Homework 1

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UCLA - 2016

ECON 221C MONETARY ECON III

Liquidity and Financial Friction in Macroeconomics

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Excercise 1. Read the course lecture notes and report any typos.

Excercise 2. Read Section 5.2 in Acemoglu. Solve Question 5.2 and 5.3.

Excercise 3. Aggregation with GHH preferences. A household has the following preferences over consumption and labor:

$$U(c,h) = \frac{\left(c - \frac{h^{1+v}}{1+\nu}\right)^{1-1/\theta}}{1-1/\theta}.$$

Their intertemporal utility is given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$$

and the satisfy the following sequence of budget constraints given by:

$$c_t + W_{t+1} = w_t h_t + R_t W_t.$$

The sequences $\{R_t, w_t\}$ are deterministic and convergent. Also, assume the following borrowing limit

$$W_{t+1} \geq -H_{t+1}.$$

where we have the following:

$$H_{t+1} = \sum_{s=0}^{\infty} \frac{w_{t+1+s}^{(1+v)/\nu}}{1+\nu} \frac{1}{\prod_{\tau=s}^{\infty} R_{t+1+\tau}}.$$

Show that the problem where the agent wants to maximize his expected utility admits aggregation. Also, discuss the connection with Gorman aggregation.

(Hint: you will have to transform the problem by performing change of variables multiple times. Also, guess and verify that the borrowing constraint is not binding).

Solution. The foc for labor solves $w = h^{\nu}$. Thus, $\frac{h^{1+v}}{1+\nu} = \frac{w^{(1+v)/\nu}}{1+\nu}$. Treat $\frac{w_t^{(1+v)/\nu}}{1+\nu}$ as a bliss point. Change variables in the utility function to obtain:

$$U(c,h) = \frac{x^{1-1/\theta}}{1-1/\theta} \text{ for } x = c - g.$$

Thus, the budget constraint satisfies:

$$x_t - \frac{w_t^{(1+v)/\nu}}{1+\nu} + W_{t+1} = w_t h_t + R_t W_t$$

The budget constraint can be written as:

$$x_t + W_{t+1} = \frac{w_t^{(1+v)/\nu}}{1+\nu} + w_t h_t + R_t W_t.$$

Observe that since $w_t h_t = w_t^{(1+v)/\nu}$, we can express the two terms in the RHS as $g_t = \frac{\nu}{(1+\nu)} w_t^{(1+v)/\nu}$. Since the sequence of w_t is known, the sequence of g_t is known. Now define the following artifical variable:

$$H_t = \frac{g_t + H_{t+1}}{R_t}.$$

Thus, the budget constraint can be written as:

$$x_t + W_{t+1} + H_{t+1} = g_t + R_t \left(W_t + H_t \right).$$

Thus, since $W_{t+1} + H_{t+1}$ can be defined in a different way, we have that the original problem can be written as:

$$\begin{aligned} & \max \frac{x^{1-1/\theta}}{1-1/\theta} \\ s.t.\ x_t + M_t &= R_t M_t \\ M_t &\geq 0. \end{aligned}$$

Intertemporal Elasticity of Substitution and the Effects of Idiosyncratic Risk.

This questions asks you to write a code based on the Algorithm the code that solves for the steady state of Angeletos's model. The sketch of the algorithm is presented towards the end of the lecture notes. You can also consulto the original paper.

Perform the following tasks:

1. Build a 3-D graph where you summarize what occurs to the capital stock, the risk-free rate, expected return to capital, and wages as you vary the IES and the RA parameters —$1/\theta$ and γ— in the class notes. Explain your intuition.

2. Now fix a given value of γ. Then, change the model parameters to induce a mean-preserving increase in the return to capital —TFP. What happens to the capital stock in steady state as you increase risk? How does your answer depend on whether $1/\theta$ is greater or less than 1. Provide intuition