Online Appendix to "Repurchase Options in the Market for Lemons"

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B Repurchase Options in Competitive Search Market

We introduce repo contracts in the asset market with competitive search presented in Guerrieri, Shimer and Wright (2010) (henceforth GSW). Relative to the environment in Section 2, here lenders can each serve at most one contract, markets clear by adjusting the trading probabilities, and the equilibrium concept is modified accordingly. In this section, we elaborate on the result provided in Section 5, which shows that the equilibrium features a unique pooling repo contract, $\boldsymbol{p}^{GSW} = \{\underline{\lambda}, \underline{\lambda}\}$, with full trading probability.

B.1 Environment

Timing. At $t = 1$, the markets for repo contracts open. All agents enter the markets simultaneously. We assume that there are no fundamental search frictions and there are no contract posting costs.[1] The only innovation relative to Guerrieri et al. (2010) is that we enrich the contract space by adding the second leg of the contract—the repurchase option.

The market works as follows. On one side of the market, lenders decide which market to enter by posting a contract $\boldsymbol{p} = \{p_s, p_r\} \in \Lambda \times \Lambda$.[2] Once the contract is posted and signed, the lender commits to making the loan in amount p_s and returning the asset if p_r is repaid. Importantly, each lender is capacity constrained and can at most serve one borrower

[1] Our results carry through to a more general setting that allows for contract posting cost.

[2] As in Guerrieri et al. (2010), it is without loss of generality to assume that each lender posts a single contract. If we were to allow lenders to post mechanisms, the corresponding equilibrium outcome is payoff-equivalent to the contract posting equilibrium.

and fulfill one contract. On the other side of the market, each borrower decides whether to participate in the market and, if so, which contract to apply for.

As is standard in Walrasian markets with adverse selection (Gale, 1992, 1996), markets clear not by adjusting prices, but by adjusting trading probabilities. Rationing may occur if one side of the market exceeds the other side. The short-side of the market gets perfectly matched while the long-side is rationed. Let $\Theta : \Lambda^2 \to \mathbb{R}_+ \cup \{\infty\}$ denote the lender-to-borrower ratio in the market for contract $\boldsymbol{p}$. The trading probabilities are $\min\{\Theta(\boldsymbol{p}), 1\}$ for borrowers and $\min\left\{\Theta(\boldsymbol{p})^{-1}, 1\right\}$ for lenders. The functions Θ and Γ are determined endogenously. We define an active repo market as one in which the lender-to-borrower ratio is strictly positive and finite $\Theta(\boldsymbol{p}) \in (0, \infty)$. The set of active markets is $\mathbb{P} = \{\boldsymbol{p} \in \Lambda^2 : \Theta(\boldsymbol{p}) \in (0, \infty)\}$. The cumulative measure of entered contracts is denoted by $G : \mathbb{P} \to [0, 1]$. That is, the amount of transactions in market $\boldsymbol{p}$ is $g(\boldsymbol{p})$.

At $t = 2$, borrowers decide whether to default on repurchasing their assets.

Borrower's Problem. As in the main text, the borrower decides whether to participate in the repo markets:

$$\max\{0, v(\lambda)\}, \tag{B.1}$$

where the maximum value obtained from participating in a repo transaction and and choosing the optimal contract is

$$v(\lambda) = \max_{\boldsymbol{p} \in \Lambda^2} \{\min\{\Theta(\boldsymbol{p}), 1\}((1+r)p_s - \min\{\lambda, p_r\})\}. \tag{B.2}$$

Similar to (1), the expression in (B.1) encodes the borrower's participation constraint: the borrower only brings the asset to the repo markets if $v(\lambda)$ is positive. Otherwise, the borrower opts out. The value function in (B.2) resembles (2): if the borrower enters contract $\boldsymbol{p}$, her payoff equals the investment payoff, $(1+r)p_s$, minus the costs of repayment, $\min\{\lambda, p_r\}$. One difference is that now, when the borrower chooses a market, she takes into consideration the chances that she is indeed matched with a lender are $\min\{\Theta(\boldsymbol{p}), 1\}$. Another difference is that here the borrowers and lenders enter the markets simultaneously, therefore the borrowers choose among all contracts in Λ^2, taking as given the equilibrium market tightness. In contrast, in the setup that follow the Netzer and Scheuer (2014) timing, the borrowers choose among the contracts offered by lenders. The set of contracts chosen by borrowers with an asset of quality λ is denoted by $P : \Lambda \rightrightarrows \Lambda^2$, where $P(\lambda) = \{P_s(\lambda), P_r(\lambda)\}$ is a two-valued set mapping.[3] We denote by $\Gamma(\lambda; \boldsymbol{p})$ the distribution of assets that apply to contract $\boldsymbol{p}$.

[3]The borrower can be indifferent among multiple contracts, but ultimately must chose one contract.

Lender's Profit. The lender's expected profit by posting a repo contract $\boldsymbol{p}$ is

$$\Pi(\boldsymbol{p}) = \min\left\{\Theta(\boldsymbol{p})^{-1}, 1\right\}\left\{\int \min\{\lambda, p_r\}\, d\Gamma(\lambda; \boldsymbol{p}) - p_s\right\}. \tag{B.3}$$

Similar to equation (3), the expected profit in equation (B.3) considers that if the lender enters into contract $\boldsymbol{p}$, she will first pay p_s for the asset and later be repaid $\min\{\lambda, p_r\}$ if she transacts with a borrower with asset quality λ. However, different from equation (3), here the lender builds an expectation with respect to the distribution of borrowers that participate in the $\boldsymbol{p}$ market $\Gamma(\lambda; \boldsymbol{p})$. Hence, $\int \min\{\lambda, p_r\}\, d\Gamma(\lambda; \boldsymbol{p})$ is the borrower's expected repayment. Another difference is that here the lender takes into account the trading probability. Posting in market $\boldsymbol{p}$ doesn't guarantee that the contract is signed. Instead, the lender takes into consideration that by posting contract $\boldsymbol{p}$, she is matched with probability $\min\left\{\Theta(\boldsymbol{p})^{-1}, 1\right\}$.

Equilibrium Concept. Next, we define the equilibrium.

Definition B.1 (Competitive Equilibrium)**.** A competitive equilibrium consists of a measure of contracts G with support over the set of active repo markets $\mathbb{P}$, a market tightness function Θ, and a distribution of asset qualities for each market Γ, the borrowers' value function and decisions $v(\lambda)$, and $P(\lambda)$ such that:

(i) Lenders profit maximization and free entry:

$$\Pi(\boldsymbol{p}) \leq 0,\ \forall \boldsymbol{p} \in \Lambda^2 \text{ and } \Pi(\boldsymbol{p}) = 0,\ \forall \boldsymbol{p} \in \mathbb{P}. \tag{B.4}$$

(ii) Borrowers' optimization:

  (i) If $\Theta(\boldsymbol{p}) \in (0, \infty)$ and $\gamma(\lambda; \boldsymbol{p}) > 0$, then $v(\lambda) \geq 0$ and $\boldsymbol{p} \in P(\lambda)$;

  (ii) If $v(\lambda) < 0$ or $\boldsymbol{p} \notin P(\lambda)$, then either $\Theta(\boldsymbol{p}) \notin (0, \infty)$ or $\gamma(\lambda; \boldsymbol{p}) = 0$.

(iii) Market clearing:

$$\int_{\boldsymbol{p} \in \mathbb{P}} \frac{\gamma(\lambda; \boldsymbol{p})}{\min\{\Theta(\boldsymbol{p}), 1\}} dG(\boldsymbol{p}) \leq f(\lambda), \tag{B.5}$$

with equality if $v(\lambda) \geq 0$.

The equilibrium concept follows Guerrieri et al. (2010). First, equation (B.4) is the lender's zero-profit condition. If there were a contract posting cost, $k > 0$, the profit would be modified to be equal to k in active markets. Second, regarding the borrowers optimal choice, item (ii).(a) says that, in an active market $\boldsymbol{p}$, $\Theta(\boldsymbol{p}) \in (0, \infty)$, and if asset λ is in that market, $\gamma(\lambda; \boldsymbol{p}) > 0$, then it must be that borrowers with asset λ choose to participate in market $\boldsymbol{p}$. Conversely, item (ii).(b) says that, if borrowers with asset λ opt out or if they do not choose market $\boldsymbol{p}$, then either that market is not open or the density of asset λ is zero in that market. Third, equation (B.5) is a market clearing condition. The right-hand side, $f(\lambda)$, is the measure of assets of quality λ. On the left-hand side, $\gamma(\lambda; \boldsymbol{p}) / \min\{\Theta(\boldsymbol{p}), 1\}$ is the amount of assets of quality λ in market $\boldsymbol{p}$ per each transaction. Summing across the transactions in all markets cannot exceed the total number of assets of quality λ.

The rationality of off-equilibrium beliefs is embedded in the equilibrium definition: If a contract is not posted in equilibrium, the belief regarding the asset distribution is consistent with the actual distribution attracted to that contract, if it were indeed posted. In precise terms, lenders expect the types with the greatest incentive to search for this deviating contract to show up to their contract, should there be a deviation. The limit to one contract only per lender, effectively a capacity constraint on the side of lenders, plays a crucial role in this refinement. If a lender decides to post a single deviating contract, if it attracts any type, she will only serve one of them. Hence she serves the type willing to bear the lowest trading probability, and that borrower is the type with the highest incentive to deviate. The market tightness, in that case, would adjust such that this type is just indifferent between the current contract and the deviating contract.

B.2 Characterization

In this section we characterize the repo equilibrium under competitive search. The steps taken follow closely the ones in Section 3. In particular, the borrower's incentives for participating in repo contracts and for defaulting are very similar, with only minor differences in details.

First of all, Lemma 1 still applies. We show the arguments heuristically, without fully reproducing the proof. We start by making the same observation as in the main text: the borrower's value of participating $v(\lambda)$ is weakly decreasing. Again, the "weakly" decreasing property is due to the repurchase option in the contract. To be specific, we can reproduce the algebras in Proof A.1 after augmenting the trading probabilities in the value function in equation (B.2). Thus, we obtain a unique default threshold λ_d. To show full participation, we argue that if for the highest quality assets, their value function from participating is $v(\lambda) < 0$, deviating to the $\{\underline{\lambda}, \underline{\lambda}\}$ contract yields a positive value for them. In this case, lenders should from a belief that these highest quality assets will come to them, since these assets are willing to bear the lowest trading probability. Hence, lenders are willing to deviate as well..

Building on Lemma 1, we provide an additional intermediate step on the equilibrium trading probability. The next proposition states that, unlike in the competitive search model with asset sales, when we expand the contract space to include the repurchase option, rationing never occurs in equilibrium.

Proposition B.1 (No Rationing)**.** *The trading probability for borrowers is 1 in all active markets, i.e.,*

$$\Theta(\boldsymbol{p}) \geq 1, \forall \boldsymbol{p} \in \mathbb{P}.$$

Proof. The strategy for establishing that rationing does not occur in equilibrium is as follows. Because there's a unique default threshold λ_d, we analyze the behavior of market tightness and trading probabilities among default and nondefault assets. Step 1 shows that, if there is rationing in equilibrium, default and nondefault assets do not pool in a rationed contract. Step 2 shows that rationing cannot occur for contracts for default assets. Step 3 shows that rationing cannot occur for contracts for nondefault assets. Combining all threes steps, we conclude that the equilibrium contracts do not feature rationing.

Step 1. Separation of default and nondefault assets in any rationed contract. We first show that if there is a contract with rationing, there cannot be pooling of default and nondefault assets in that specific contract. The argument is by contradiction. Suppose there is a contract $\boldsymbol{p}$ such that $\Theta(\boldsymbol{p}) < 1$ and default and nondefault assets are pooled together. Given that the default assets are worth less than the repurchase price, in order for lenders to break even, the sales price must be below the repurchase price, $p_s < p_r$. In the following steps, we show another lender will deviate to cream-skim the nondefault assets by improving the trading probability.

Consider the deviating contract that lowers the sales price and the repurchase price slightly, i.e., $\tilde{\boldsymbol{p}} = (p_s - \varepsilon, p_r - (1+r)\varepsilon)$, where ε is a small positive value. Conditional on trading, the deviating contract delivers a lower payoff to the default assets but keeps the payoff of nondefault assets unchanged:

$$(1+r)\tilde{p}_s - \tilde{p}_r = (1+r)p_s - p_r.$$

Therefore the nondefault assets have the most incentive to search for this deviating contract.The market tightness that makes the nondefault assets just indifferent is $\tilde{\theta} = \Theta(\boldsymbol{p}) < 1$. Hence, lenders would form an off-equilibrium belief that the nondefault assets will come to the contract. Therefore, the deviating contract is expected to be profitable. This is because no assets default in this contract, but the sales price is lower than the repurchase price.

Step 2. No rationing in the default interval. We have showed in step 1 that no equilibrium can feature a contract that rations and pools default and nondefault assets. Next, we show that there is no rationing in the default interval $[\underline{\lambda}, \lambda_d]$. Before proceeding further, we first show that the trading probability $\min\{\Theta(P(\lambda)), 1\}$ must be weakly decreasing in quality λ in the default interval. Consider any two nondefault qualities λ_1 and λ_2 such that $\lambda_1 < \lambda_2 < \lambda_d$. In equilibrium, the following conditions must hold

$$\min\{\Theta(P(\lambda_1)), 1\}((1+r)P_s(\lambda_1) - \lambda_1) \geq \min\{\Theta(P(\lambda_2)), 1\}((1+r)P_s(\lambda_2) - \lambda_1) \tag{B.6}$$

$$\min\{\Theta(P(\lambda_2)), 1\}((1+r)P_s(\lambda_2) - \lambda_2) \geq \min\{\Theta(P(\lambda_1)), 1\}((1+r)P_s(\lambda_1) - \min\{\lambda_2, P_r(\lambda_1)\}). \tag{B.7}$$

Combining the two inequalities (B.6) and (B.7), we obtain:

$$\min\{\Theta(P(\lambda_1)), 1\}(\min\{\lambda_2, P_r(\lambda_1)\} - \lambda_1) \geq \min\{\Theta(P(\lambda_1)), 1\}(\lambda_2 - \lambda_1).$$

Since, $0 < \min\{\lambda_2, P_r(\lambda_1)\} - \lambda_1 < \lambda_2 - \lambda_1$, then,

$$\min\{\Theta(P(\lambda_1)), 1\} \geq \min\{\Theta(P(\lambda_2)), 1\}.$$

Thus, the trading probability is weakly decreasing in the default interval $[\underline{\lambda}, \lambda_d]$.

Now suppose there is some rationing among contracts that attract default assets. Because trading probability is decreasing in quality, the postulate implies that there exists a highest non-rationed quality, λ_0. That is, there is an interval $[\underline{\lambda}, \lambda_0]$ in which default assets are not rationed, and an interval in which default assets $(\lambda_0, \lambda_d]$ are rationed:

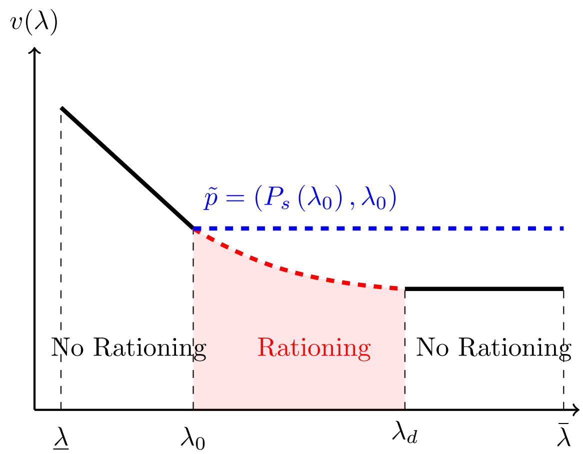
$$\Theta(P(\lambda)) \geq 1, \ \forall \lambda \in [\underline{\lambda}, \lambda_0] \text{ and } \Theta(P(\lambda)) < 1, \ \forall \lambda \in (\lambda_0, \lambda_d].$$

As noted earlier, default value is strictly decreasing. The value of assets with quality λ above λ_0 satisfies:

$$v(\lambda) < v(\lambda_0) = (1+r)P_s(\lambda_0) - \lambda_0, \ \forall \lambda \in \left(\lambda_0, \bar{\lambda}\right].$$

The result in step 1 implies that the rationed default assets in the interval $(\lambda_0, \lambda_d]$ will not pool with any of the nondefault assets. Hence, among the set of defaulters, the equilibrium must resemble the direct sales equilibrium in GSW. As established in GSW, these contracts must be fully separating and each asset must participate in its own contract. Figure B.1 illustrates this possibility. The zero-profit condition implies that, for any $\lambda \in (\lambda_0, \lambda_d]$, $P_s(\lambda) = \lambda$.

Figure B.1: Deviating contract when rationing occurs



Furthermore, for any $\lambda \in (\lambda_0, \lambda_d]$ not to participate in $P(\lambda_0)$, we obtain a condition,

$$\Theta(P(\lambda))((1+r)\lambda - \lambda) > (1+r) P_s(\lambda_0) - \lambda,$$

which implies that $P_s(\lambda_0) \leq \lambda_0$.

After concluding that $P_s(\lambda_0) \leq \lambda_0$, we now consider the following deviating contract $\tilde{\boldsymbol{p}} = (P_s(\lambda_0), \lambda_0)$, which has the same sales price as the contract for quality λ_0 but a lower repurchase price, $\lambda_0 \leq P_r(\lambda_0)$. For assets in $(\lambda_0, \bar{\lambda}]$, if they stay with their original contract, they obtain a lower value than the value for asset λ_0. However, if they move to the deviating contract, they obtain the same value as asset λ_0. Therefore, these assets are the ones with higher incentives to deviate. Regardless of which has the most incentive the deviate and hence is willing to bear the lowest trading probability, it wouldn't default after deviating. Therefore the deviating contract is profitable given the corresponding lender's off-equilibrium belief.The profit is

$$\Pi(\tilde{\boldsymbol{p}}) = \Theta(\tilde{\boldsymbol{p}})^{-1}(\lambda_0 - P_s(\lambda_0)) \geq 0.$$

Since the contract is profitable, lenders will indeed deviate and post the contract. If the contract is indeed posted, it does not feature rationing in equilibrium, i.e., $\Theta(\tilde{\boldsymbol{p}}) \geq 1$. Thus we can conclude that rationing cannot occur among default assets.

Step 3. No rationing in the nondefault interval. Finally, we show that there is no rationing in the nondefault interval $(\lambda_d, \bar{\lambda}]$. Suppose otherwise: Then, there exists a contract $\boldsymbol{p}$ with trading probability $\theta < 1$ and nondefault value

$$\bar{v} = \theta((1+r) p_s - p_f).$$

The result in step 1 implies that $\boldsymbol{p}$ attracts only nondefault assets. For the lenders to break even, the sales price must equal the repurchase price,

$$p_s = p_r = \frac{\bar{v}}{\theta r}.$$

Further, it must be that $p_s > \underline{\lambda}$, since rationing can never occur for the $\{\underline{\lambda}, \underline{\lambda}\}$ contract.

Now, consider a deviating contract with lower sales price, $\tilde{\boldsymbol{p}} = (p_s - \varepsilon, p_s - \varepsilon)$, where ε is small positive value. For the nondefault assets to be just indifferent, the market tightness $\tilde{\theta}$ should equal

$$\tilde{\theta}\left((1+r)\,\tilde{p}_s - \tilde{p}_f\right) = \bar{v} \rightarrow \tilde{\theta} = \theta \frac{p_s}{p_s - \varepsilon} < 1.$$

At this market tightness, none of the existing defaults assets can obtain a higher payoff by deviating. Since the contract does not feature default assets, the contract is profitable. This deviation will occur until $\tilde{\theta} = 1$. Therefore rationing cannot occur in the nondefault interval either. □

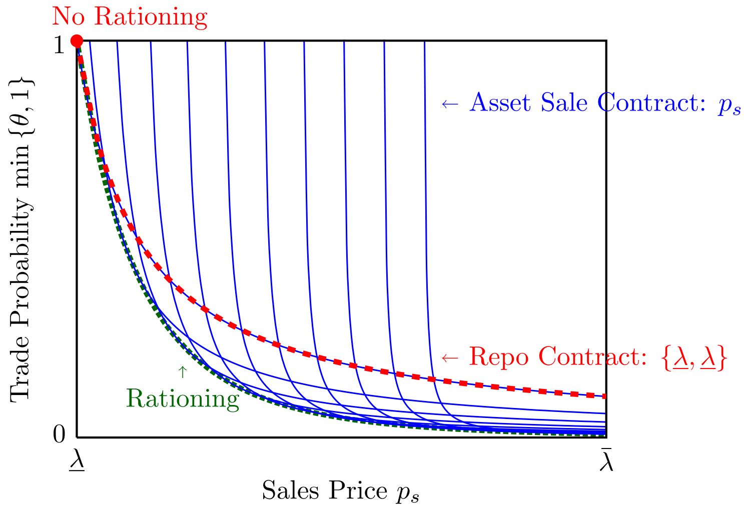
The intuition behind Proposition B.1 is that the repurchase option breaks the use of rationing as a screening device. First, the trading probability has to be weakly decreasing in the asset quality in equilibrium. This results from the incentive compatibility constraints of borrowers, because low-quality assets can always imitate high-quality ones. Second, if rationing were to occur, we show that there exists a deviating contract which attracts all rationed assets in the hypothetical equilibrium. This deviating contract offers a sales price equal to that of the highest quality among the non-rationed asset and, crucially, a repurchase price equal to the quality of that asset. This contract improves the payoff of all and only rationed assets and doesn't require rationing. Thus, a hypothetical equilibrium with rationing is ruled out.

Another way to understand the no rationing result is through the single crossing property, which is critical for obtaining a separating equilibrium with outright sales. When the repurchase option is allowed, the single crossing property no longer hold. This insight is illustrated in Figure B.2. We start from an asset-sale equilibrium, in which the value function of selling an asset of quality λ is

$$v_a(\lambda) = \max_{p_s \in \Lambda} \left\{\min\left\{\Theta(p_s), 1\right\}\left((1+r)\,p_s - \lambda\right)\right\}. \tag{B.8}$$

Here, a separating equilibrium is obtained, in which higher quality assets are sold at higher price but with lower probability. The trading probability is a screening device for quality and rationing occurs in equilibrium. Graphically, the blue solid lines are the isovalue curves for a given asset quality λ, captured by $\min\{\theta, 1\}\left((1+r)\,p_s - \lambda\right) = v_a(\lambda)$. The single crossing

Figure B.2: Repurchase option and the single crossing property



property holds here. The envelope curve in dotted green represents the resulting separating equilibrium with all active markets of price p_s and trading probability θ. The only non-rationed asset is the lowest quality one $\underline{\lambda}$ sold at price $\underline{\lambda}$. Now we introduce the repurchase option and consider a repo contract $\{\underline{\lambda}, \underline{\lambda}\}$. This contract allows all rationed assets to mimic the non-rationed asset $\underline{\lambda}$ and obtain the same payoff. Going back to borrower's payoff with repo contract in equation (B.2), since the payoff can be flat in type due to the repurchase option, the single crossing property does not hold any more.

The no rationing result resonates an earlier result by Gale (1996), who showed that rationing will not occur in equilibrium if the contract space is sufficiently rich. We discuss later that no rationing is a key force that make repo contracts a welfare improving innovation in this framework.

The pooling result here relates to other pooling outcomes in competitive search models. In Chang (2017) and Guerrieri and Shimer (2018), sellers possess multidimensional private information, which breaks the single-crossing property that allows separation in Guerrieri et al. (2010). In Auster and Gottardi (2019), multiple interactions break separation. Here, multidimensional contracting breaks the single-crossing property and hence the possibility of separating equilibria.

Having established that rationing doesn't occur in equilibrium, we also obtain Lemma 2 here, following the exact same reasoning as in the main text. Thus we can restrict our attention to two (sets of) contracts: the highest sales price contract for default assets and the highest nondefault value contract for nondefault assets. Furthermore, Proposition 1 on pooling is obtained.

Proposition B.2 (Uniqueness)**.** *There exists a unique equilibrium featuring a single zero-profit pooling contract:*

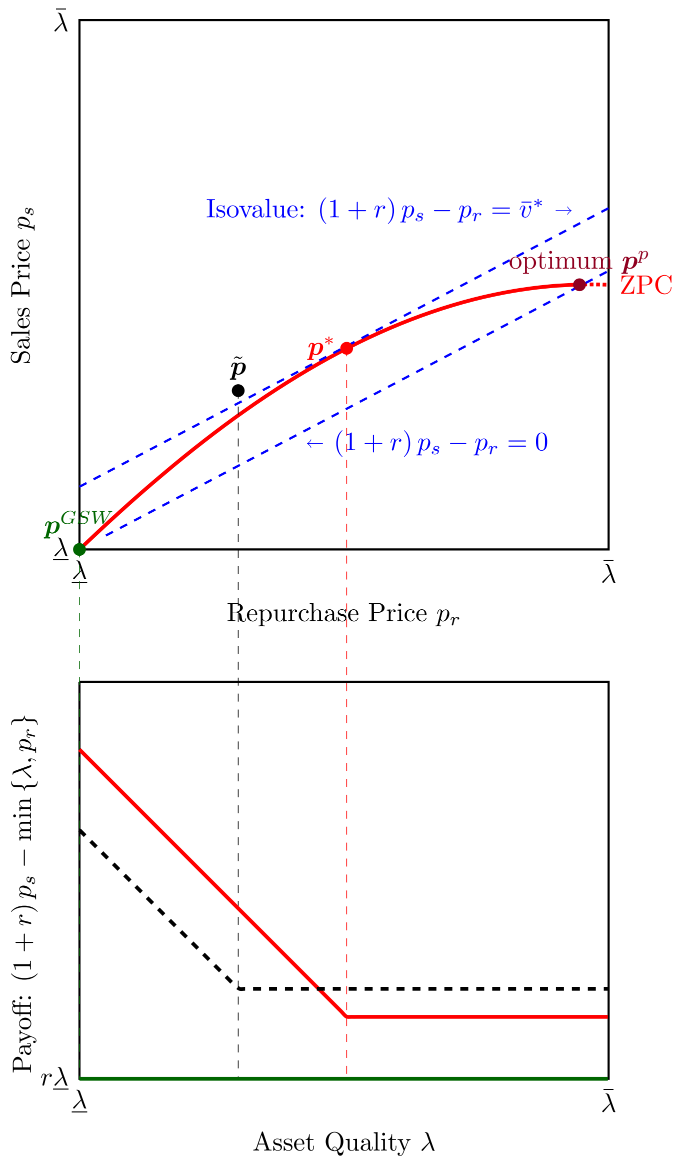
$$\boldsymbol{p}^{GSW} = \{\underline{\lambda}, \underline{\lambda}\}\,.$$

Proof. The equilibrium refinement hinges on, in our equilibrium definition, lenders form the off-equilibrium belief that any deviating contract attracts the borrowers who has the most to gain from searching for this contract. We first show that contracts with sales prices above $\underline{\lambda}$ cannot be the basis of an equilibrium due to cream skimming of nondefault assets. Suppose we were in an equilibrium in which the sales price is above $\underline{\lambda}$, for example say the equilibrium contract is $\boldsymbol{p}^*$ depicted in the upper panel of Figure B.3. Consider a deviating contract with slightly lower sales price and much lower repurchase price such that it improves the nondefault value $\bar{v}$. In the graph, the deviating contract is denoted by $\tilde{\boldsymbol{p}}$. The lower panel of the figure plots the payoff functions for assets conditional on trading. It shows that the deviating contract improves the payoff of all of the nondefault assets. In contrast, the low-quality default assets however obtain a lower payoff with the new contract. Accordingly, the nondefault assets are the ones willing to bear the lowest trading probability when deviating from $\boldsymbol{p}^*$ to $\tilde{\boldsymbol{p}}$. Hence lenders should form the belief that only the high-quality nondefault assets would come to this deviating contract and hence find it profitable. In a way, lenders are optimistic about the composition of asset quality when they deviate to lower sales price contracts. Therefore cream skimming of high-quality nondefault assets is possible for any contract that offers a sales price higher than $\underline{\lambda}$.

On the contrary, when lenders deviate to higher sales price contracts, they form pessimistic beliefs about the composition of asset quality. This is because increase in sales price improves the payoff of low-quality default assets more than the payoff of high-quality nondefault assets. Hence low-quality assets have a higher incentive to search for higher sales price contracts and are willing to bear a lower trading probability. Lenders find it unprofitable to deviate to a higher sales price contract given their belief that it will attract only quality $\underline{\lambda}$ asset. Despite the deviating contract will eventually improve the payoff of all assets, in the adjustment process, the lender expect it to first attract low-quality assets. As a consequence, the adjustment process does not occur. Hence the equilibrium with sales price $\underline{\lambda}$ and repurchase price survives. □

Proposition B.2 characterizes the equilibrium repo contract under competitive search: it offers a sales price equals to the lowest quality. Figure B.3 positions the contract $\boldsymbol{p}^{GSW}$ in relation to the equilibrium contract under the MWS equilibrium notion $\boldsymbol{p}^*$ and the optimal repo contract $\boldsymbol{p}^p$.

Figure B.3: Equilibrium refinement



The competitive search solution has an unappealing feature: if we introduce an arbitrary small amount of assets with zero quality, i.e., $\underline{\lambda} = 0$, we reach a "no-trade" equilibrium and the aggregate liquidity vanishes to zero. In this case, repos and outright sales both produce zero liquidity, something that does not occur under the MWS equilibrium notion. In addition, the contract $\boldsymbol{p}^{GSW}$ features zero default, which is difficult to reconcile with the empirical facts. For the reasons above, we set up the repo market according to the MWS equilibrium notion in our baseline formulation.

B.3 Repo vs. Outright Sales: Efficiency Comparison

In this section, we finalize with the efficiency implications of the repo contract in the competitive search framework. For the purpose of comparison, we first solve for the asset-sale equilibrium in the competitive search environment.

Solution for Asset-Sale Equilibrium. Since we assumed away the contract posting cost, the zero-profit condition on the side of lenders (buyers) implies that the selling price equals the asset quality in that submarket. The value function of borrowers (sellers) with asset sales in (B.8) is characterized by the following ordinary differential equation:

$$\frac{\partial v_a(\lambda)}{\partial \lambda} = -\frac{1}{r\lambda} v_a(\lambda),$$

with an initial condition:

$$v_a(\underline{\lambda}) = r\underline{\lambda}.$$

We obtain a closed-form solution for the value function:

$$v_a(\lambda) = r\underline{\lambda}\left(\frac{\underline{\lambda}}{\lambda}\right)^{\frac{1}{r}}.$$

The trading probability in the submarket for quality λ is

$$\min\{\Theta(\lambda), 1\} = \left(\frac{\underline{\lambda}}{\lambda}\right)^{\frac{1+r}{r}} \leq 1.$$

Comparing the asset-sale equilibrium and the repo equilibrium, we obtain the following welfare proposition.

Proposition B.3 (Repo vs. Outright Sales)**.** *Under competitive search, the repo equilibrium Pareto dominates the asset-sale equilibrium.*

Proof. Given the equilibrium repo contract $\boldsymbol{p}^{GSW}$, the value function for lender with asset quality λ is

$$v(\lambda) = r\underline{\lambda} \geq v_a(\lambda) = r\underline{\lambda}\left(\frac{\underline{\lambda}}{\lambda}\right)^{\frac{1}{r}}.$$

The repo contract allows higher quality asset to imitate the lowest quality asset $\underline{\lambda}$ and obtain the same payoff. This is in contrast with outright sales, where all higher quality are distorted downward to induce separation. Thus, in the repo equilibrium, all borrowers are better off than with asset sales.

Further, the repo equilibrium generates a higher level of aggregate liquidity than the asset-sale equilibrium:

$$\underline{\lambda} \geq \mathbb{E}\left[\left(\frac{\underline{\lambda}}{\lambda}\right)^{\frac{1+r}{r}} \lambda\right].$$

□

Proposition B.3 shows that repo is always a welfare improving innovation over asset sales under competitive search. The key for the welfare improvement is that the repurchase option breaks rationing as a screening device for quality, allowing for more gains from trade to be realized. While the introduction of the repurchase option increases the trading volume under both equilibrium concepts, the driving forces are different. Under the MWS equilibrium notion, repo induces full participation at the expense of cream skimming. Therefore it is welfare improving if and only if when the gains from increased participation outweighs the potential cost of cream skimming, as show in Proposition 5. In the competitive search framework, repo is always welfare improving by preventing rationing to take place. To further illustrate this distinction, we reproduce the uniform distribution example in Section 4.2 below.

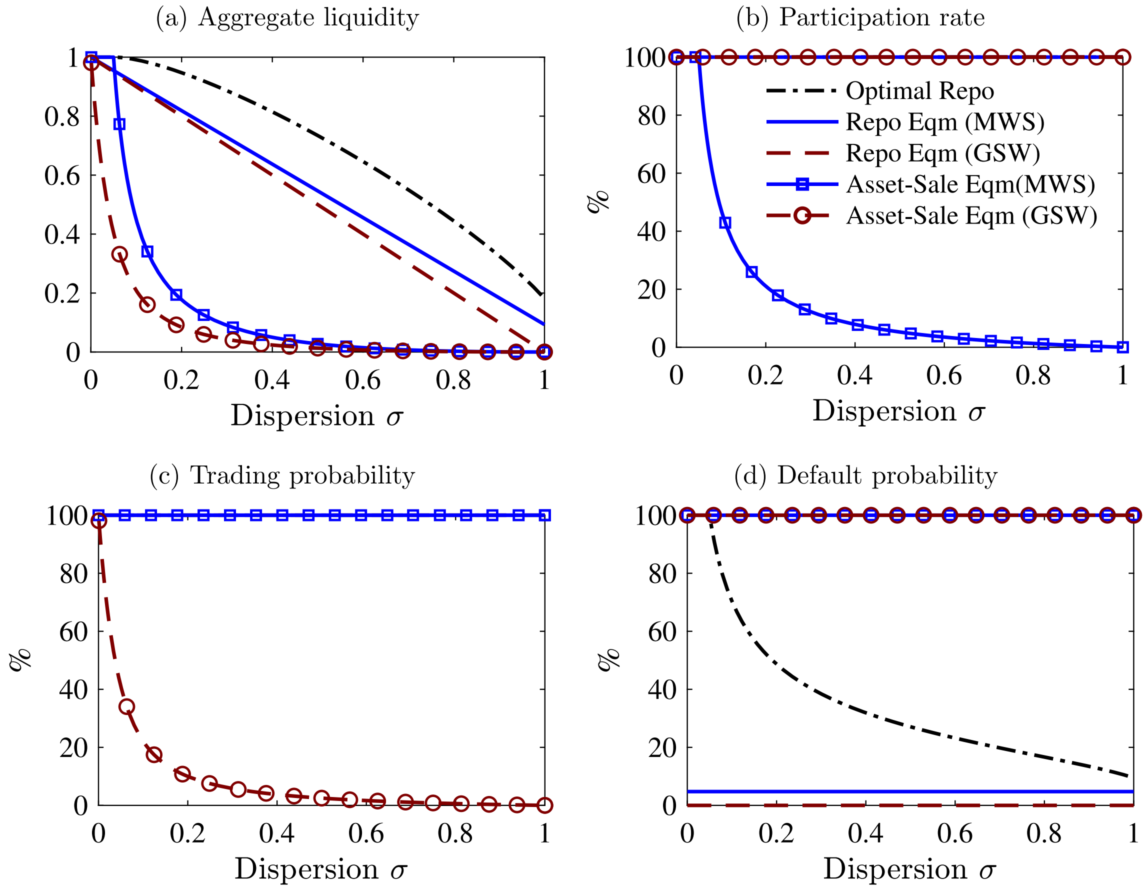
Example: Uniform Distribution. The asset quality $\lambda \sim U(1-\sigma, 1+\sigma)$, where the bound of distribution $\sigma \in [0,1]$. In connection to the closed-form solutions in Appendix A.9, we derive the corresponding expressions in the competitive search equilibria. In the asset-sale equilibrium under competitive search, the aggregate liquidity is

$$\int \min\{\Theta(\lambda), 1\} \lambda dF(\lambda) = \frac{r}{1-r}\frac{(1-\sigma)^2}{2\sigma}\left[1 - \left(\frac{1-\sigma}{1+\sigma}\right)^{\frac{1-r}{r}}\right].$$

Applying Bernoulli's inequality, $\left(\frac{1-\sigma}{1+\sigma}\right)^{\frac{1}{r}} \geq 1 - \frac{1}{r}\frac{2\sigma}{1+\sigma}$. Hence, the aggregate liquidity is bounded above by the level in the repo equilibrium $1-\sigma$.

Figure B.4 compares repos and asset sales under both the competitive search equilibrium

Figure B.4: The welfare improvements of repo contracts over asset sales



and the MWS equilibrium. Panel (a) shows that, repo always dominates sales in a competitive search environment. As the dispersion level σ increases, both the repo equilibrium and the sales equilibrium experience a decline in aggregate liquidity. However the decline with asset sales is much more precipitous. In Panel (b), we see that under the MWS equilibrium, repo induces full participation, which is key for its improvement over sales. While in the competitive search equilibrium, both repo and asset sales have full participation. In contrast, as shown by Panel (c), many trades cannot be realized due to rationing of sales contract in competitive search. In that setup, by utilizing the repurchase option and thus pooling assets together, repo avoids rationing and realizes more gains from trade. The predictions for default probability is plotted in Panel (d). In the MWS equilibrium, the default probability is constant and depends on the return of investment, while in the competitive search equilibrium default never takes place.

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