tilde{\omega})\right].
\end{aligned}$$

B.4.1 First point of Proposition 4: condition for a non-empty constraint set

Clearly, the constraint set is not empty if and only if it is not empty for choices of the form $c(\omega)=0$ and $I(\omega)=\bar{R}(\omega)-D(\omega)$, which maximize the left-hand side of the participation constraint given $D(\omega)$. Therefore, we obtain that the constraint set is not empty if and only if there exists some $D(\omega)\geq 0$ such that:

$$r(\omega_h)\mathbb{E}\left[D(\tilde{\omega})\right]\geq D(\omega_h) \text{ and } \mathbb{U}\left[D(\tilde{\omega})\right]+\mathbb{E}\left[\bar{R}(\tilde{\omega})-D(\tilde{\omega})\right]\geq\bar{U}.$$

The result follows because the left-hand side of the above participation constraint is largest when $D(\omega)$ solves the deposit maximization problem.

B.4.2 Some elementary results about the bank's problem

We prove in this paragraph a series of Lemma about the bank's problem. First, quite obviously, we have that:

Lemma B2. *In an optimal contract with a representative depositor, the resource constraint and the participation constraint are binding.*

Proof. The resource constraints are binding in all states because otherwise one could increase the value of the bank by increasing $c(\omega)$ and keeping everything else the same.

The participation constraint is binding because otherwise the bank could scale down $D(\tilde{\omega})$ and $I(\tilde{\omega})$ by a common $\lambda < 1$ until the participation constraint binds, and increase its consumption by $(1-\lambda)\left[D(\tilde{\omega}) + I(\tilde{\omega})\right]$. ■

Next,

Lemma B3. *In any solution of the bank problem, of the deposit minimization problem, or of the deposit maximization problem,* $\mathbb{E}\left[D(\tilde{\omega})\right] > 0$.

Proof. Consider first the bank problem consider any feasible contract c, D, I such that $\mathbb{E}\left[D(\tilde{\omega})\right] = 0$. Since $D(\omega) \geq 0$ for all ω, this implies that $D(\omega) = 0$ for all ω. Moreover, from the participation constraint, we have that $\mathbb{E}\left[I(\tilde{\omega})\right] > 0$. Consider, for ε small enough, the following contract is feasible and strictly increase the bank's value:

$$\hat{D}(\omega) = \varepsilon,\ \hat{I}(\omega) = I(\omega)\left(1 - \varepsilon\frac{\mathbb{E}\left[r(\tilde{\omega})\right]}{\mathbb{E}\left[I(\tilde{\omega})\right]}\right), \hat{c}(\omega) = c(\omega) + \varepsilon\left(\frac{I(\omega)}{\mathbb{E}\left[I(\tilde{\omega})\right]}\mathbb{E}\left[r(\tilde{\omega})\right] - 1\right).$$

Hence, in any optimal contract, $\mathbb{E}\left[D(\tilde{\omega})\right] > 0$. Next consider the deposit minimization problem. In that case, the participation constraint implies that $\mathbb{E}\left[D(\tilde{\omega})\right]$. Finally, consider the deposit maximization problem. Clearly, the deposit $D(\omega) = \varepsilon$ is feasible and yield a strictly positive value. Hence, any optimal deposit yield a strictly positive value, which implies that $\mathbb{E}\left[D(\tilde{\omega})\right] > 0$. ■

Lemma B4. *If* (10) *holds, then there exists a solution to the bank's problem such that* $I(\omega_\ell) = 0$.

Proof. Under (10), there exists a solution, (c, D, I). Consider the alternative contract $(\hat{c}, \hat{D}, \hat{I})$ such that $\hat{c}(\omega) = c(\omega)$, $\hat{I}(\omega_\ell) = 0$ and $\hat{D}(\omega_\ell) = D(\omega_\ell) + I(\omega_\ell)$. The feasibility constrained is satisfied. The the liquidity constraint is relaxed because the payoff of the liquid security, $D(\omega)$, increases in the low state. The participation constraint is also satisfied because $\mathbb{U}_\ell^+ \geq \pi(\omega_\ell)$. ■

Lemma B5. *Consider any solution of the bank's problem. Then, if* $I(\omega_h) > 0$, $D(\omega_h) = r(\omega_h)\mathbb{E}\left[D(\tilde{\omega})\right]$ *and* $D(\omega_\ell) = R(\omega_\ell)$.

Proof. Suppose, towards a contradiction, that the liquidity constraint does not bind, $D(\omega_h) < \frac{r(\omega_h)[1-\pi(\omega_h)]}{1-r(\omega_h)\pi(\omega_h)} D(\omega_\ell)$. Consider then, the alternative contract:

$$D(\omega_h) + \varepsilon, I(\omega_h) - \varepsilon\frac{\mathbb{U}_h^+}{\pi(\omega_h)}, \text{ and } c(\omega_h) + \varepsilon\left[\frac{\mathbb{U}_h^+}{\pi(\omega_h)} - 1\right].$$

The objective of the bank increases by $\mathbb{U}_h^+ - 1$, which is strictly positive in this region of the payoff space.

Next, suppose that $D(\omega_\ell) < R(\omega_\ell)$. If $I(\omega_\ell) > 0$, then the following deviation is feasible and strictly increases the bank's objective:

$$D(\omega_\ell) + \varepsilon, I(\omega_\ell) - \frac{\mathbb{U}_\ell^+}{\pi(\omega_\ell)}\varepsilon, c(\omega_\ell) + \varepsilon\left(\frac{\mathbb{U}_\ell^+}{\pi(\omega_\ell)} - 1\right)$$
$$D(\omega_h) + \varepsilon, I(\omega_h) - \frac{\mathbb{U}_h^+}{\pi(\omega_h)}\varepsilon, c(\omega_\ell) + \varepsilon\left(\frac{\mathbb{U}_h^+}{\pi(\omega_h)} - 1\right).$$

If $I(\omega_\ell) = 0$, then by our maintained assumption that $D(\omega_\ell) < R(\omega_\ell)$ and since the resource constraint binds, we must have that $c(\omega_\ell) > 0$. In this case, the following deviation is feasible and strictly increases the bank's objective:

$$D(\omega_\ell) + \varepsilon, c(\omega_\ell) - \varepsilon, D(\omega_h) + \varepsilon, I(\omega_h) - \frac{\mathbb{U}_\ell^+ + \mathbb{U}_h^+}{\pi(\omega_h)}\varepsilon, c(\omega_h) + \varepsilon\left(\frac{\mathbb{U}_\ell^+ + \mathbb{U}_h^+}{\pi(\omega_h)} - 1\right),$$

and keep $I(\omega_\ell) = 0$, for some small $\varepsilon > 0$. ■

A corollary of this Lemma is:

Corollary B6. *If $I(\omega_h) > 0$, then $\bar{R}(\omega_h) > \bar{R}(\omega_\ell)$.*

Proof. Indeed, if $I(\omega_h) > 0$, we have that $D(\omega_h) > D(\omega_\ell) = \bar{R}(\omega_\ell)$ since the liquidity constraint is binding and all resources are used in the low state. But, by resource feasibility, $D(\omega_h) \leq \bar{R}(\omega_h)$, so the result follows. ■

B.4.3 Proof of Proposition 4 when $\bar{R}(\omega_h) > \bar{R}(\omega_\ell)$, and of Proposition 9

Lemma B7. *If $\bar{R}(\omega_h) > \bar{R}(\omega_\ell)$ then in any solution of the bank's problem, of the deposit minimization, or of deposit maximization problems, $D(\omega_h) > D(\omega_\ell)$.*

Proof. Consider first the bank's problem and suppose, towards a contradiction, that $D(\omega_h) \leq D(\omega_\ell)$. Since $D(\omega_\ell) \leq \bar{R}(\omega_\ell) < \bar{R}(\omega_h)$ and since the resource constraint is binding, it follows that either $I(\omega_h) > 0$ or $c(\omega_h) > 0$. If $I(\omega_h) > 0$, then the following deviation is feasible and strictly improves the bank's objective:

$$D(\omega_h) + \varepsilon, I(\omega_h) - \varepsilon\frac{\mathbb{U}_h^+}{\pi(\omega_h)}, c(\omega_h) + \varepsilon\left(\frac{\mathbb{U}_h^+}{\pi(\omega_h)} - 1\right),$$

for some small $\varepsilon > 0$, and keep every else the same. As before $\mathbb{U}_h^+$ denotes the right derivative of $\mathbb{U}$ with respect to $D(\omega_h)$. If $c(\omega_h) > 0$, then the following deviation is feasible increases the bank's objective:

$$D(\omega_h) + \varepsilon, c(\omega_h) + \varepsilon, D(\omega_\ell) - \varepsilon\frac{\mathbb{U}_h^+}{\mathbb{U}_\ell^-}, c(\omega_\ell) + \varepsilon\frac{\mathbb{U}_h^+}{\mathbb{U}_\ell^-},$$

and keep everything else the same. Notice that the feasibility of this deviation follows because $D(\omega_\ell) \geq \mathbb{E}[D(\tilde{\omega})]$ and, by Lemma B3, $\mathbb{E}[D(\tilde{\omega})] > 0$. This deviation strictly increases the bank's objective because one easily verifies that, for any deposit such that $D(\omega_h) \leq D(\omega_\ell)$, $\frac{\pi(\omega_\ell)\mathbb{U}_h^+}{\pi(\omega_h)\mathbb{U}_\ell^-} > 1$.

Next consider either the deposit minimization or maximization problems and take any $D(\omega_h) \leq D(\omega_\ell)$. Since $D(\omega_\ell) \leq \bar{R}(\omega_\ell) < \bar{R}(\omega_h)$, the following deviation is feasible: $D(\omega_h) + \varepsilon$ and $D(\omega_\ell) - \varepsilon \frac{\mathbb{U}_h^+}{\mathbb{U}_\ell^-}$. By direct calculations, one verifies that this deviation strictly improves the objective, making it strictly smaller in the case of deposit maximization, and strictly larger in the case of deposit maximization. ■

We are now able to prove Step 1 of Proposition 4.

Lemma B8. *If $\bar{R}(\omega_h) > \bar{R}(\omega_\ell)$ then any solution of the deposit minimization problem also solves the bank's problem.*

Proof. Compare any solution of the deposit minimization problem, $D_{\min}(\omega)$, to a solution of the bank's problem, $D(\omega)$ and $I(\omega)$. By the previous two results, we have that $\mathbb{U}\left[D_{\min}(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D_{\min}(\tilde{\omega})\right]$ and $\mathbb{U}\left[D(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D(\tilde{\omega})\right]$. Since the participation constraint binds in both problems, we have that:

$$\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D(\tilde{\omega})\right] + \mathbb{E}\left[I(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D_{\min}(\tilde{\omega})\right],$$

and since $\mathbb{E}\left[r(\tilde{\omega})\right] > 1$, this implies that $\mathbb{E}\left[D(\tilde{\omega}) + I(\tilde{\omega})\right] \geq \mathbb{E}\left[D_{\min}(\tilde{\omega})\right]$. Hence, using the resource constraint, we obtain that $\mathbb{E}\left[c(\omega)\right] \leq \mathbb{E}\left[\bar{R}(\tilde{\omega}) - D_{\min}(\tilde{\omega})\right]$, and we are done. ■

Finally, we obtain Step 2 of the Proposition.

Lemma B9. *Suppose that $\bar{R}(\omega_h) > \bar{R}(\omega_\ell)$, that the deposit minimization problem has no solution, but that the bank's problem has a solution, i.e., that (10) holds. Then, $D^\star(\omega)$ and $I^\star(\omega)$ solve the bank's problem if and only if $D^\star(\omega)$ solves the deposit maximization problem, $I^\star(\omega) = \alpha\left[\bar{R}(\omega) - D^\star(\omega)\right]$, and α is chosen so that the participation constraint binds.*

Proof. Before proving the if and only if statement, let us consider any solution of the bank's problem, $D^\star(\omega)$, $I^\star(\omega)$. Then, the the participation and resource constraint binds. Combining these two condition we find that:

$$\mathbb{E}\left[c(\tilde{\omega})\right] = \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \bar{U} + \mathbb{U}\left[D(\tilde{\omega})\right] - \mathbb{E}\left[D(\tilde{\omega})\right].$$

Since $D^\star(\omega)$ is feasible for the deposit maximization problem, it follows that the bank's problem is bounded above by

$$\mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \bar{U} + \mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right] - \mathbb{E}\left[D^\star_{\max}(\tilde{\omega})\right]$$

for any $D^\star_{\max}(\omega)$ solving the deposit maximization problem. To see that this upper bound is achieved, let $D^\star(\omega) = D^\star_{\max}(\omega)$ and $I^\star(\omega) = \alpha\left\{\bar{R}(\omega) - D^\star_{\max}(\omega)\right\}$, and pick α so that the participation constraint of the representative depositor holds with equality:

$$\mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right] + \alpha\left\{\mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \mathbb{E}\left[D^\star_{\max}(\omega)\right]\right\} = \bar{U} \tag{B.8}$$

By construction, the contract achieves the upper bound and $D^\star_{\max}(\tilde{\omega})$ satisfies the liquidity constraint. We only need to check positivity and resource feasibility, which is equivalent to $\alpha \in [0, 1]$.

We have that $\alpha \geq 0$ because for two reasons. On the one hand, the deposit minimization problem has no solution, which implies that $\mathbb{E}\left[D^\star_{\max}(\tilde{\omega})\right] < \bar{U}$. On the other hand, by its construction, $D^\star(\omega) \leq \bar{R}(\omega)$.

Evaluating the right-hand side of (B.8) at $\alpha = 1$, one sees that:

$$\alpha \leq 1 \Leftrightarrow \mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right] + \left\{\mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \mathbb{E}\left[D^\star_{\max}(\omega)\right]\right\} \geq \bar{U}$$

which is the same as condition (10) for the existence of a solution to the bank's problem. This shows that if $D^\star(\omega)$ solves the deposit maximization problem, then it also solves the bank's problem in combination with $I^\star(\omega)$.

Now we can prove the if and only if statement of the Lemma. Consider any solution $D^\star_{\max}(\omega)$ of the deposit maximization problem. Then we have just shown that the bank's problem is solved by $D^\star(\omega) = D^\star_{\max}(\omega)$, and $I^\star(\omega) = \alpha\left[\bar{R}(\omega) - D^\star_{\max}(\omega)\right]$ for some $\alpha \in [0,1]$. Conversely, consider any solution $D^\star(\omega)$ and $I^\star(\omega)$ of the bank's problem. Then $D^\star(\omega)$ is feasible for the deposit maximization problem, and we have just shown that

$$\mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \bar{U} + \mathbb{U}\left[D^\star(\tilde{\omega})\right] - \mathbb{E}\left[D^\star(\tilde{\omega})\right] = \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \bar{U} + \mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right] - \mathbb{E}\left[D^\star_{\max}(\tilde{\omega})\right]$$

for any solution $D^\star_{\max}(\omega)$ of the deposit maximization problem. Hence, $D^\star(\omega)$ achieves the value of the deposit maximization problem. It remains to show that there exists some α such that $\alpha\left[\bar{R}(\omega) - D^\star(\omega)\right]$. But this follows by our preliminary results. Since the deposit minimization problem has no solution, then we must have that $I^\star(\omega_h) > 0$ – otherwise the constraint set of the deposit minimization problem would not be empty. By Lemma B5, this implies that $D(\omega_\ell) = \bar{R}(\omega_\ell)$. Hence, the result follows with α such that $I(\omega_h) = \alpha\left[\bar{R}(\omega_h) - D^\star(\omega_h)\right]$. ■

Next we turn to Proposition 9. We first show that:

Lemma B10. *When $\bar{R}(\omega_h) > \bar{R}(\omega_\ell)$, the solution of the deposit maximization problem is*

$$D^\star_{\max}(\omega_\ell) = \bar{R}(\omega_\ell) \text{ and } D^\star_{\max}(\omega_h) = \min\left\{\bar{R}(\omega_h), \frac{r(\omega_h)\left[1 - \pi(\omega_h)\right]}{1 - r(\omega_h)\pi(\omega_h)}\bar{R}(\omega_\ell)\right\},$$

and the value of the deposit maximization problem is:

$$\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D^\star_{\max}(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right]\min\left\{\mathbb{E}\left[\bar{R}(\tilde{\omega})\right], \frac{1 - \pi(\omega_h)}{1 - r(\omega_h)\pi(\omega_h)}\bar{R}(\omega_\ell)\right\}.$$

Proof. We know from from Lemma B7 that, when $\bar{R}(\omega_h) > \bar{R}(\omega_\ell)$, the deposit maximization problem is solved by some deposit such that $D(\omega_h) > D(\omega_\ell)$, hence $\mathbb{U}\left[D(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D(\tilde{\omega})\right]$. Hence, the deposit maximization problem reduces to $\max \mathbb{E}\left[D(\tilde{\omega})\right]$, with respect to $D(\omega) \geq 0$ and subject to $D(\omega_\ell) \leq D(\omega_h) \leq \frac{r(\omega_h)[1-\pi(\omega_h)]}{1-r(\omega_h)\pi(\omega_h)}D(\omega_\ell)$. This problem is easily represented graphically in Figure **??**. The blue vertical line is the resource constraint in the low state. The blue horizontal line is the resource constraint in the high state. The diagonal lines represent the information-sensitivity constraints. The shaded area is the constraint set. The green dotted line represent an iso-value line for the deposit security – a set of $D(\omega_\ell)$ and $D(\omega_h)$ that have the same expected value. Clearly, the optimum is to move that iso-value line to the north-east corner of the constraint set. ■

Next, we have:

Corollary B11. *Suppose that $\bar{R}(\omega_h) > \bar{R}(\omega_\ell)$. Then, the bank's problem has a solution if and only if $\left\{\mathbb{E}\left[r(\tilde{\omega})\right] - 1\right\}\mathbb{E}\left[D^\star_{\max}(\tilde{\omega})\right] + \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] \geq \bar{U}$. The deposit minimization problem has a solution if and only if $\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D^\star_{\max}(\tilde{\omega})\right] \geq \bar{U}$.*

Proof. For the "if" statement, suppose that the bank's problem has a solution $(c^\star, D^\star, I^\star)$. Then, $D^\star(\omega)$ is feasible for the deposit maximization problem. Moreover, substituting the resource constraint into the participation constraint of the producer, we obtain that:

$$\mathbb{U}\left[D(\tilde{\omega})\right] - \mathbb{E}\left[D(\tilde{\omega})\right] + \mathbb{E}\left[\bar{R}(\tilde{\omega})\right] - \bar{U} \geq \mathbb{E}\left[c(\tilde{\omega})\right] \geq 0.$$

Hence, the inequality is also satisfied for any solution $D^\star_{\text{max}}(\omega)$ of the deposit maximization problem. Conversely, suppose that inequality is satisfied for a solution of the deposit maximization problem, $D^\star_{\text{max}}(\omega)$. Then, $c(\omega) = 0$, $D(\omega) = D^\star_{\text{max}}(\omega)$ and $I(\omega) = \bar{R}(\omega) - D^\star_{\text{max}}(\omega)$ is feasible for the bank's problem, hence the bank's problem has a solution.

The second statement follows because, by Lemma B7 the deposit minimization problem is solved by a security such that $D(\omega_h) > D(\omega_\ell)$, and so it has a solution if and only if the largest such pooling security achieves a value greater than $\bar{U}$. ■

With this in mind we can conclude:

Lemma B12. *Suppose $\bar{R}(\omega_h) > \bar{R}(\omega_\ell)$ and that the bank's problem has a solution.*

- *If $\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D^\star_{\text{max}}(\tilde{\omega})\right] \geq \bar{U}$, then the bank's problem is solved by*

$$D^\star(\omega) = \lambda D^\star_{\text{max}}(\omega) \text{ and } I^\star(\omega) = 0, \text{ where } \lambda = \frac{\bar{U}}{\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D^\star_{\text{max}}(\tilde{\omega})\right]}.$$

- *If $\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D^\star_{\text{max}}(\tilde{\omega})\right] < \bar{U} \leq \left\{\mathbb{E}\left[r(\tilde{\omega})\right] - 1\right\}\mathbb{E}\left[D^\star_{\text{max}}(\tilde{\omega})\right] + \mathbb{E}\left[\bar{R}(\tilde{\omega})\right]$, then the bank's problem is solved by*

$$D^\star(\omega) = D^\star_{\text{max}}(\omega) \text{ and } I^\star(\omega) = \alpha\left[\bar{R}(\omega) - D^\star_{\text{max}}(\omega)\right],$$

and α is chosen so that the participation constraint binds. In all cases, the value of the bank's problem is:

$$V(\mu) = \mathbb{E}\left[\bar{R}(\omega)\right] - \frac{\bar{U}}{\mathbb{E}\left[r(\tilde{\omega})\right]} + \left(1 - \frac{1}{\mathbb{E}\left[r(\tilde{\omega})\right]}\right)\min\left\{\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D^\star_{\text{max}}(\omega)\right] - \bar{U}, 0\right\}.$$

Proof. The first bullet point arises when the deposit minimization problem has a solution. Moreover, by Lemma B7, $D^\star_{\text{min}}(\omega)$ satisfies $D^\star_{\text{min}}(\omega_h) > D^\star_{\text{min}}(\omega_\ell)$ so that $\mathbb{U}\left[D^\star_{\text{min}}(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D^\star_{\text{min}}(\tilde{\omega})\right]$. Using the binding participation constraint, it follows that the value of the deposit minimization problem is $\mathbb{E}\left[D^\star_{\text{min}}(\omega)\right] = \bar{U}/\mathbb{E}\left[r(\tilde{\omega})\right]$. Hence, by scaling down $\lambda D^\star_{\text{max}}(\omega)$ by λ so that $\mathbb{U}\left[\lambda D^\star_{\text{max}}(\omega)\right] = \lambda\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D^\star_{\text{max}}(\tilde{\omega})\right] = \bar{U}$, one obtains a solution of the debt minimization problem. The rest follows from Proposition 4.

The second bullet point arises when the deposit minimization problem has no solution, and follows directly from Proposition 4.

The formula for the value of the bank's problem follows by direct computations. ■

B.4.4 Proof of Proposition 7

Case 1. Suppose that the deposit minimization problem has no solution. Then an illiquid liability is issued, $I^\star(\omega) > 0$, and the proposed liability design is simply $\mu^\star_0 = 1$, $D^\star(\omega) = D^\star_{\text{max}}(\omega)$, and $I^\star(\omega) = \alpha\left[\bar{R}(\omega) - I^\star(\omega)\right]$, which we know from Proposition 4 achieves an optimum.

Case 2. Suppose that the deposit minimization problem has a solution. Then we know that $I^\star(\omega) = 0$ so that $\alpha = 0$, i.e., there is no illiquid liability. All we need to prove, then, is that there exists a net worth contribution, $\mu_0^\star \in [0, \mu_0]$, and a liability of the form stated in the proposition, $D^\star(\omega)$, which is feasible, liquid, and achieves the optimum of the bank problem. To this end let $\mathcal{D}$ denote the set of liabilities of the form stated in the proposition. The payoff of any liability $D^\star(\omega)$ in the set $\mathcal{D}$ is

$$
\begin{aligned}
&D^\star(\omega_\ell) = R^\star(\omega_\ell) \\
&D^\star(\omega_h) \in \left[R^\star(\omega_\ell), \min\left\{R^\star(\omega_h), \frac{r(\omega_h)\left[1 - \pi(\omega_h)\right]}{1 - r(\omega_h)\pi(\omega_h)} R^\star(\omega_\ell)\right\}\right],
\end{aligned}
$$

where $R^\star(\omega) = \mu_0^\star + \sum_{j \in J} \mu_j R_j(\omega)$, for some $\mu_0^\star \in [0, \mu_0]$. Any such liability is clearly resource feasible. It is also liquid since the constraint on $D^\star(\omega_h)$ can be shown to imply that $r(\omega_h)\mathbb{E}\left[D^\star(\omega)\right] \geq D^\star(\omega_h)$. Hence, only thing we need to establish is that there exists some $D^\star(\omega) \in \mathcal{D}$ that achieves the optimum of the banker's problem. Given that the deposit minimization problem has a solution, this is equivalent to showing that there exists some $D^\star(\omega) \in \mathcal{D}$ which brings the representative producer against its participation constraint, that is, such that $\mathbb{U}\left[D^\star(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right] \mathbb{E}\left[D^\star(\tilde{\omega})\right] = \bar{U}$.

To show that this is the case, we consider the largest and smallest liability in $\mathcal{D}$. The largest liability is the solution of the deposit maximization problem, $D^H(\omega) = D^\star_{\max}(\omega)$, as it is obtained by setting $\mu_0^\star = \mu_0$ and $D^\star(\omega_h)$ to its upper bound given $\mu_0^\star$. By our maintained assumption that the deposit minimization problem has a solution, we know from Lemma B13 that $\mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right] > \bar{U}$. The smallest liability is $D^L(\omega)$ obtained by setting the net worth contribution of the banker to $\mu_0^\star = 0$, and equalizing the payoff in the high and the low state $D^L(\omega_h) = D^L(\omega_\ell) = \sum_{j \in J} \mu_j R_j(\omega_\ell)$. We have:

$$
\mathbb{U}\left[D^L(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right] \sum_{j \in J} \mu_j R_j(\omega_\ell). \tag{B.9}
$$

On the other hand:

$$
\bar{U} = \sum_{j \in L} \mu_j \mathbb{E}\left[r(\tilde{\omega})\right] \mathbb{E}\left[R_j(\tilde{\omega})\right] + \sum_{j \in I} \mu_j \mathbb{E}\left[R_j(\tilde{\omega})\right], \tag{B.10}
$$

where L denote the subset of liquid securities, and I the subset of illiquid securities. Since all securities have positively correlated payoffs, $\mathbb{E}\left[R_j(\tilde{\omega})\right] \geq R_j(\omega_\ell)$. For illiquid security, the condition $r(\omega_h)\mathbb{E}\left[R(\tilde{\omega})\right] < R(\omega_h)$ can be written:

$$
\mathbb{E}\left[R_j(\tilde{\omega})\right] \geq \frac{1 - \pi(\omega_h)}{1 - r(\omega_h)\pi(\omega_h)} R_j(\omega_\ell).
$$

Plugging these inequalities back into (B.10), we obtain:

$$
\bar{U} > \sum_{j \in L} \mu_j \mathbb{E}\left[r(\tilde{\omega})\right] R_j(\omega_\ell) + \frac{1 - \pi(\omega_h)}{1 - r(\omega_h)\pi(\omega_h)} \sum_{j \in I} \mu_j R_j(\omega_\ell).
$$

Comparing with (B.9), we obtain that

$$
\bar{U} > \mathbb{U}\left[D^L(\tilde{\omega})\right] \Leftarrow \frac{1 - \pi(\omega_h)}{1 - r(\omega_h)\pi(\omega_h)} > \mathbb{E}\left[r(\tilde{\omega})\right],
$$

which can be readily verified. Hence, we have found that $\mathbb{U}\left[D^H(\omega)\right] > \bar{U}$ and $\bar{U} > \mathbb{U}\left[D^L(\omega)\right]$. Since the set $\mathcal{D}$ is convex, these two liabililites can be connected by a continuous path in $\mathcal{D}$. Applying the Intermediate Value Theorem along the path delivers a liability such that $\mathbb{U}\left[D^\star(\omega)\right] = \bar{U}$. ■

B.4.5 Proof of Proposition 4 and solution to the banker's problem when $\bar{R}(\omega_\ell) \geq \bar{R}(\omega_h)$

Consider first Proposition 4. By Lemma B4, we can set $I(\omega_\ell) = 0$. By the contrapositive of Corollary B6, we must have that $I(\omega_h) = 0$. Hence, $I(\omega) = 0$ and so the bank's problem must coincide with the solution of the deposit minimization problem. This shows that the bank's problem is indeed solved in Step 1 of the Proposition.

First we note that:

Lemma B13. *When $\bar{R}(\omega_\ell) \geq \bar{R}(\omega_h)$, the following statements are equivalent:*

- *The bank's problem has a solution;*
- *The deposit minimization problem has a solution;*
- $\mathbb{U}\left[\bar{R}(\tilde{\omega})\right] = \mathbb{E}\left[r(\omega)R(\omega)\right] \geq \bar{U}$.

Proof. Consider the first two bullet points. We have just shown that, when $\bar{R}(\omega_\ell) \geq \bar{R}(\omega_h)$, then the bank's problem has a solution if and only if the deposit minimization problem has a solution. Indeed, if the deposit minimization problem has a solution, then this solution is feasible for the bank's problem, hence the bank's problem has a solution. Conversely, if the bank's problem has a solution, then it is such that $I(\omega) = 0$, and so it is feasible for the deposit minimization problem.

Next, we note that $\mathbb{U}\left[D(\tilde{\omega})\right]$ is strictly increasing in both its arguments. Moreover, since $\bar{R}(\omega_\ell) \geq \bar{R}(\omega_h)$, we have that $\bar{R}(\tilde{\omega})$ satisfies the liquidity constraint and hence is feasible for the deposit minimization problem. Hence, $\mathbb{U}\left[\bar{R}(\tilde{\omega})\right]$ is the highest utility that can be generated by designing deposit securities satisfying the feasibility constraint and the liquidity constraint. The result follows. ■

Next we characterize the bank's value and a solution of the deposit minimization problem. We start with:

Lemma B14. *If $\mathbb{U}\left[R(\tilde{\omega})\right] \geq \bar{U}$, then there exists a solution to the deposit minimization problem such that $D(\omega_\ell) \geq D(\omega_h)$.*

Proof. Take any $D(\omega_\ell) < D(\omega_h) \leq R(\omega_h) \leq R(\omega_\ell)$ and the deviation: $D(\omega_\ell) + \varepsilon\mathbb{U}_h^-$, $D(\omega_h) - \mathbb{U}_\ell^+$. This deviation are feasible since $D(\omega_\ell) < R(\omega_\ell)$. It keeps the participation constraint the same. It also keeps the objective the same because, in this region of the payoff space, $\mathbb{U}_\ell = \pi(\omega_\ell)\mathbb{E}\left[r(\tilde{\omega})\right]$ and $\mathbb{U}_h = \pi(\omega_h)\mathbb{E}\left[r(\tilde{\omega})\right]$. ■

Next:

Lemma B15. *Consider a solution of the deposit minimization problem. If $D(\omega_\ell) > D(\omega_h)$, then $D(\omega_h) = \bar{R}(\omega_h)$ and $\mathbb{E}\left[r(\tilde{\omega})\right]\bar{R}(\omega_h) < \bar{U}$.*

Proof. Suppose $D(\omega_\ell) > D(\omega_h)$. If $D(\omega_h) < \bar{R}(\omega_h)$, then the following feasible deviation would strictly reduce the expected value of the deposit: $D(\omega_h) + \varepsilon\mathbb{U}_\ell^-$ and $D(\omega_\ell) - \varepsilon\mathbb{U}_h^+$, since in this region of the payoff space $\mathbb{U}_\ell^- = \pi(\omega_\ell)$ and $\mathbb{U}_h^+ = \pi(\omega_h)r(\omega_h)$.

For the second part of the Lemma note that since the participation constraint binds, we have that $\mathbb{U}\left[D(\tilde{\omega})\right] = \bar{U}$. But $\mathbb{U}$ is strictly increasing: hence, if we reduce $D(\omega_\ell)$ to $\bar{R}(\omega_h)$ we obtain that $\mathbb{E}\left[r(\tilde{\omega})\right]\bar{R}(\omega_h) = \mathbb{U}\left[\bar{R}(\omega_h)\right] < \bar{U}$. ■

Lemma B16. *Consider a solution of the deposit minimization problem. If* $D(\omega_\ell) = D(\omega_h)$*, then* $\mathbb{E}[r(\tilde{\omega})]\,\bar{R}(\omega_h) \geq \bar{U}$*.*

Proof. The result follows because the participation constraint binds at $D(\omega_\ell) = D(\omega_h)$, and because $D(\omega_h) \leq \bar{R}(\omega_h)$, and because $\mathbb{U}$ is increasing. ■

Taken together, we obtain:

Lemma B17. *A solution to the deposit minimization problem is*

$$D_{\min}(\omega_\ell) = \frac{\bar{U}}{\mathbb{E}[r(\tilde{\omega})]} + \frac{r(\omega_h)\pi(\omega_h)}{1-\pi(\omega_h)} \max\left\{\frac{\bar{U}}{\mathbb{E}[r(\tilde{\omega})]} - \bar{R}(\omega_h), 0\right\}$$
$$D_{\min}(\omega_h) = \frac{\bar{U}}{\mathbb{E}[r(\tilde{\omega})]} - \max\left\{\frac{\bar{U}}{\mathbb{E}[r(\tilde{\omega})]} - \bar{R}(\omega_h), 0\right\}$$

and the value of the deposit minimization problem is:

$$\mathbb{E}[D_{\min}(\tilde{\omega})] = \frac{\bar{U}}{\mathbb{E}[r(\tilde{\omega})]} - (\mathbb{E}[r(\tilde{\omega})] - 1) \max\left\{\frac{\bar{U}}{\mathbb{E}[r(\tilde{\omega})]} - \bar{R}(\omega_h), 0\right\}$$

Proof. First, Lemma B15 and B16 show that, for a solution such that $D(\omega_\ell) \geq D(\omega_h)$, we have $D(\omega_\ell) > D(\omega_h)$ if and only if $\bar{R}(\omega_h) < \bar{U}$. Consider a solution of the deposit minimization problem such that $D(\omega_\ell) \geq D(\omega_h)$. Such a solution exist by Lemma B14. If $D(\omega_\ell) > D(\omega_h)$, then we know by Lemma B16 that $D(\omega_h) = \bar{R}(\omega_h)$, and the result follows by solving the participation constraint at equality. If $D(\omega_\ell) = D(\omega_h)$, then the result follows as well by solving the participation constraint at equality. ■

B.5 Proof of Proposition 8

The proof involves several steps. Step one shows that producers that obtain collateralized loans default only in the low state. This can only be true for trees whose risk is above χ. Step two shows that under the conjectured terms of the loan, they receive their outside option when trading with the bank. Step three shows that producers that sell trees also obtains their outside option. That step also shows that the interest rate on savings accounts makes them indifferent between holding checking or savings deposits. In that case, any mix of checking or savings deposits is consistent with their portfolio problem. Step four shows that banks collect enough resources to pay their depositors only in the high state, and in the low state, they can only honor pay checking deposits. This rationalize the wage and verifies that the the liquidity of checking deposits satisfies incentive compatibility.

Step 1. We begin with a description of defaults.

Lemma B18. *Let* $\{L_j, F_j\}$ *be the terms of a loan collateralized by tree* j*. Under these terms, the producer repays the loan in the high state and defaults in the low state if and only if the tree satisfies condition (12).*

Proof. Consider a loan with a unit of tree j as collateral. If the producer repays the face value of his debt, he keeps his tree. If he defaults, the producer loses the tree, but avoids the repayment. By construction, the value of repaying debt is $H_j \geq 0$ in the high state. Thus, all j-types weakly prefer not to default in the high state. We must show that producers prefer to default on loans in the low state only for trees that satisfy condition 12. To see this, note that the value of paying back the debt is:

$$R_j(\omega_\ell) - F_j = -\left(R_j(\omega_h) - R_j(\omega_\ell)\right) + H_j.$$

There are two cases.

Case 1. First, suppose the aggregate tree is s-illiquid or liquid. Then, $\eta = 0$. In that case, all trees with payoffs positively correlated with $r(\omega)$ satisfy condition (12). Since $H_j = 0$, $R_j(\omega_\ell) - F_j = -\left(R_j(\omega_h) - R_j(\omega_\ell)\right)$. Thus, the value of repayment is negative if and only if a trees have payoffs positively correlated $r(\omega)$. This implies that producers default in the low state if and only all trees that satisfy condition (12)

Case 2. Suppose that the aggregate tree is f-illiquid. In that case, $H_j > 0$. We need to verify that the value of repayment is not positive,

$$-\left(R_j(\omega_h) - R_j(\omega_\ell)\right) + \alpha_j \bar{I}(\omega_h) \leq 0,$$

if and only if a tree satisfies condition (12). Substitution of the definition of α_j, yields:

$$-\left(R_j(\omega_h) - R_j(\omega_\ell)\right) + \eta \mathbb{U}\left[R_j(\tilde{\omega})\right] \leq 0.$$

For illiquid trees $\mathbb{U}\left[R_j(\tilde{\omega})\right] = \mathbb{E}\left[R(\tilde{\omega})\right]$. Thus, for an illiquid asset, the producer default if:

$$R_j(\omega_h) \geq \left[1 + \frac{\eta}{1 - \eta\pi(\omega_h)}\right] R_j(\omega_\ell).$$

If the asset is liquid, $\mathbb{U}\left[R_j(\tilde{\omega})\right] = \mathbb{E}\left[R(\tilde{\omega})\right]\mathbb{E}\left[r(\tilde{\omega})\right] = \mathbb{E}\left[R(\tilde{\omega})\right]\rho$. The condition for default is:

$$R_j(\omega_h) \geq \left[1 + \frac{\eta}{1 - \pi(\omega_h)\eta\rho}\right] R_j(\omega_l).$$

Assets are illiquid when $R_j(\omega_h) > (1+\psi) R_j(\omega_\ell)$. Thus, if $\psi \leq \frac{\eta}{1-\eta\pi(\omega_h)}$, there is default only among those for which is risk is higher that χ. If instead $\frac{\eta}{1-\eta\pi(\omega_h)} \leq \psi$, all illiquid trees feature default in the low sate. Among the liquid trees, those for which risk is above $\frac{\eta}{1-\pi(\omega_h)\eta\rho}$ feature default. Thus, there is default in the low state only when the tree satisfies, $R_j(\omega_h) \geq (1+\chi)R_j(\omega_\ell)$. ■

Step 2. Next, we show that a producer that obtains a collateralized debt contract obtains his outside option. The producer obtains L_j in deposits and with this he can buy labor in the amount L_j/w. Thus, this transaction provides him with an expected return of $\rho L_j/w = \rho\alpha_j\mathbb{E}\left[\bar{D}(\tilde{\omega})\right]$. The producer can also repay his debt and keep his tree. The expected value of this amount is $\pi(\omega_h) H_j$ since he defaults in the low state earning 0 and yields H_j in the high state. Since $H_j = \alpha_j \bar{I}(\omega_h)$, this portion has an expected value of $\alpha_j\mathbb{E}\left[\bar{I}(\tilde{\omega})\right]$. Thus, the producer obtains $\rho\alpha_j\mathbb{E}\left[\bar{D}(\tilde{\omega})\right] + \alpha_j\mathbb{E}\left[\bar{I}(\tilde{\omega})\right] = \mathbb{U}\left[R_j(\tilde{\omega})\right]$ per tree.

Step 3. Next, we analyze the decision to sell trees by producers and their corresponding investments into savings accounts. The expected return to using the checking deposit to buy hours has an expected return of ρ/w. As long as $\rho/wp_j \geq \mathbb{U}\left[R_j\left(\tilde{\omega}\right)\right]$, which yields the outside option.

Producers must be indifferent between using the checking deposits to hire workers or earning the interest of the savings account. We verify that time deposits pay zero in the low state in the next step of the proof. For now, assume this is true. Thus indifference between time and checking deposits requires $\pi\left(\omega_h\right)(1+i) = \rho/w$. Thus,

$$i = \frac{\rho \mathbb{E}\left[\bar{D}\left(\tilde{\omega}\right)\right]}{\pi\left(\omega_h\right)\bar{D}\left(\omega_h\right)} - 1.$$

Given this rate, producers are indifferent between different amounts of savings and checking deposits. In the implementation, the amounts of checking and savings deposits are:

$$L_j = \alpha_j \bar{D}\left(\omega_h\right) \text{ and } S_j = \frac{\alpha_j \bar{I}\left(\omega_h\right)}{(1+i)}$$

per tree.

Hiring workers by using his checking deposits the producer obtains an expected return:

$$\rho w^{-1} L_j = \rho \alpha_j \mathbb{E}\left[\bar{D}\left(\tilde{\omega}\right)\right].$$

From time deposits he obtains an expected consumption of

$$\pi\left(\omega_h\right)(1+i) S_j = \alpha_j \mathbb{E}\left[\bar{I}\left(\omega_h\right)\right].$$

Since $\rho\alpha_j\mathbb{E}\left[\bar{D}\left(\tilde{\omega}\right)\right] + \pi\left(\omega_h\right)\alpha_j\mathbb{E}\left[\bar{I}\left(\omega_h\right)\right] = \mathbb{U}\left[R_j\left(\tilde{\omega}\right)\right]$, the producer gets his outside option.

Finally, we verify that $L_j + S_j = p_j$, so that the amount of deposits are consistent with the initial budget. Note then that since $\pi\left(\omega_h\right)(1+i) = \rho w^{-1}$, we have that $\rho w^{-1} L_j + \pi\left(\omega_h\right)(1+i) S_j = \mathbb{U}\left[R_j\left(\tilde{\omega}\right)\right] = \rho w^{-1} p_j$. Thus, p_j is consistent with the budget $p_j = L_j + S_j$.

Step 4. We show now that allocations are the same as in the original problem. Lemma B18 states that all collateralized debt contracts feature default only in the low state. Thus, in the high state, banks collect $F_j = R_j\left(\omega_h\right) - \alpha_j \bar{I}\left(\omega_h\right)$ goods from loans contracts and $R_j\left(\omega_h\right)$ from all their direct purchases. Let J_c and J_p be the collections of trees used as collateral and those purchased respectively. The total resources in hands of the bank in the high state are:

$$\mu_0 + \sum_{j\in J_c} \mu_j \left[R_j\left(\omega_h\right) - \alpha_j \bar{I}\left(\omega_h\right)\right] + \sum_{J_p} \mu_j \left[R_j\left(\omega_h\right)\right] = \bar{R}\left(\omega_h\right) - \sum_{j\in J_c} \mu_j \alpha_j \bar{I}\left(\omega_h\right).$$

The amount of consumption goods promised as checking deposits:

$$\sum_{j\in J} \mu_j L_j = \sum_{j\in J_c} \mu_j \alpha_j \bar{D}\left(\omega_h\right) = \bar{D}\left(\omega_h\right).$$

The amount of promised to savings deposits accounts is:

$$\sum_{J_p} \mu_j (1+i) S_j = \sum_{J_p} \mu_j \alpha_j \bar{I}\left(\omega_h\right).$$

Note that $\sum_{j\in J_c} \mu_j \alpha_j \bar{I}\left(\omega_h\right) + \sum_{j\in J_c} \mu_j \alpha_j \bar{I}\left(\omega_h\right) = \bar{I}\left(\omega_h\right)$. Since we know that $\bar{R}\left(\omega_h\right) \geq \bar{I}\left(\omega_h\right) + \bar{D}\left(\omega_h\right)$ in the primitive problem, this proofs that the bank is solvent in the high state and thus, can honor all its debts. Furthermore, by construction $c(\omega_h) = \bar{R}(\omega_h) - \bar{I}(\omega_h) - \bar{D}(\omega_h)$ as in the primitive contract.

In the low state, banks collect $R(\omega_\ell)$ goods per loan contract because all loans feature default. In turn, they collect $R(\omega_\ell)$ from purchased trees. In the low state, the bank has:

$$\mu_0 + \sum_{j \in J_c} \mu_j R_j(\omega_\ell) + \sum_{J_p} \mu_j [R_j(\omega_\ell)] = \bar{R}(\omega_\ell).$$

Thus, in the low state, the bank owes $\bar{D}(\omega_h)$ in checking deposits. However, since $\bar{D}(\omega_\ell) = \bar{R}(\omega_\ell) < \bar{D}(\omega_h)$, checking deposits cannot pay in full in the low state. Given seniority, savings deposits and bank equity pay zero in the low state. Given that the payoffs of checking deposits are the same as deposits in the primitive contract, this guarantees that their circulation satisfies incentive compatibility.

Next, we show that the wage is consistent with the workers decision. We already saw producers use all their deposits to buy hours. From the above, we know that workers receive $\left[\bar{D}(\omega_\ell) / \bar{D}(\omega_h)\right]$ goods per deposit in the low state and one good per deposit in the high state. Also j is leads to $L_j = \alpha_j \bar{D}(\omega_h)$ checking deposits. The amount of hours worked from a tree j are $w^{-1} L_j = \mathbb{E}\left[\bar{D}(\omega_\ell)\right] / \bar{D}(\omega_h) \alpha_j \bar{D}(\omega_h) = \alpha_j \mathbb{E}\left[\bar{D}(\omega_\ell)\right]$. This is the same amount of hours worked per tree j in the primitive problem.

The allocation of goods to producers given a mass μ_j is:

$$\begin{aligned} & r(\omega_h) \mu_j \alpha_j \mathbb{E}\left[\bar{D}(\tilde{\omega})\right] + \mu_j R_j(\omega_h) - \mu_j F_j \\ = \quad & \mu_j \alpha_j r(\omega_h) \mathbb{E}\left[\bar{D}(\omega_\ell)\right] + \mu_j \alpha_j \bar{I}(\omega_h). \end{aligned}$$

in the high state and and in the low state: $r(\omega_\ell) \mu_j \alpha_j \mathbb{E}\left[\bar{D}(\omega_\ell)\right]$ as in the primitive problem. Producer and banker allocations are the same as in the primitive problem, and labor is the same, all allocations are identical. ■

B.6 Proof of Proposition 11

From the argument shown in the text, we obtain a characterization of the optimum in two cases:

Lemma B19. *Let* $\mu_j^{LS} \equiv M_j \Phi\left(\partial_j V^{LS}\right)$ *and* $\mu_j^F \equiv M_j \Phi\left(\partial_j V^F\right)$. *Then:*

- $V^{LS}(\mu^\star) < V^F(\mu^\star)$ *if and only if* $\mu^\star = \mu^{LS}$ *and* $V^{LS}(\mu^{LS}) < V^F(\mu^{LS})$.
- $V^F(\mu^\star) < V^{LS}(\mu^\star)$ *if and only if* $\mu^\star = \mu^F$ *and* $V^F(\mu^F) < V^{LS}(\mu^F)$.

If $V^{LS}(\mu^{LS}) \geq V^F(\mu^{LS})$ $V^F(\mu^F) \geq V^{LS}(\mu^F)$, the optimum must lie at the boundary between the F and the LS region, and so it must solve the auxiliary problem:

$$\max V^{LS}(\mu),$$

with respect to $\mu_j \in [0, M_j]$ for all $j \in J$ and subject to the constraint that $V^F(\mu) = V^{LS}(\mu)$ – that is, subject to making asset choices at the boundary between the LS and the F region. Notice that, under (16), the constraint set is not empty. Indeed, by an application of the Intermediate Value Theorem, (16) implies that there exists a convex combination of μ^{LS} and μ^F such that $V^F(\mu) = V^{LS}(\mu)$.

Letting λ denote the multiplier on the constraint $V^F(\mu) = V^{LS}(\mu)$ first-order necessary and sufficient condition with respect to μ_j now implies that::

$$\partial_j V^{LS} + \lambda \left(\partial_j V^F - \partial_j V^{LS}\right) - \Phi^{-1}\left(\frac{\mu_j^\star}{M_j}\right) = 0 \Rightarrow \mu_j^\star = M_j \Phi\left[\lambda \partial_j V^F + (1-\lambda)\partial_j V^{LS}\right]$$

Notice that this formula also holds in the LS region, with $\lambda = 0$, and in the F region, with $\lambda = 1$. Now plugging the above expression into the equation $V^F(\mu^\star) = V^{LS}(\mu^\star)$, we obtain a one-equation-in-one-unknown problem for λ: $\Delta V(\lambda) = 0$, where

$$\Delta(\lambda) = \left(\partial_0 V^F - \partial_0 V^{LS}\right)\mu_0 + \sum_{j \in J}\left(\partial_j V^F - \partial_j V^{LS}\right) M_j \Phi\left[\lambda \partial_j V^F + (1-\lambda)\partial_j V^{LS}\right]. \quad \text{(B.11)}$$

Notice that, under maintained assumption for this third case that (16), it follows that, $\Delta V(0) \leq 0$ and $\Delta V(1) \geq 0$. Moreover, as long as $\partial_j V^{LS} \neq \partial_j V^F$ for either $j = 0$ or some $j \in J$, then $\Delta V(\lambda)$ is strictly increasing. The case $\partial V_j^{LS} = \partial V_j^F$ for all j is non-generic and can be dealt with separately. It implies that $\Delta V(\lambda) = 0$ and $\mu_j^\star = \mu^{LS}$, which is equal to $M_j \Phi\left[\lambda \partial_j V^F + (1-\lambda)\partial_j V^{LS}\right]$ for all $\lambda \in [0,1]$. ■

B.7 Proof of Proposition 12

The Proposition follows from an application of the Implicit Function Theorem to the equation $\Delta(\lambda) = 0$. We note that:

$$\frac{\partial \Delta}{\partial \lambda} = \sum_{j \in J}\left(\partial_j V^F - \partial_j V^{LS}\right)^2 M_j \Phi'\left[\lambda \partial_j V^F + (1-\lambda)\partial_j V^{LS}\right]$$

$$\frac{\partial \Delta}{\partial M_j} = \left(\partial_j V^F - \partial_j V^{LS}\right)\Phi\left[\lambda \partial_j V^F + (1-\lambda)\partial_j V^{LS}\right].$$

Therefore,

$$\frac{\partial \lambda^\star}{\partial M_j} = -\frac{\partial \Delta / \partial M_j}{\partial \Delta / \partial \lambda} = -\frac{\left(\partial_j V^F - \partial_j V^{LS}\right)\Phi\left[\lambda^\star \partial_j V^F + (1-\lambda^\star)\partial_j V^{LS}\right]}{\sum_{\ell \in J}\left(\partial_\ell V^F - \partial_j V^{LS}\right)^2 M_\ell \Phi'\left[\lambda^\star \partial_\ell V^F + (1-\lambda^\star)\partial_\ell V^{LS}\right]},$$

which is strictly positive if $j \in L \cup S$, and strictly negative if $j \in F$. For $k \neq j$, we thus obtain that:

$$\frac{\partial \mu_k^\star}{\partial M_j} = M_k \Phi'\left[\lambda^\star \partial_k V^F + (1-\lambda^\star)\partial_k V^{LS}\right]\left(\partial_k V^F - \partial_k V^{LS}\right)\frac{\partial \lambda^\star}{\partial M_j}.$$

This means that, for $k \neq j$:

$$\text{sign}\frac{\partial \mu_k^\star}{\partial M_j} = \text{sign}\left[\left(\partial_k V^F - \partial_k V^{LS}\right) \times \left(\partial_j V^F - \partial_j V^{LS}\right)\right].$$

Using Lemma B2, this implies that, if $j \in L \cup S$, then $\mu_k^\star$ decreases for all $k \in L \cup S$, $k \neq j$, and increases for all $k \in F$. The opposite comparative statics obtains for $j \in F$.

Now let us turn to $k = j$. In this case:

$$\begin{aligned}\frac{\partial \mu_j^\star}{\partial M_j} =& M_j \Phi' \left[\lambda^\star \partial_j V^F + (1-\lambda^\star)\partial_j V^{LS}\right] \left(\partial_j V^F - \partial_j V^{LS}\right) \frac{\partial \lambda^\star}{\partial M_j} + \Phi\left[\lambda^\star \partial_j V^F + (1-\lambda^\star)\partial_j V^{LS}\right] \\ =& \frac{\mu_j^\star}{M_j}\left[1 - \frac{\left(\partial_j V^F - \partial_j V^{LS}\right)^2 M_j \Phi'\left[\lambda^\star \partial_j V^F + (1-\lambda^\star)\partial_j V^{LS}\right]}{\sum_{\ell \in J}\left(\partial_\ell V^F - \partial_j V^{LS}\right)^2 M_\ell \Phi'\left[\lambda^\star \partial_\ell V^F + (1-\lambda^\star)\partial_\ell V^{LS}\right]}\right],\end{aligned}$$

where we used that $\mu_j^\star / M_j = \Phi\left[\lambda^\star \partial_j V^F + (1-\lambda^\star)\partial_j V^{LS}\right]$ and collected terms. ■

C Construction of Figure ??

This section explains in details how Figure **??** is constructed.

Admissible pairs of μ_I and $\bar{R}(\omega_\ell)$. Notice that, given $\mathbb{E}\left[R_j(\tilde{\omega})\right] = 1$, there is a maximum and a minimum payoff for liquid and illiquid assets in the low state. Namely, for aliquid asset, we have that:

$$r(\omega_h)\mathbb{E}\left[R_j(\tilde{\omega})\right] \geq R_j(\omega_h) \Longleftrightarrow r(\omega_h) \geq \frac{1 - \left[1 - \pi(\omega_h)\right] R_j(\omega_\ell)}{\pi(\omega_h)}$$

$$\Longleftrightarrow R(\omega_\ell) \geq \frac{1 - r(\omega_h)\pi(\omega_h)}{1 - \pi(\omega_h)}.$$

Vice versa, for an illiquid asset, we have $R(\omega_\ell) < \frac{1 - r(\omega_h)\pi(\omega_h)}{1 - \pi(\omega_h)}$. Taken together, the constraints on payoff are:

$$\forall j \in L : \frac{1 - r(\omega_h)\pi(\omega_h)}{1 - \pi(\omega_h)} \leq R_j(\omega_\ell) \leq 1$$

$$\forall j \in I : 0 \leq R_j(\omega_\ell) < \frac{1 - r(\omega_h)\pi(\omega_h)}{1 - \pi(\omega_h)}.$$

Hence, given a fraction μ_I of illiquid asset in the pool, we have:

$$(1 - \mu_I)\frac{1 - r(\omega_h)\pi(\omega_h)}{1 - \pi(\omega_h)} \leq \bar{R}(\omega_\ell) \leq (1 - \mu_I) + \frac{1 - r(\omega_h)\pi(\omega_h)}{1 - \pi(\omega_h)}\mu_I.$$

Conversely, for any μ_I and $\bar{R}(\omega_\ell)$ satisfying the above inequality, one can construct an asset pool such that the fraction of illiquid asset and the aggregate payoff in the low state are exactly equal to μ_I and $\bar{R}(\omega_\ell)$.

The boundary between liquid and illiquid pools of assets. From the above calculation, we know that the pool of asset is liquid if

$$\mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[\bar{R}(\omega)\right] \geq \bar{R}(\omega_h) \Longleftrightarrow R_j(\omega_\ell) \geq \frac{1 - r(\omega_h)\pi(\omega_h)}{1 - \pi(\omega_h)},$$

which is the horizontal lines separating the green from the red area in the figure.

The boundary between structurally and fundamentally illiquid pools of assets. Given our maintained assumption that $R_j(\omega_h) > R_j(\omega_\ell)$ and $\mathbb{E}\left[R_j(\tilde{\omega})\right] = 1$ for all $j \in J$, we have that

$$\mathbb{U}\left[D^{\star}_{\max}(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right]\mathbb{E}\left[D_{\max}(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right]\min\left\{1, \frac{1 - \pi(\omega_h)}{1 - r(\omega_h)\pi(\omega_h)}\bar{R}(\omega_\ell)\right\}.$$

On the other hand:

$$\bar{U} = \sum_{j \in L}\mu_j\mathbb{E}\left[r(\tilde{\omega})\right] + \sum_{j \in I}\mu_j = (1 - \mu_I)\,\mathbb{E}\left[r(\tilde{\omega})\right] + \mu_I, \text{ where } \mu_I \equiv \sum_{j \in I}\mu_j,$$

is the fraction of illiquid assets on the balance sheet. Notice that if $\mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right] = \mathbb{E}\left[r(\tilde{\omega})\right]$, then $\mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right] > \bar{U}$. It thus follows that $\mathbb{U}\left[D^\star_{\max}(\tilde{\omega})\right] \leq \bar{U}$ if and only if:

$$\begin{aligned}
&\mathbb{E}\left[r(\tilde{\omega})\right] \frac{1-\pi(\omega_h)}{1-r(\omega_h)\pi(\omega_h)} \bar{R}(\omega_\ell) \leq \bar{U} = (1-\mu_I)\,\mathbb{E}\left[r(\tilde{\omega})\right] + \mu_I \\
\Longleftrightarrow &\bar{R}(\omega_\ell) \leq \frac{1-r(\omega_h)\pi(\omega_h)}{1-\pi(\omega_h)} \left[(1-\mu_I) + \frac{\mu_I}{\mathbb{E}\left[r(\tilde{\omega})\right]}\right],
\end{aligned}$$

which is represented by the line separating the red from the green area in the figure.